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**Question Paper Code : 57502**

**B.E./B. Tech. DEGREE EXAMINATION, MAY/JUNE 2016**

**Third Semester**

**Civil Engineering**

**MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS**

**(Common to all branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)**

**(Regulations 2013)**

**Time : Three Hours**

**Maximum : 100 Marks**

**Answer ALL questions.**

**PART – A (10 × 2 = 20 Marks)**

1. Form the partial differential equation by eliminating the arbitrary functions from  $f(x^2 + y^2, z - xy) = 0$ .
2. Find the complete solution of the partial differential equation  $p^3 - q^3 = 0$ .
3. Find the value of the Fourier series of  $f(x) = \begin{cases} 0 & \text{in } (-c, 0) \\ 1 & \text{in } (0, c) \end{cases}$  at the point of discontinuity  $x = 0$ .
4. Find the value of  $b_n$  in the Fourier series expansion of  $f(x) = \begin{cases} x + \pi & \text{in } (-\pi, 0) \\ -x + \pi & \text{in } (0, \pi) \end{cases}$

5. Classify the partial differential equation  $u_{xx} + u_{yy} = f(x, y)$ .
6. Write down all the possible solutions of one dimensional heat equation.
7. State Fourier integral theorem.
8. Find the Fourier transform of a derivative of the function  $f(x)$  if  $f(x) \rightarrow 0$  as  $x \rightarrow \pm \infty$ .
9. Find  $Z \left\{ \frac{1}{n!} \right\}$
10. Find  $Z \{ (\cos \theta + i \sin \theta)^n \}$ .

**PART - B (5 × 16 = 80 Marks)**

11. (a) (i) Solve the equation  $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ . (8)
- (ii) Find the singular integral of the equation  $z = px + qy + \sqrt{1 + p^2 + q^2}$ . (8)

**OR**

- (b) (i) Solve :  $(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$ . (8)
  - (ii) Solve :  $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$  (8)
12. (a) (i) Find the Fourier series of  $f(x) = x$  in  $-\pi < x < \pi$ . (6)
  - (ii) Find the Fourier series expansion of  $f(x) = |\cos x|$  in  $-\pi < x < \pi$ . (10)

**OR**

- (b) (i) Find the half range sine series of  $f(x) = x \cos \pi x$  in  $(0, 1)$ . (8)

- (ii) Find the Fourier cosine series up to third harmonic to represent the function given by the following data : (8)

$$x: 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

$$y: 4 \quad 8 \quad 15 \quad 7 \quad 6 \quad 2$$

13. (a) Find the displacement of a string stretched between two fixed points at a distance of  $2l$  apart when the string is initially at rest in equilibrium position and points of

$$\text{the string are given initial velocities } v \text{ where } v = \begin{cases} \frac{x}{l} & \text{in } (0, l) \\ \frac{2l-x}{l} & \text{in } (l, 2l) \end{cases}, x \text{ being the}$$

distance measured from one end. (16)

OR

- (b) A long rectangular plate with insulated surface is  $l$  cm wide. If the temperature along one short edge is  $u(x, 0) = k(lx - x^2)$  for  $0 < x < l$ , while the other two long edges  $x = 0$  and  $x = l$  as well as the other short edge are kept at  $0^\circ\text{C}$ , find the steady state temperature function  $u(x, y)$ . (16)

14. (a) Find the Fourier cosine and sine transform of  $f(x) = e^{-ax}$  for  $x \geq 0$ ,  $a > 0$ . Hence

$$\text{deduce the integrals } \int_0^{\infty} \frac{\cos sx}{a^2 + s^2} ds \text{ and } \int_0^{\infty} \frac{s \sin sx}{a^2 + s^2} ds. \quad (16)$$

OR

- (b) (i) Find the Fourier transform of  $f(x) = e^{-\frac{x^2}{2}}$  in  $(-\infty, \infty)$ . (8)
- (ii) Find the Fourier transform of  $f(x) = 1 - |x|$  if  $|x| < 1$  and hence find the

$$\text{value of } \int_0^{\infty} \frac{\sin^4 t}{t^4} dt. \quad (8)$$

15. (a) (i) Find the Z-transforms of  $\cos \frac{n\pi}{2}$  and  $\frac{1}{n(n+1)}$ . (8)

(ii) Using convolution theorem, evaluate  $Z^{-1} \left\{ \frac{z^2}{(z-a)^2} \right\}$ . (8)

OR

(b) (i) Find the inverse Z-transform of  $\frac{z}{z^2 - 2z + 2}$  by residue method. (8)

(ii) Solve the difference equation  $y_{n+2} + y_n = 2$ , given that  $y_0 = 0$  and  $y_1 = 0$  by using Z-transforms. (8)