

**MA6351 TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS NOV/DEC 2014****PART – A**

- Form the partial differential equation by eliminating the arbitrary function  $f$  from  $z = f\left(\frac{y}{x}\right)$ .
- Find the complete solution of  $p+q=1$ .
- State the sufficient conditions for existence of Fourier series.
- If  $(\pi - x)^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{\cos(nx)}{n^2}$  in  $0 < x < 2\pi$ , then deduce that the value of  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ .
- Classify the partial differential equation  $(1 - x^2)Z_{xx} - 2xyZ_{xy} + (1 - y^2)Z_{yy} + xZ_x + 3x^2yZ_y - 2z = 0$ .
- Write down the various possible solutions of one dimensional heat flow equation.
- State and prove modulation theorem on Fourier Transforms.
- If  $F[f(x)] = F(s)$ , then find  $F\{e^{iax}.f(x)\}$ .
- Find the Z-transform of  $n$ .
- State initial value theorem on Z-transforms.

**PART – B**

- (i) Find the singular solution of  $z = px + qy = p^2 - q^2$ .
  - (ii) Solve  $(D^2 - 2DD')z = x^3y + e^{2x-y}$ .

(OR)

  - (i) Solve  $x(y-z)p + y(z-x)q = z(x-y)$ .
  - (ii) Solve  $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y)$ .
- (i) Find the fourier series of  $f(x) = x^2$  in  $-\pi < x < \pi$ . Hence deduce that the value of  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ .
  - (ii) Find the half range cosine series expansion of  $(x-1)^2$  in  $0 < x < 1$ .

(OR)

  - (i) Compute the first two harmonics of the Fourier series of  $f(x)$  from the table given:

$x$	0	$\pi/3$	$2\pi/3$	$\pi$	$4\pi/3$	$5\pi/3$	$2\pi$
$y=f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0
  - (ii) Obtain the Fourier cosine series expansion of  $f(x) = x$  in  $0 < x < 4$ . Hence deduce the value of  $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$
- (a) If a tightly stretched string of length  $l$  is initially at rest in equilibrium position and each point of it is given the velocity  $\left(\frac{\partial y}{\partial t}\right)_{t=0} = v_0 \sin^3\left(\frac{\pi x}{l}\right), 0 < x < l$ , determine the transverse displacement  $y(x,t)$ .

(OR)

(b) A square plate is bounded by the lines  $x=0$ ,  $x=a$ ,  $y=0$  and  $y=b$ . Its surfaces are insulated and the temperature along  $y=b$  is kept at  $100^\circ\text{C}$ , while the temperature along other three edges are at  $0^\circ\text{C}$ . Find the steady state temperature at any point in the plate.

14. (a) Find the Fourier transform of  $f(x) = \begin{cases} 1-|x|, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$ . Hence deduce that

$$(i) \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2} \quad (ii) \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}.$$

(OR)

(b) (i) Find the Fourier transform of  $f(x) = e^{-a^2 x^2}$ . Show that  $e^{-x^2/2}$  is self reciprocal under Fourier Transform.

(ii) Evaluate  $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$  using transforms.

15. (a) (i) Find  $Z[\cos(n\theta)]$  and  $Z[\sin(n\theta)]$ .

(ii) Using Z-transforms, solve  $u_{n+2} - 3u_{n+1} + 2u_n = 3^n$  given that  $u_0=0$  and  $u_1=1$ .

(OR)

(b) (i) Find the Z-transform of  $\left[\frac{1}{n(n+1)}\right]$ , where  $n \geq 1$

(ii) Find the inverse Z-transform of  $\frac{z^2 + z}{(z-1)(z^2 + 1)}$  using partial fraction method.