MA6351 TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS MAY/JUNE 2016

PART –A

1. Form the partial differential equation by eliminating the arbitrary functions from $f(x^2+y^2, z-xy)=0$.
2. Find the complete solution of the partial differential equation $p^3-q^3=0$.
3. Find the value of the Fourier series of $f(x) = \begin{cases} 0 & in\,(-c,0) \\ 1 & in\,(0,c) \end{cases}$ at the point of discontinuity x=0.
4. Find the value of b_n in the Fourier series expansion of $f(x) = \begin{cases} x+\pi & in\,(-\pi,0) \\ -x+\pi & in\,(0,\pi) \end{cases}$.
5. Classify the partial differential equation $u_{xx}+u_{yy}=f(x,y)$.
6. Write down all the possible solutions of one dimensional heat equation.
7. State Fourier integral theorem.
8. Find the Fourier transform of derivative of the function f(x) if $f(x)\to 0$ as $x\to\pm\infty$.
9. Find $Z\left[\frac{1}{n!}\right]$
10. Find $Z\{(\cos\theta+i\sin\theta)^n\}$.

PART – B

11. (a) (i) Solve $(x^2-yz)p+(y^2-zx)q=(z^2-xy)$.

(ii) Find the singular solution of $z=px+qy+\sqrt{1+p^2+q^2}$

(OR)

(b) (i) Solve $(D^3-2D^2D')z=2e^{2x}+3x^2y$.

(ii) Solve $(D^2+2DD'+D'^2-2D-2D')z=\sin(x+2y)$.

12 (a) (i) Find the Fourier series of $f(x) = x$ in $-\pi<x<\pi$.

(ii) Find the Fourier series of $f(x) = |\cos x|$ in $-\pi<x<\pi$.

(OR)

(b) (i) Find the half range sine series of $f(x) = x.\cos(\pi x)$ in (0,1).

(ii) Find the Fourier cosine series up to third harmonic to represent the function given by the following data:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

13 (a) Find the displacement of a string stretched between two fixed points at a distance of 2l apart when the string is initially at rest in equilibrium position and points of the string are given initial velocities v where

$$v = \begin{cases} \frac{x}{l} \text{ in } (0,l) \\ \frac{(2l-x)}{l} \text{ in } (l,2l) \end{cases}$$, x being the distance measured from one end.

(OR)

(b) A long rectangular plate with insulated surface is l cm wide. If the temperature along one short edge is $u(x,0) = k(lx-x^2)$ for $0<x<l$, while the other two long edges x=0 and x=l as well as the other short edge are kept at 0^0C, find the steady state temperature function u(x,y).

14. (a) Find the Fourier cosine and sine transform of $f(x) = e^{-ax}$ for x>0 . Hence deduce the integrals $\int_0^\infty \frac{\cos(sx)}{a^2+s^2}ds$ and $\int_0^\infty \frac{s.\sin(sx)}{a^2+s^2}ds$

(OR)

(b) (j) Find the Fourier transform of f(x) = $e^{-x^2/2}$ in (-∞,∞).

(ii) Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, |x|<1 \\ 0, |x| \geq 1 \end{cases}$. Hence deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$.

15 (a) (i) Find the Z transform of $\cos\left(\frac{n\pi}{2}\right)$ and $\frac{1}{n(n+1)}$

(ii) Using Convolution theorem, evaluate $Z^{-1}\left[\frac{z^2}{(z-a)^2}\right]$

(OR)

(b) (i) Find the inverse Z transform of $\frac{z}{z^2-2z+2}$ by residue method.

(ii) Solve the difference equation $y_{n+2}+y_n=2$, given that $y_0=0$ and $y_1=0$ by using Z-transforms.