MA6351 TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS NOV/DEC 2015

PART –A

1. Construct the partial differential equation of all spheres whose centres lie on the Z-axis, by the elimination of arbitrary constants.
2. Solve (D+D'-1)(D-2D'+3)z=0.
3. Find the root mean square value of f(x)=x(l-x) in 0≤x≤l.
4. Find the sine series of the function f(x) =1, 0≤x≤π.
5. Solve $3x\frac{\partial u}{\partial x}-2y\frac{\partial u}{\partial y}=0$ by method of separation of variables.
6. Write all possible solutions of two dimensional heat equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.
7. If F(s) is the Fourier transform of f(x), prove that F[f(ax)]= $\frac{1}{a}F\left[\frac{s}{a}\right], a\neq 0$.
8. Evaluate $\int_0^\infty \frac{s^2 ds}{(s^2+a^2)(s^2+b^2)}$ using Fourier transforms.
9. Find the Z-transform of $\frac{1}{n+1}$.
10. State the final value theorem in Z transform.

PART – B

11. (a) (i) Find the complete solution $z^2(p^2+q^2)=(x^2+y^2)$.
 (ii) Find the general solution of $(D^2+2DD'+D'^2)z=2\cos y-x.\sin y$.

(OR)

 (b) (i) Find the general solution of $(z^2-y^2-2yz)p+(xy+zx)q=(xy-zx)$.
 (ii) Find the general solution of $(D^2+D'^2)z=x^2y^2$.

12. (a) (i) Find the Fourier series expansion of the following periodic function of period 4,
 $f(x)=\begin{cases}2+x, -2\leq x\leq 0\\ 2-x, 0\leq x\leq 2\end{cases}$. Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$
 (ii) Find the complex form of Fourier series of f(x) $=e^{ax}$ in the interval (-π,π), where a is a real number.
 Hence deduce that $\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{a^2+n^2}=\frac{\pi}{a\sinh(a\pi)}$.

(OR)

 (b) (i) Find the half range cosine series of f(x) $=(\pi-x)^2$, 0<x<π. Hence find the sum of the series
 $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots=\frac{\pi^4}{90}$

(ii) Determine the first two harmonics of Fourier series for the following data.

x	0	π/3	2π/3	π	4π/3	5π/3
y=f(x)	1.98	1.30	1.05	1.30	-0.88	-.25

13. (a) A tightly stretched string with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity v= $\begin{cases} \frac{2kx}{l} \text{ in } 0 < x < \frac{l}{2} \\ \frac{2k(l-x)}{l} \text{ in } \frac{l}{2} < x < l \end{cases}$. Find the displacement of the string at any distance x from one end at any time t.

(OR)

(b) A bar 10 cm long with insulated sides has its ends A and B maintained at temperature 50^0C and 100^0C respectively, until steady state conditions prevails. The temperature at A is suddenly raised to 90^0C and at the same time lowered to 60^0C at B. Find the temperature distributed in the bar at time t.

14. (a) (i) Find the Fourier sine integral representation of the function f(x) =e^{-x}sinx.

(ii) Find the Fourier cosine transform of the function f(x) = $\frac{e^{-ax} - e^{-bx}}{x}, x > 0$

(OR)

(b) (i) Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, |x| < 1 \\ 0, |x| \geq 1 \end{cases}$. Hence deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$.

(ii) Verify the convolution theorem for Fourier transform if f(x) =g(x) = e^{-x^2}

15. (a) (i) If U(z) = $\frac{z^3 + z}{(z-1)^3}$, find the value of u_0; u_1 and u_2.

(ii) Use Convolution theorem to evaluate $Z^{-1}\left\{\frac{z^2}{(z-3)(z-4)}\right\}$

(OR)

(b) (i) Using the inversion integral method(Residue theorem), find the inverse Z-transform of U(z) = $\frac{z^2}{(z+2)(z^2+4)}$.

(ii) Using the Z-transform solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ given that u_0=0 and u_1=1.