Unit - IV

Torsion

Introduction:-

Torsion refers to twisting of a straight member under action of a turning moment or torque which tends to produce a rotation or twist about the longitudinal axis.

In engineering problems, many members are subjected to torsion, shafts transmitting power from engine to the rear axle of automobile, from a motor to machine tool and from a turbine to electric motors, propeller shaft, steering rods of automobile are common examples of member in torsion.

Pure torsion:-

A member is said to be pure torsion when its cross sections are subjected to only torsional moments and not accompanied by axial forces or bending moment.

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Assumption in theory of pure torsion:-

The theory of pure torsion is based on the following assumptions.

1) The material homogeneous and isotropic.

2) The twist along the shaft is uniform.

3) The shaft is uniform circular section throughout.

Elastic theory of Torsion or Derivation of Torsional equations:-

Consider a shaft of length L and radius R fixed at one end and subjected to a torque T at the other end as shown in the figure.

Let 'O' be the centre of circular section and B be a point on surface. AB is the line on the shaft parallel to the axis of the shaft. When torque T is applied, the point B moves to B'. If ϕ is shear strain

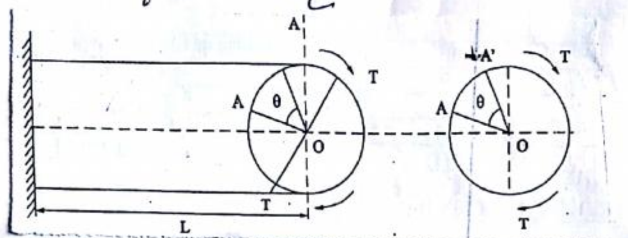
(2)

and θ is the angle of twist in length L

Then $R\theta = BB' = L\phi$

If τ is shear stress and C modulus of rigidity then,

$$\phi = \frac{\tau}{C}$$



$$R\theta = L \times \frac{\tau}{C}$$

$$\text{or } \frac{\tau}{R} = \frac{C\theta}{L} \quad \rightarrow \text{eqn. } 4.7$$

Similarly if the point B considered is at any distance from the centre instead of on the surface, it can be shown that

$$\tau_x = \frac{C\theta}{L}$$

$$\frac{\tau}{R} = \frac{\tau_x}{x} \quad \rightarrow \text{eqn. } 4.8$$

Thus shear stress increases linearly from zero at axis to the maximum value τ at surface.

We know that $T = \frac{\pi}{16} \times \tau \times D^3$

$$\tau = \frac{16 \times T}{\pi D^3}$$

(3)

Substitute τ value in equation (4.7)

$$\Rightarrow \frac{C\theta}{L} = \frac{\frac{16 \times T}{\pi D^3}}{R}$$

$$= \frac{\frac{16 \times T}{\pi D^3}}{\frac{D}{2}}$$

$$\frac{C\theta}{L} = \frac{32\,T}{\pi D^3 \times D}$$

$$= \frac{32\,T}{\pi D^4}$$

$$\frac{C\theta}{L} = \frac{T}{\pi/32 \times D^4}$$

$$= \frac{C\theta}{L} = \frac{T}{J}$$

Where J = Polar moment inertia $= \pi/32\, D^4$

Torsional equation $= \frac{T}{J} = \frac{C\theta}{L} = \frac{\tau}{R}$

$\lfloor$ eqn 4.9

above equations (4.9) is called as Torsional equations.

For hallow shaft Polar moment of Inertia

$$= J = \pi/32\,(D^4 - d^4)$$

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Power transmitted to shaft :-

Consider a shaft subjected to a torque T and rotating at N revolutions per minute power is defined as the rate of doing work. Taking second as the unit of time, Angle through which torque moves $= \frac{N}{60} \times 2\pi = \frac{2\pi N}{60}$

Power = Work done per second.

$$P = T \times \frac{2\pi N}{60}$$

$$\text{Power} = \frac{2\pi N T}{60}$$

Where, N = No. of revolution per minute

T = Torque in kNm

P = power in kW or kNm /sec.

Shaft in series and parallel :-

Two shafts may be joined in series or parallel. Let 1 and R denote various parameters of two shafts.

(a) Shaft in parallel :-

When two shafts are joined in

(5)

parallel, torque applied to the composite shaft is the same as the torque on the two shafts. (i.e.,)

$$T = T_1 + T_2 = \frac{C_1 J_1 \theta_1}{L_1} + \frac{C_2 J_2 \theta_2}{L_2}$$

If angular twist and the length $= \frac{\theta}{L}$ are the same, T

$$\times (C_1 J_1 + C_2 J_2)$$

Thus angular twist $= \theta = \dfrac{TL}{C_1 J_1 + C_2 J_2}$

Shaft in series:-

When two shaft are joined in series and torque is applied both shafts are subjected to the same torque. Thus

$$T = J_1 \frac{\tau_1}{R_1} = J_2 \frac{\tau_2}{R_2}$$

Also $$T = \frac{C_1 J_1 \theta_1}{l_1} = \frac{C_2 J_2 \theta_2}{l_2}$$

The angle of twist is the same of angle of twist of each shaft i.e.,

$$\theta = \frac{TL_1}{C_1 J_2} + \frac{TL_2}{C_2 J_2}$$

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(6)

Problem in torsion:—

(1) A solid circular shaft transmit 75kW power at 200rpm. Calculate the shaft diameter, if the twist in the shaft is not exceed 1° in 2 meters length of shaft, and Shear stress is limited to 50MN/m². Take $C = 100 GN/m^2$. [Nov. / Dec 2013]

Solution:—

Power $P = 75\,kW = 75 \times 10^3\,W$ $C = 100\,GN/m^2$ $= 100 \times 10^3\,N/mm^2$, Speed $N = 200\,rpm$, $f_s = 50\,MN/m^2$ $= 50\,N/mm^2$, $l = 2000\,mm$ $\theta = 1^\circ = \pi/180$ radian = Angle of twist

$$Power = \frac{2\pi N T}{60}$$

$$75 \times 10^3 = \frac{2 \times \pi \times 200 \times T}{60}$$

$$T = 3580.99\,Nm$$

i) Shear stress consideration:—

$$T = \pi/16 \times f_s \times D^3$$

$$3580.99 \times 10^3 = \pi/16 \times 50 \times D^3$$

$$D = 72\,mm$$

(7)

ii) Twist consideration: -

$$\frac{T}{J} = \frac{C\theta}{l} = \frac{f_s}{R}$$

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\frac{2580.99 \times 10^3}{\pi/32 \times D^4} = \frac{100 \times 10^3 \times \pi/180}{2000}$$

$$D = 80.4mm$$

The suitable diameter of the shaft is higher of two values $D = 80.4mm$

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(2) A solid steel shaft subjected to a torque of 45kNm. If the angle of twist is 0.5° per meter length of the shaft and the shear stress is not to be allowed to exceed $90MN/m^2$ Find (i) suitable diameter for the shaft (ii) Final maximum shear stress (iii) Maximum shear strain in shaft Take $C = 80 GN/m^2$ May/June 2013

Solution: -

$$T = 45\,kNm = 45 \times 10^6 Nmm$$

$$\theta = 45° = 0.5 \times \pi/180 = 8.72 \times 10^{-3}$$

(8)

Shear stress $= 90\ MN/m^2$

$= 90\ N/mm^2$

Shear modulus $C = 80\ GN/m^2$

$= 80 \times 10^3\ N/mm^2$

$l = 1m = 1000mm$

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\frac{45 \times 10^6}{\pi/32 \times D^4} = \frac{80 \times 10^3 \times 8.72 \times 10^{-3}}{1000}$$

$$\frac{45 \times 10^6}{0.098 D^4} = \frac{697.6}{1000}$$

$$D = 160.19mm$$

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(3) A hollow steel shaft 5m long is to transit 160kN of power at 120rpm The total angle of twist is not to exceed 2° in this length and allowable shear stress is $50\ N/mm^2$. Determine the inside and outside diameters of the shaft $N = 0.8 \times 10^5\ N/mm^2$ and $d/D = 0.4$ [NOV./Dec 2012]

Solution:—

(9)

$$P = \frac{2\pi N T}{60}$$

$$160 = \frac{2 \times \pi \times 120 \times T}{60}$$

$T = 12.73$ kNm

$P = T_{mean} = 12.73 \times 10^6$ Nmm

Assume $T_{max} = 1.2\, T_{mean}$

$T_{max} = 1.2 \times 12.73 \times 10^6$ Nmm

Shear stress and angle of twist both are given

First case: considering shear stress (τ) for hollow shaft

Torque, $T_{max} = \pi/16 \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$

$$15.28 \times 10^6 = \pi/16 \times 50 \times \left(\frac{D^4 - (0.4D)^4}{D}\right)$$

$$1.56 \times 10^6 = \frac{0.9744\, D^4}{D}$$

$D = 116.98$ mm

$d = 46.8$ mm

Second case: considering angle of twist θ

$$\frac{T_{max}}{J} = \frac{C\theta}{l}$$

$J = \pi/32 \times (D^4 - d^4)$

$$\frac{15.28\times10^6}{\pi/32\times(D^4-d^4)} = \frac{0.8\times10^5\times0.035}{5000}$$

$$D^4-(0.4D^4) = 277.93\times10^6$$

$$0.9744\,D^4 = 277.93\times10^6$$

$$D = 129.96\text{mm}$$

$$d = 51.98\text{mm}$$

From the above two cases, the suitable external and internal diameter of the shaft is the greater value only.

External diameter $= D = 129.96\text{mm}$

Internal diameter $= d = 51.98\text{mm}$

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Helical spring:-

A spring is a device which is used to absorb energy by taking very large change in its form without permanent deformation and then realease the same when it's required. For example, the carriage springs or leaf spring in an automobile which absorb road shock (11) by continuously absorbing energy due to shock by their deformation and then dissipating the same by vibrating.

Types of Springs :-

1. Torsion Spring
2. Bending Spring.

Stiffness :-

Stiffness of the spring is defined as the load required to produce unit deflection.

Closed coiled helical spring :-

It is a type spring in which the wire is turned so closely that each turn is nearly right angle to the axis of the spring and the gap between two consecutive turns is small. In closed coil springs, an axial pull or thrust produces only torsion on the material of the spring. The types of stresses that are

(12)

produced in a spring are,

i) Direct shear stress

ii) Torsional " "

iii) Bending " "

Formula used for closed coil helical spring.

1) Shear stress $= \tau = \frac{8WD}{\pi d^3}$ (or) $\frac{16WR}{\pi d^3}$ N/mm^2

2) Deflection $= \delta = \frac{8WD^3 n}{C d^4}$ (or) $\frac{64WR^3 n}{C d^4}$ mm

3) Stiffness $= K = \frac{W}{\delta}$ N/mm or $\frac{Cd^4}{64R^3 n}$

Open coiled Helical Spring:-

In open coiled helical spring there is a large gap between two consecutive turns. Here the helix angle 'α' plays an important role. As a result of large gap between consecutive coils, these springs can take compressive as well as tensile loads.

(13)

Open coil Helical spring - Formula:-

1. Deflection

$$mm = \delta = \frac{64WR^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E}\right]$$

Where α = Helix angle

R = Mean radius of spring

2. Bending stress $= \sigma_b = \frac{32WR\sin\alpha}{\pi d^3}$

$= N/mm^2$

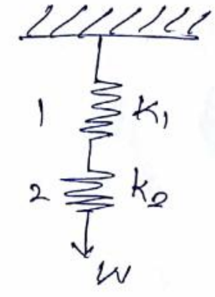
3. Shear stress $(\tau) = \frac{16WR\cos\alpha}{\pi d^3} = N/mm^2$

Springs in series and parallel:-

In many situations, the combination of two or more springs either may be Connected or parallel are required.

1. Spring in series:-

Two Spring of stiffness K_1 and K_2 are connected in series and loaded with W as shown in figure. In this case each Spring is subjected to same load applied at end of one Spring. Therefore the total (14) deflection of assembly is equal to the algebraic sum of deflection of two Springs.



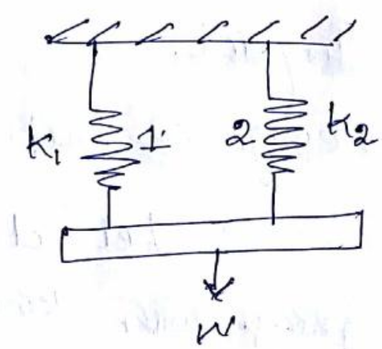
Total deflection $\frac{W}{K} = \frac{W}{K_1} = \frac{W}{K_2}$

$$1/K = 1/K_1 = 1/K_2$$

Combined stiffness $= K = \frac{K_1 K_2}{K_1 + K_2}$

2) Springs in parallel:-

Two Springs of stiffness K_1 and K_2 are connected in parallel, loaded with W. Let load shared by two springs be W_1 and W_2. Therefore deflection of each Spring is the same.



Total load $= W = W_1 + W_2$

Common deflection

$$\delta = \frac{W}{K} = \frac{W_1}{K_1} = \frac{W_2}{K_2}$$

$$W_1 = \frac{WK_1}{K}$$

$$W_2 = \frac{WK_2}{K}$$

(15)

Total load $= W = W_1 + W_2$

$$W = \frac{W k_1}{k} + \frac{W k_2}{k}$$

$$W = \frac{W}{k}(k_1 + k_2)$$

$$\boxed{k = k_1 + k_2}$$

Buffer spring:—

Buffer spring is most commonly used in the Railway wagons. The shock between two colliding bodies may be softened or cushioned by means of buffers. The purpose of the buffers is to increase the duration of impact.

Design of buffer spring:—

Let d be the diameter of the spring with kinetic energy absorbed by the buffer spring $= \frac{1}{2} m v^2 \quad \ldots \to (1)$

$$= \frac{1}{2} \times \frac{W_w}{g} v^2$$

(16)

Where m = mass of wagon

W_W = Weight of wagon

Let W be the equivalent load which when applied gradually on each spring causes a deflection.

∴ Energy absorbed in the springs

$$= \frac{1}{2} \times W \times \delta \quad \text{-----} \rightarrow \text{(ii)}$$

equating equation (i) and equation (ii) we find the value of W

$$\text{Torque} = T = W \times D/2$$

we also know that torque transmitted by the spring

$$T = \frac{\pi}{16} \times f_s d^3$$

Let n be number of active turns of the spring coil

$$\delta = \frac{64 W R^3 n}{G d^4}$$

$$\delta = \frac{8 W D^3 x}{G d^4}$$

Solid length = Total No. of coils × diameter of wire
= $n \times d$

(17)

Free length = Solid length + Max. compression + Clearance between adjacent coil

$= nd + \delta_{max} + 0.15 \times \delta_{max}$

Pitch of the coil $P = \frac{\text{Free length}}{n-1}$

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Probelems on Spring:—

(1) A closed coil spring (helical) is made out of 10mm diameter steel rod. The coil consist of 10 complete turns with a mean diameter of 120mm. The spring carries an axial pull of 200N. Find the maximum shear stress induced in the section of the rod. If $C = 80 GN/m^2$, find the deflection in the spring, the stiffness and strain energy stored in the spring. [Nov. / Dec. 2013]

Load on spring = $W = 200N$

Dia of wire $d = 10mm$

Mean dia of coil ; $D = 120$, $R = \frac{120}{2} = 60mm$

(18)

No. of coils $n = 10$,

$C = 80 \times 10^3 \, N/mm^2$

$$f_{s(max)} = \frac{16 WR}{\pi d^3}$$

$$= \frac{16 \times 200 \times 60}{\pi \times 10^3}$$

$$= 61.12 \, N/mm^2$$

$$\delta = \frac{64 WR^3 n}{G d^4}$$

$$= \frac{64 \times 200 \times (\frac{120}{2})^3 \times 10}{80 \times 10^3 \times 10^4}$$

$$= 34.56 mm$$

Energy Stored $= \frac{1}{2} W\delta$

$= \frac{1}{2} \times 200 \times 34.56$

$= 3456$ Nmm

$= 3.456$ Nm.

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(2) An open coiled spring (helical) of wire diameter 12mm, mean coil radius 84mm, helix angle 20° carries an axial load of 480 N. Determine the shear stress and direct stress developed at inner (19)

radius of the coil. [May/June 2013]

Given data

dia of wire $d = 12mm$

Coil dia $D = 84mm$

helix angle $\theta = 20^\circ$

Axial load $P = 480N$

Solution:-

$$\text{Shear stress} = \tau = \frac{16WR\cos\alpha}{\pi d^3}$$

$$= \frac{16 \times 480 \times 42\cos 20^\circ}{\pi \times 12^3}$$

$$= 55.83\ N/mm^2$$

$$\text{Direct stress} = \frac{16WR}{\pi d^3}(\sin\alpha + 1)$$

$$= \frac{16 \times 480 \times 42}{\pi \times 12^3}(\sin 20^\circ + 1)$$

$$= 79.73\ N/mm^2$$

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A closed coil helical compression spring is made of 10mm steel wire closely coiled to a mean diameter of 100mm with 20 coils. A weight of 100N is dropped on to the spring. If the maximum instantaneous compression (20)

60mm. Calculate the height of the drop. Take $N = 0.85 \times 10^5 \, N/mm^2$

[NOV./DEC 2012]

Given data:-

$P = 100N$

$d = 10mm$

$R = 50mm$

$C = N = 0.85 \times 10^5 \, N/mm^2$

$\delta = 60mm$

$n = 20$ coils.

To find: Height of spring.

Solution:-

Let W be the equivalent gradually applied load to produce the same deflection (60mm) as given load

$$\delta = \frac{64 W R^3 n}{C d^4}$$

$$60 = \frac{64 \times W \times 50^3 \times 20}{0.85 \times 10^5 \times 10^4}$$

$$W = 318.75 \, N$$

Work stored in spring as strain energy $= \frac{1}{2} W \delta$

The work done by falling weight $= P(h+\delta)$

(21)

$= 100 \times (h + 60)$

$\therefore \frac{1}{2} \times W \delta = (100 \times (h + 60))$

$\frac{1}{2} \times 318.75 \times 60 = 100\ (h + 60)$

$\boxed{h = 35.625\,m}$

(22)