MA8452-STATISTICS & NUMERICAL METHODS

Unit-IV INTERPOLATION , NUMERICAL DIFFERENTIATION AND INTEGRATION

Two mark Question and answer

1. **What is the nature of the n^{th} divided difference of a polynomial of the n^{th} degree**?

   The n^{th} divided difference of a polynomial of the n^{th} degree is constant.

2. **State Newton's divided difference formula**.

   $$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2)f(x_0, x_1, x_2, x_3) + \cdots$$

3. **State the properties of divided difference**.

   1.The divided difference are symmetrical in all their arguments.
   2.The divided difference of sum (or) difference of two functions is equal to the sum (or) difference of the corresponding separate divided difference.
   3. The n^{th} divided difference of a polynomial of the n^{th} degree is constant.
   4.The divided difference operator is linear.

4. **Show that the divided difference are symmetrical in all their arguments.(or) The value of any difference is independent of the order of the argument**.

   We know that $f(x_0, x_1) = \frac{f(x_1)-f(x_0)}{x_1-x_0}$

   $$= \frac{f(x_0) - f(x_1)}{x_0 - x_1} = f(x_1, x_0)$$

   Therefore $f(x_0, x_1) = f(x_1, x_0)$

5.Find the divided difference table for $\boldsymbol{x}$: $-1 \quad 1 \quad 2 \quad 4$
$\boldsymbol{y}$: $-1 \quad 5 \quad 23 \quad 119$

x	y	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$
-1	-1			
		2		
1	5		16/3	
		18		.94
2	23		10	
		48		
4	119			

6.Using Lagrange's formula , to find the quadratic polynomial that takes these value.

$\boldsymbol{x}$: **0 1 3** **then find y(2).**
$\boldsymbol{y}$: **0 1 0**

$$y(2) = 1\frac{(2-0)(2-3)}{(1-0)(1-3)} = 1$$

7. Define forward difference and backward difference.

Forward

$$\Delta f(x) = f(x + h) - f(x)$$

$$\Delta^2 f(x) = f(x + 2h) - 2f(x + h) + f(x)$$

Backward

$$\nabla f(x) = f(x) - f(x - h)$$

$$\Delta^2 f(x) = f(x) - 2f(x - h) + f(x - 2h)$$

8.Define central difference and divided difference.

central difference

$$\delta f(x) = f\left(x + \frac{h}{2}\right) - f(x - \frac{h}{2})$$

Divided difference

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_2)}{x_2 - x_0}$$

9. Evaluate $\Delta^{10}(1-x)(1-2x)(1-3x)\dots(1-10x)$ $by\ taking\ h = 1$

$= \Delta^{10}[10!\ x^{10}] = 10!\,10! = (10!)^2$

10.Evaluate $\Delta^3(1-x)(1-2x)(1-3x)$ $by\ taking\ h = 1$

$= \Delta^3[-6\,x^3] = (-6)3! = 36$

11.Find the sixth term of the sequences 8,12,19,29,42…

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	8			
		4		
1	12		3	
		7		0
2	19		3	
		10		0
3	29		3	
		13		
4	42			

$$y_5 = E^5 y_0 = (1+\Delta)^5 y_0$$
$$= y_0 + 5C_1\,\Delta y_0 + 5C_2\,\Delta^2 y_0 + 5C_3\,\Delta^3 y_0 + \cdots$$
$$= 8 + 5(4) + 10(3) + 10(0) = 58$$

The sixth term is 58.

12.Find the second divided difference with arguments a,b. If $f(x) = \frac{1}{x}$

$f(a,b) = \frac{\frac{1}{b} - \frac{1}{a}}{b-a} = \frac{-1}{ab}$, Similarly $f(b,c) = \frac{-1}{bc}$ $and\ f(a,c) = \frac{-1}{ac}$

$$f(a,b,c) = \frac{f(b,c) - f(a,b)}{c-a} = \frac{\frac{-1}{bc} + \frac{1}{ab}}{c-a} = \frac{1}{abc}$$

13. State Newton's formula to find f'(x) using the forward differences.

Let y=f(x) be a function taking the values $y_0, y_1, y_2 \dots . y_n$. corresponding to $x_0, x_1, x_2 \dots . x_n$. of the independent variable x. Let the values of x be at equidistant intervals of size h.

Then $f'(x) = \frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \left(\frac{2p-1}{2!}\right)\Delta^2 y_0 + \left(\frac{3p^2 - 6p + 2}{3!}\right)\Delta^3 y_0 + \cdots .\right]$Where $p = \frac{x - x_0}{h}$--------(1)

(1) gives the value of $\frac{dy}{dx}$ at any x, which is a non tabular value.

In particular, at $x = x_0$, u=0. Then putting u=0 in (1), we have

$$\left(\frac{dy}{dx}\right)_{x=x_0} = f'(x) = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} \dots \dots\right]$$

14.Find $\frac{dy}{dx}$ at x=1 from the following table:

x	**1**	**2**	**3**	**4**
y	**1**	**8**	**27**	**64**

The forward difference table is as follows.

x	y	Δ	Δ^2	Δ^3
1	1			
		7		
2	8		12	
		19		6
3	27		18	
		37		
4	64			

$$\left(\frac{dy}{dx}\right)_{x=x_0} = f'(x) = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} \dots \dots\right]$$

Here h=1,$x_0 = 1, \Delta y_0 = 7, \Delta^2 y_0 = 12, \Delta^3 y_0 = 6$

$$\left(\frac{dy}{dx}\right)_{x=1} = f'(x) = \frac{1}{1}\left[7 - \frac{12}{2} + \frac{6}{3}\right] = 3$$

15.Find $\frac{dy}{dx}$ at x=2 from the following data

x:	**2**	**3**	**4**
y:	**26**	**58**	**112**

$$\Delta y_0 = 32, \Delta y_1 = 54, \Delta^2 y_0 = 22$$
$$\frac{dy}{dx} = 32 - \frac{1}{2}(22) = 21$$

16.Find $\frac{dy}{dx}$at x=6 from the following data

x:	**2**	**4**	**6**
y:	**3**	**11**	**27**

$$\nabla y_n = 16, \nabla y_{n-1} = 8, \nabla^2 y_n = 16 - 8 = 8$$
$$\left(\frac{dy}{dx}\right)_{at\ x=6} = \frac{1}{2}\left[16 + \frac{8}{2}\right] = 10$$

17.A curve passing through the points (1,0),(2,1) and (4,5).Find the slope of the curve at x=3

$$f(a,b) = \frac{f(b) - f(a)}{b - a}$$
$$f(1,2) = \frac{f(2) - f(1)}{2 - 1} = \frac{1 - 0}{2 - 1} = 1, f(2,4) = \frac{f(4) - f(2)}{4 - 2} = \frac{5 - 1}{4 - 2} = 2$$
$$f(1,2,4) = \frac{f(2,4) - f(1,2)}{4 - 1} = \frac{2 - 1}{3} = \frac{1}{3}$$
$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2)$$
$$= 0 + (x - 1)(1) + (x - 1)(x - 2)\tfrac{1}{3} = x - 1 + \tfrac{1}{3}(x^2 - 3x + 2)$$
$$f'(x) = 1 + \frac{2x}{3} - 1 = \frac{2x}{3}$$

18.State the basic principle for deriving Simpson's $\frac{1}{3}$ rule.(or) When does Simpson's $\frac{1}{3}$ rule give exact result?

Simpson's $\frac{1}{3}$ rule will give exact result, if the entire curve y=f(x) is itself a Para bola.

19.state the order of error in Simpson's $\frac{1}{3}$ rule.

Error in Simpson's $\frac{1}{3}$ rule is of order h^4.

20.Using Simpson's $\frac{1}{3}$ rule, find $\int_0^4 e^x dx$ given $e^0 = 1, e^1 = 2.72, e^2 = 7.39, e^3 = 20.09$ *and* $e^4 = 54.6$

$$\int_0^4 e^x dx = \frac{1}{3}[(1 + 54.6) + 4(2.72 + 20.09) + 2(7.39)] = 53.873$$

21.A curve passing through (2,8),(3,27),(4,64) and (5,125). Find the area of the curve between x-axis and the lines x=2 and x=5, by Trapezoidal rule.

$$\int_2^5 ydx = \frac{1}{2}[(8 + 125) + 2(27 + 64)] = 157.5 sq.units$$

22.Evaluate $\int_{-2}^{2} x^4 dx$by Simpson's rule, taking h=1

x:	-2	-1	0	1	2
y:	16	1	0	1	16

$$\int_{-2}^{2} x^4 dx = \frac{1}{3}[(16 + 16) + 4(2)] = 13.3\ sq.units$$

23.Why is Trapezoidal rule so called?

The trapezoidal rule is so called because it approximates the integral By the sum of a trapezoids.

24.How the accuracy can be increased in Trapezoidal rule of evaluating A given definite integral?

If the number of points of the base segment b-a, (the range of Integration) is increased, a better approximation to the area given by the definite integral will be obtained.

25.Evaluate$\int_{\frac{1}{2}}^{1} \frac{1}{x} dx$by Trapezoidal rule,dividing the range into 4 equal parts.

Here $h = \frac{1-\frac{1}{2}}{4} = \frac{1}{8}$

x	$\frac{1}{2} = \frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$
f(x)=$\frac{1}{x}$	$\frac{8}{4}$	$\frac{8}{5}$	$\frac{8}{6}$	$\frac{8}{7}$	$\frac{8}{8}$

$$\int_{x_0}^{x_n} y dx = \frac{h}{3}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \cdots \ldots \ldots \ldots y_{n-1}]$$

$$\int_{\frac{1}{2}}^{1} \frac{1}{x} dx = \frac{1}{16}\left[\left(\frac{8}{4} + \frac{8}{8}\right) + 2\left(\frac{8}{5} + \frac{8}{6} + \frac{8}{7}\right)\right] = \frac{1}{16}\left[3 + 2\left(\frac{856}{210}\right)\right] = \frac{1171}{1680} = 0.6971$$

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26. Using Trapezoidal rule, find $\int_0^6 f(x)dx$ from the following set of Values of x and f(x)

x	0	1	2	3	4	5	6
f(x)	1.56	3.64	4.62	5.12	7.08	9.22	10.44

Here h=1

$$\int_0^6 f(x)dx = \frac{1}{2}[(1.56 + 10.44) + 2(3.64 + 4.62 + 5.12 + 7.08 + 9.22)]$$

$$= \frac{1}{2}[12 + 2(29.68)] = \frac{1}{2}(71.36) = 35.68$$

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27. What is the local error term in Trapezoidal formula?

Principal part of the error in the interval $(x_1, x_2) = \frac{h^2}{12} y_1''$

Where y_1 is the value of y and y_1'' is the value of the second derivative of y at x=x_1.

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28. State the local error term in Simpson's one third rule.

Principal part of the error in the interval $(x_1, x_3) = \frac{h^5}{90} y_1^{iv}$

Where y_1 is the value of y and y_1^{iv} is the value of the fourth Order of y at x=x_1

29.What are the errors in Trapezoidal and Simpson's rule of numerical Integration?

Error in Trapezoidal rule $|E| < \frac{(b-a)}{12} h^2 M \quad h = \frac{b-a}{n}$

Error in Simpson's rule $|E| < \frac{(b-a)}{180} h^4 M$

30.In order to evaluate $\int_{x_0}^{x_n} y dx$ by Simpson's $\frac{1}{3}$ what is the restriction on the number of intervals?

Let n=interval

Simpson's $\frac{1}{3}$rule : The number of ordinates is odd or the intervals Number is even.

$f(x) = f(x_0) + (x-x_0) f(x_0, x_1 \ldots$

$+ (x-x_0)(x-x_1)(x- \ldots$

$+ (x-x_0)(x-x_1)(x- \ldots$

$f(x) = 48 + 9(x-4) \ldots$

$+ (x-4)(x-5)(x- \ldots$

$f(8) = 448$

(5) Using Newton's divided [illegible]

Compute $f(5)$ from [illegible]

x	1	2	4	7	[illegible]
y	22	30	82	106	[illegible]

Hints:

x	y	Δ	Δ^2	Δ^3
1	22			
2	30	8		
4	82	26		
7	106	8	[illegible]	[illegible]
12	[illegible]	22	[illegible]	[illegible]

Using Newton forward interpolation formula, find the polynomial $f(x)$ satisfying the following data and hence find the value of y for $x = 5$

x	4	6	8	10
y	1	3	8	16

x	y	Δ	Δ^2	Δ^3
4	1			
		2		
6	3		3	
		5		0
8	8		3	
		8		
10	16			

$$u = \frac{x-x_0}{h} = \frac{10-10}{5} = 0.4$$

$$y(x) = y_0 + [illegible] + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$f(5) = $ [illegible]

(7) Using Newton's forward interpolation formula, find the cubic polynomial which takes the values

x	0	1	2	3
$f(x)$	1	2	1	10

Hints

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}$$

$x_0 = 0 \quad y_0 = 1 \quad h = 1 \quad u = \frac{x - x_0}{h} = x$

$$y(x) = 2x^3 - 7x^2 + 6x + 1$$

$f(4) = 41$

From the given data, find the number of Students whose weight is between 60 and 70

weight in lbs	0-40	40-60	60-80	80-100	100-120
No of Students	250	120	100	70	50

x	y	cumulative frequency	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250	250				
			120			
60	120	370		-20		
			100		-10	
80	100	470		-30		20
			70		10	
100	70	540		-20		
			50			
120	50	590				

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$u = \frac{x-x_0}{h} = \frac{70-40}{20} = 1.5$$

$$y(70) = 423.5937$$

$x = 60$

$u = \frac{60-40}{20} = 1$

$y(60) = 370$

no. of students whose weight is between 60 & 70 $= 54$

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(9) Fit a Natural cubic Spline to the following data

x	1	2	3	4
y	1	2	5	11

Also compute $y(1.5)$ and $y'(3)$

Hints:

Here $h = 1$

Let $M_0 = M_3 = 0$

The cubic Spline in $x_{i-1} \le x \le x_i$ is gn by we have

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]$$

$$M_0 + 4M_1 + M_2 = 6(2) = 12$$
$$M_1 + 4M_2 + M_3 = 18$$

$4M_1 + M_2 = 12$ —— (1)

$M_1 + 4M_2 = 18$ —— (2)

Solve (1) & (2)

$M_1 = 2 \; , \; M_2 = 4$

$$f(x) = y(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right] + \frac{1}{h}\left[(x_i - x)\left[y_{i-1} - \frac{h^2}{6}M_{i-1}\right]\right] + \frac{1}{h}\left[(x - x_{i-1})\left[y_i - \frac{h^2}{6}M_i\right]\right] \quad \text{--- (3)}$$

$i=1$

$$f(x) = \frac{1}{3}\left[x^3 - 3x^2 + 5x\right], \quad 1 \le x \le 2 \quad \text{--- (4)}$$

$i=2$

$$f(x) = \frac{1}{3}\left[x^3 - 3x^2 + 5x\right]; \quad 2 \le x \le 3 \quad \text{--- (5)}$$

$i=3$

$$f(x) = \frac{1}{3}\left[-2x^3 + 24x^2 - 76x + 81\right], \quad 3 \le x \le 4 \quad \text{--- (6)}$$

(4), (5) & (6)

$f(1.5) = 1.875$

$f'(x) = \frac{1}{3}\left[3x^2 - 6x + 5\right], \quad 2 \le x \le 3$

$f'(3) = 4.667$

(2) From the given data

x	1	2	3
y	-8	-1	18

compute $f(1.5)$, $f(1.75)$ & $f'(1)$ using ...

$d_1 = \frac{-1+8}{1} = 7, \quad d_3 = \frac{18+1}{1} = 19 \quad t = \frac{x - x_{i-1}}{h_i}$

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} k_0 \\ k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 21 \\ 78 \\ 57 \end{pmatrix}$$

$k_0 = 4 \quad k_1 = 13 \quad k_2 = 22$

$x_0 = 1 \quad h_1 = 1 \quad x_1 = 2$

$f_0 = -8 \quad f_1 = -1$

$S_i(x) = (1-t) f_{i-1} + t f_i + h_i t (1-t) [k_{i-1} \ldots$

$(1-t) - (k_i \ldots$

$$t = \frac{x - x_{i-1}}{h_i}$$

Put $i = 1$, $\quad t = \frac{x - x_0}{h_1} = x - 1$

$S_1(x) = 8x + (x-1)(-1) + (1)(x-1)(\ldots$

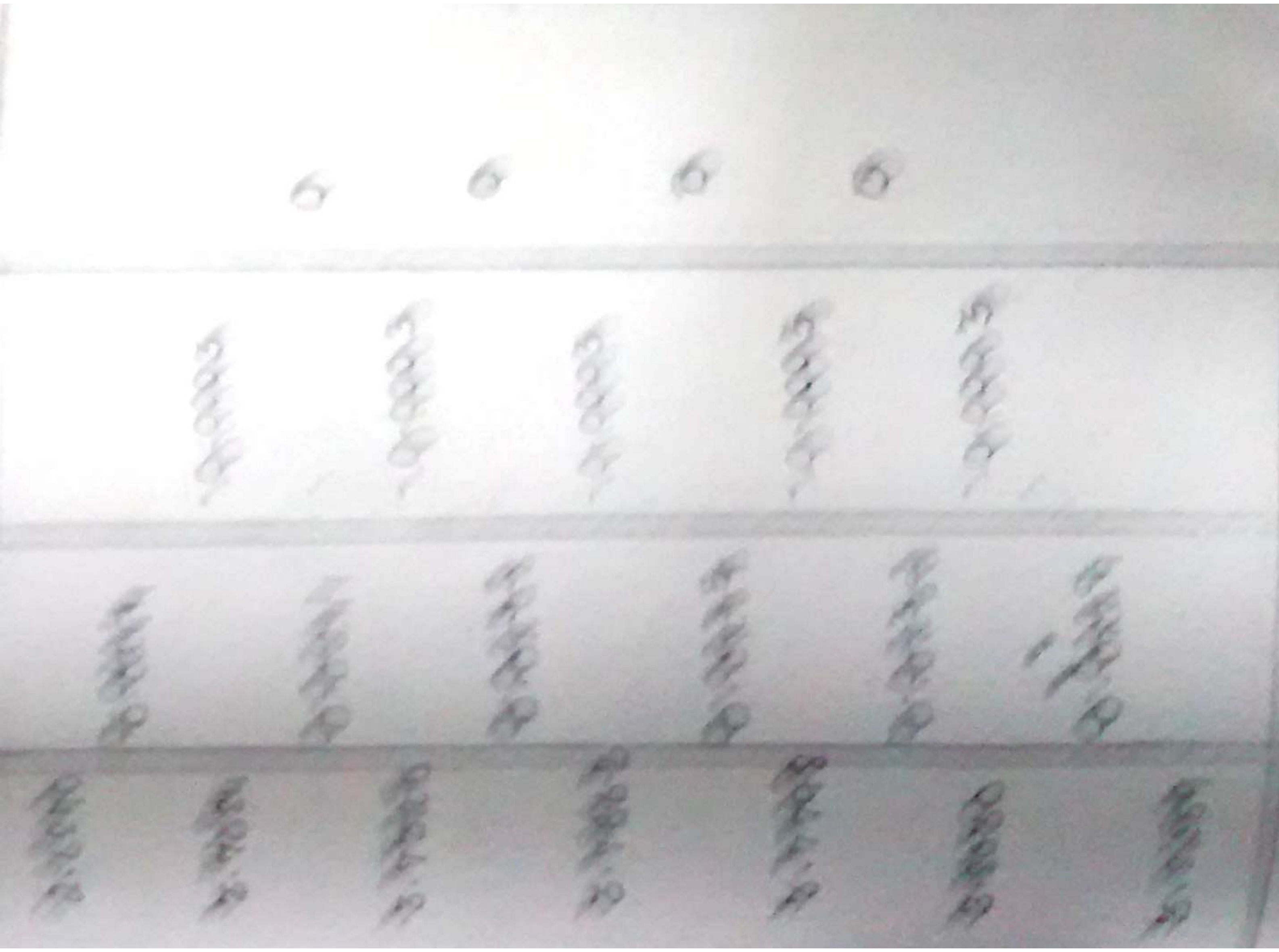
$[(4-7)(-x) - (13 \ldots$

$S_1(x) = 3x^3 - 9x^2 + 13x - 15$

$S_1(x) = 9x^2 - 18x + 13$

$S_1(1.5) = -5.625$

$S_1(1.75) = -3.7343$



Find the value of sec 31° from the following data

θ	31	32	33	34
$\tan\theta$	0.6008	0.6249	0.6494	0.6745

Hints : The difference table is

θ	$\tan\theta$	Δy	$\Delta^2 y$	$\Delta^3 y$

Newton's formula

$\left[\frac{dy}{dx}\right]_{x=x_0} = \left[\frac{dy}{dx}\right]_{u=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + \cdots\right]$

$\left[\frac{dy}{dx}\right]_{x=x_0} = \left[\frac{dy}{dx}\right]_{u=0} = \frac{1}{h}\left[0.0244 - \frac{1}{2}(0.0014) + \frac{1}{3}(0.0002) + \cdots\right]$ [illegible]

$\frac{\pi}{180}$

$(\sec 31^\circ) = \sec^2 31^\circ = 1.36063$

$\sec 31^\circ = 1.1702$

From the following data, find $f'(6)$ and the maximum value of $f(x)$

x: 0 2 3 4 7 9

$f(x)$: 4 26 58 112 466 922

Hints

Since the arguments are not equally spaced, we will use Newton's divided difference formula

By Newton's divided difference,

$y = f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) \ldots$

x	$y = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	4				
		11			
2	26		7		
		32		1	
3	56		11		0
		54		1	
4	112		16		0
		118		1	
7	466		22		
		228			
9	922				

$f(x) = x^3 + 2x^2 + 3x + 4$

$f'(x) = 3x^2 + 4x + 3$

$f'(6) = 135$

Maximum Value

$f(x)$ is maximum if $f'(x) = 0 \Rightarrow 3x^2 + 4x + 3 = 0$

But root are imaginary. Therefore, there is no extreme

Value in the range

[illegible] $\int_0^1 xe^x dx$ using Simpson's $\frac{1}{3}$ rule, choosing $h = 0.25$

[illegible]

$y = xe^x$

x	0	0.25	0.5	0.75	1
y	0	0.321	0.824	1.588	2.718

By Simpson's rule

$$I = \frac{h}{3}\left[(y_0 + y_4) + 2(y_2) + 4(y_1 + y_3)\right]$$

$= 1$

[illegible] the value of $\log 2^{1/3}$ from $\int_0^1 \frac{x^2}{1+x^3}$ using Simpson's $\frac{1}{3}$ rule with $h = 0.25$

[illegible]:

$h = 0.25$

$y = \frac{x^2}{1+x^3}$

x	0	0.25	0.5	0.75	1
y	0	0.0615	0.2222	0.3956	0.5

By Simpson's rule,

$$I = \frac{h}{3}\left[(y_0 + y_4) + 2(y_2) + 4(y_1 + y_3)\right]$$

$= 0.2311$

Actual Integration:

$$\int_0^1 \frac{x^2}{1+x^3} = \left[\frac{1}{3}\log(1+x^3)\right]_0^1 = \frac{1}{3}\log 2 = 0.2311$$

... the x axis.

Soln:-

Area $= \int y\,dx$ Volume $= \pi \int y^2 dx$

Find Area:

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

By Simpson's rule,

$I = \frac{h}{3}\left[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)\right]$

Area = 7.7833 sq. units

Find Volume:

x	1	1.5	2	2.5	3	3.5	4
y	4	5.76	7.29	7.84	9	6.76	4.41

Volume $= \pi \int y^2 dx$

$= 3.14\ (122.43)$

Volume $=$ 384.4 cubic units.

Evaluate $\int_0^{\frac{\pi}{2}}$ sin t dt ... point formula

Hints

Transform the variable from x to t by the transformation

$$t = \left(\frac{b-a}{2}\right)x + \left(\frac{b+a}{2}\right)$$

Here $a = 0$, $b = \frac{\pi}{2}$

$$t = \frac{\pi}{4}x + \frac{\pi}{4} = \frac{\pi(x+1)}{4} \text{ and } dt = \frac{\pi\,dx}{4}$$

Limit

t	0	$\frac{\pi}{2}$
x	-1	1

$$\int_0^{\frac{\pi}{2}} \sin t\,dt = \int_{-1}^{1} \sin\frac{\pi}{4}(1+x)\,dx$$

where $f(x) = \sin\frac{\pi}{4}(1+4)$

$$I = \frac{\pi}{4}\left[f\left(\frac{1}{\sqrt{3}}\right) + f\left(\frac{-1}{\sqrt{3}}\right)\right]$$

$$\int_0^{\frac{\pi}{2}} \sin t\,dt = 0.99848$$

Gaussian Quadrature:

Note:

Transform the variable from x to t

Transformation

$$x = \left(\frac{b-a}{2}\right)t + \left(\frac{b+a}{2}\right)$$

here $a = 0.2$, $b = 1.5$

$x = 0.65t + 0.85$

$dx = 0.65\,dt$

Limit

x	0.2	1.5
t	-1	1

$$\int_{0.2}^{1.5} e^{-x^2}\,dx = \int_{-1}^{1} e^{-(0.65t+0.85)^2}(0.65)\,dt$$

$$= (0.65)\int_{-1}^{1} e^{-(0.65t+0.85)^2}\,dt$$

here

$f(t) = e^{-(0.65t+0.85)^2}$

$f(0) = 0.4855$

$f\left(-\sqrt{\frac{3}{5}}\right) = 0.8869$

Evaluate $\int_1^2 \int_3^4 \frac{1}{(x+y)^2}\,dx\,dy$ by us

rule and Trapezoidal rule ta

$k = 0.5$

Hints:

$f(x,y) = \frac{1}{(x+y)^2}$ & $h = k = 0.5$

x/y	3	3.5	4
1	0.0625	0.0494	0.04
1.5	0.0494	0.04	0.0331
2	0.04	0.0331	0.0278

$I = 0.0413$

By Simpson's rule

$I = 0.0408$

Prepared by

Verified by

Part-B

Solve $\frac{dy}{dx} = \log_{10}(x+y)$, $y(0)=2$ by Euler's Method by choosing $h = 0.2$, find $y(0.2)$ and $y(0.4)$

Soln

Hints:

$x_0 = 0$, $y_0 = 2$, $h = 0.2$

$f(x,y) = \log_{10}(x+y)$

Euler's formula

$y_{n+1} = y_n + h f(x_n, y_n)$, $n = 0,1,2,3,\ldots$

Put $n=0$ $\quad y_1 = 2 + 0.2 \log_{10}(0+2) = 2.0602 = y(0.2)$

Put $n=1$ $\quad y_2 = 2.0602 + 0.2 \log_{10}(0.2 + 2.0602)$

$= 2.1310 = y(0.4)$

---

2) Solve $\frac{dy}{dx} = -\frac{y^2}{1+x}$ $y(0)=1$ by Euler's Method find $y(0.1)$ and $y(0.2)$ by choosing $h = 0.1$,

Hints:

Given $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$f(x, y) = \frac{y^2}{1+x}$

Euler's formula is $y_{n+1} = y_n + h f(x_n, y_n)$

$\therefore n = 0, 1, 2, 3, \ldots$

Put $n = 0$ we get

$$y_1 = y(0.1) = 1 - 0.1\left(\frac{1^2}{1+0}\right) = 0.9$$

Put $n = 1$, we get

$$y_2 = y(0.2) = 0.9 - 0.1\left(\frac{0.9^2}{1+0.1}\right)$$

$$= 0.82636$$

---

(3) find the value of y at $x = 0.1$ from $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ by Taylor Series Method.

Hints

Given $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$$y' = x^2y - 1$$

Taylor Series expansion is

(x_n, y_n)

$$y_{n+1} = y_n + \frac{h}{1!} y_n' + \frac{h^2}{2!} y_n'' + \frac{h^3}{3!} y_n''' + \cdots$$

$n=0,$ $$y_1 = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \cdots$$

$y' = x^2 y - 1$ $\qquad y_0' = -1$

$y'' = 2xy + x^2 y'$ $\qquad y_0'' = 0$

$y''' = 2y + 4xy' + x^2 y''$ $\qquad y_0''' = 2$

$y^{iv} = 6y' + 6xy'' + x^2 y'''$ $\qquad y_0^{iv} = -6$

$$y(0.1) = 1 + (0.1)(-1) + \frac{(0.1)^2}{2}(0) + \frac{(0.1)^3}{6}(2) + \frac{(0.1)^4}{24}(-6)$$

$$= 0.900305$$

---

(6) Solve $\frac{dy}{dx} = \sin x + \cos y$, $y(2.5) = 0$ by Modified Euler's method by choosing $h = 0.5$, find $y(3.5)$

Soln

$x_0 = 2.5,\ y_0 = 0,\ h = 0.5$

$f(x, y) = \sin x + \cos y$