MA2211 TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS MAY/JUNE 2016

PART –A

1. Form the partial differential equations of all planes passing through the origin.
2. Find the complete integral of $\sqrt{p}+\sqrt{q}=1$
3. If **If x^2 =** $\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(nx)$ **in (-π,π), deduce that** $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$
4. State TRUE or FALSE: Fourier series of period 20 for the function f(x) =x. cos(x) in the interval (-10, 10) contains only sine terms. Justify your answer.
5. Write down the initial and boundary conditions for the boundary value problem when a string of length l is tightly fastened on both ends and the midpoint of the string is taken to height of k are released from rest.
6. The ends A and B of a rod of length 20cm have their temperature kept at 10^0C and 50^0C respectively. Find the steady state temperature distribution on the rod.
7. If F(s) is the Fourier transform of f(x), obtain the Fourier transform of f(x-2)+f(x+2).
8. Given that $F_S\{f(x)\}=\frac{s}{s^2+a^2}$, hence find $F_C\{x.f(x)\}$
9. If $Z\{n^2\}=\frac{z^2+z}{(z-1)^3}$, then find $Z(n+1)^2$.
10. State damping rule related to Z transform and then find $Z(n.a^n)$.

PART – B

11. (a) (i) Find the general solution of the equation $x(z^2-y^2)\frac{\partial z}{\partial x}+x(z^2-y^2)\frac{\partial z}{\partial y}=z(y^2-x^2)$.

(ii) Solve $(D^3-7DD'^2-6D'^3)z=\sin(x+2y)+3e^{2x+y}$.

(OR)

(b) (i) Form the partial differential equations by eliminating the arbitrary function $f(x+y+z, x^2+y^2+z^2)=0$.

(ii) Obtain the complete integral of $p^2+x^2y^2q^2=x^2z^2$.

12. (a) (i) Find the fourier series for $f(x)=\begin{cases}-x,-\pi<x<0\\ x,0<x<\pi\end{cases}$. Hence deduce the sum of the series

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$$

Find the Fourier series up to second harmonic for y=f(x) from the following table.
(ii)

x	0	π/3	2π/3	π	4π/3	5π/3	2π
y=f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(OR)

(b)(i) Find the half range Fourier cosine series expansion for the function f(x) =x in 0<x<l. Hence deduce the sum of the series $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+\cdots=\frac{\pi^4}{96}$

(ii) Find the complex form of the Fourier series of $f(x) = e^{-x}$, $-1<x<1$.

13. (a) A tightly stretched string with fixed end points x=0 and x=l is initially displace to the form $2.\sin\left(\frac{3\pi x}{l}\right).\cos\left(\frac{2\pi x}{l}\right)$ and then released. Find the displacement of the string at any distance x from one end at any time t.

(OR)

(b) A rectangular plate is bounded by the lines x=0; x=a; y=0 and y=b and the edge temperatures are $u(x,0)=10.\sin\left(\frac{3\pi x}{a}\right)+8\sin\left(\frac{5\pi x}{a}\right)$; u(0,y)=0; u(x,b)=0 and u(a,y)=0. Find the steady state temperature distribution u(x,y) at any point of the plate.

14. (a) (i) Find the Fourier transform of $f(x)=\begin{cases}1-x^2, |x|<1\\ 0, |x|\geq 1\end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin(s)-s.\cos(s)}{s^3}.\cos\left(\frac{s}{2}\right)ds$

(ii) Find the Fourier sine transform of $f(x) = e^{-ax}$ and hence find Fourier sine transform $\frac{x}{a^2+x^2}$

(OR)

(b) (i) Find the Fourier transform of $f(x)=\begin{cases}1, |x|<a\\ 0, |x|\geq a\end{cases}$. Hence deduce that $\int_0^\infty \frac{\sin x}{x}dx$

(ii) Find the Fourier Cosine transform of $f(x) = e^{-ax}$. Hence evaluate the following: $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$

15. (a) (i) Derive a difference equation by eliminating the constants from $y_n=(A+Bn)3^n$.

(ii) Use convolution theorem to find the inverse Z transform of $\frac{z^2}{(z-1/2)(z-1/4)}$.

(OR)

(b) (i) State initial value theorem. Use it to find u_0, u_1, u_2 and u_3, where U (z) = $\frac{2z^2+5z+14}{(z-1)^4}$.

(ii) Solve the equation $y_{n+2}+6y_{n+1}+9y_n=2^n$ given that $y_0=0$ and $y_1=0$.