UNIT 3
RANDOM PROCESS
TWO MARK QUESTIONS

1. Define random process?
   The sample space composed of functions of time is called a random process.

2. Define Stationary process?
   If a random process is divided into a number of time intervals, the various sections of the process exhibit essentially the same statistical properties. Such a process is said to be stationary.

3. Define Non Stationary process?
   If a random process is divided into a number of time intervals, the various sections of the process does not exhibit essentially the same statistical properties. Such a process is said to be stationary.

4. Define sample function?
   A fixed sample point s_j, the function of $X(t, s_j)$, is called a realization or a sample function of the random process. The sample function is given as
   $$X_j(t) = X(t, s_j)$$

5. Define Mean function?
   The mean of the random process is denoted by $\mu_x(t)$ the mean value is the expected value of the random process X(t).
   $$\mu_x(t) = E[X(t)]$$
   $$= \int_{-\infty}^{\infty} (\ )(\ )$$
   Where $f_{x(t)}(x)$ is the first order probability density function of the random process. For a stationary random process, $f_{x(t)}(x)$ is independent of time.

6. Define Auto Correlation function?
   It is defined as the expectation of the product of two random variables which are obtained by observing the random process X (t) at different times t_1 and t_2. The corresponding random variables are X (t_1) and X (t_2).
   The autocorrelation function is given by
   $$R_x(t_1,t_2) = E[X[t_1]X[t_2]]$$
   $$R_x(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_{x(t1)x(t2)}(x_1, x_2)dx_1dx_2$$
   Where $f_{x(t1)x(t2)}(x_1, x_2)$ is a second order probability density function of the random process.

7. List out the properties of auto correlation function.
   The autocorrelation function of a stationary process X(t) is
   R_x(ح) = E[X(t+ح)X(t)]
   Property1:

The mean square value of the process is obtained from $R_x(\tau)$ by putting $\tau = 0$

$$R_x(0) = E[X(t+0)X(t)] = E[X^2(t)]$$

Property 2:

The autocorrelation function $R_x(\tau)$ is a even function of τ

$$R_x(\tau) = R_x(-\tau)$$

Property 3:

The autocorrelation function has its maximum magnitude at $\tau = 0$

$$| R_x(\tau) | \leq R_x(0)$$

8. Define Auto Covariance?

The auto covariance function is denoted by $C_x(t_1, t_2)$ is given by

$$C_x(t_1,t_2) = E[(X(t_1) - \mu_x)(X(t_2) - \mu_x)]$$
$$C_x(t_1,t_2) = E[X(t_1) X(t_2) - X(t_1)\mu_x - \mu_x X(t_2) + \mu_x^2]$$
$$C_x(t_1,t_2) = E[X(t_1) X(t_2)] - E[X(t_1)]\mu_x - \mu_x E[X(t_2)] + \mu_x^2]$$
$$C_x(t_1,t_2) = R_x(t_1,t_2) - \mu_x^2 - \mu_x^2 + \mu_x^2$$
$$C_x(t_1,t_2) = R_x(t_1,t_2) - \mu_x^2$$

9. Define Cross Correlation?

The two cross correlation function of X(t) and Y(t) are defined by

$$R_{xy}(t, u) = E[X(t) Y(u)]$$
$$R_{yx}(t, u) = E[Y(t) X(u)]$$

Where t and u are the values of times on which process is observed.

10. List the properties of correlation function

i) It is not an even function.

ii) It does not have its maximum at origin

iii) It obeys certain symmetry relationship

$$R_{xy}(\tau) = R_{yx}(-\tau)$$

11. Define time average of Ergodic process in mean?

The mean of a random process X(t) at some fixed time t_k is the expectation of the random variable $X(t_k)$ that describes all possible values of the sample functions of the process observed at time $t=t_k$

The sample function x(t) of a stationary process X(t) at interval $-T \leq t \leq T$.

The DC value of x(t) is defined by the time average

$$\mu_x(T) = 1/2T\int_{-} ( )$$

12. Define time average of Ergodic process in auto correlation?

The time average of particular interest is the autocorrelation function $R_x(\tau, T)$ is defined in terms of the sample function x(t) of a stationary process X(t) at interval $-T \leq t \leq T$, the time averaged autocorrelation function is given by

$$R_x(\tau, T) = 1/2T\int_{-T}^{T} x(t+\tau)x(t)dt$$

13. List out the properties of power spectral density.

Property 1:

The zero value of PSD of a stationary random process equals to total area under the graph of the autocorrelation function

$$S_x(f) = \int_{-\infty}^{\infty} Rx(\tau)\exp(j2\pi f(-\tau))d\tau$$

Sub f = 0 in above equation

$$S_x(0) = \int_{-\infty}^{\infty} Rx(\tau)d\tau$$

Property 2:

The mean square value of the stationary process equals to the total area under graph of the power spectral density

$$E[X^2(t)] = \int_{-\infty}^{\infty} \quad (\;)$$

Property 3:

The power spectral density of a stationary process is always nonnegative

$S_x(f)df \geq 0$ for all f

Property 4:

The power spectral density of a real valued random process is an even function of frequency

$S_x(-f) = S_x(f)$

Property 5:

The power spectral density appropriately normalized has the properties associated with a probability density function

$$P_x(f) = \frac{(\;)}{\int_{-\infty}^{\infty} \;(\;)}$$

14. Define Gaussian Random variable

A random process X(t) is said to be Gaussian distributed if every linear function of X(t) is a Gaussian random variable. The PDF of Gaussian distributed random variable Y is given by

$$f_Y(y) = \frac{\_(\;-\;)(\;-\;)}{2}$$

15. Explain central limit theorem

An important result in probability theory that is closely related to the Gaussian distribution is the central limit theorem. Let X_1, X_2, ………X_n be a set of random variables with the following properties:

1. The X_k with k = 1, 2, 3, ……. , n are statistically independent.
2. The X_k all have the same probability density function.
3. Both the mean and the variance exist for each X_k.

We do not assume that the density function of the X_k is Gaussian. Let Y be a new random variable defined as

$$Y = \sum_{=1} X_k$$

16. Write the Einstien-Wiener-Khintchine relations

    The power spectral density $S_x(f)$ and auto correlation function $Rx(\tau)$ of a stationary process is given by

    $$S_x(f) = \int_{-\infty}^{\infty} Rx(\tau) \exp\left(j2\pi f(-\tau)\right) d\tau$$

    $$R_x(\tau) = \int_{-\infty}^{\infty} Sx(f) \exp\left(j2\pi f(\tau)\right) df$$

    The above two relation together called as Einstien Wiener Khintchine relation

17. Define transmissions of a random process through a LTI filter?

    When the random process X(t) is applied as input to a linear time – invariant filter of impulse response h(t), producing a new random process Y(t) at the filter output.

18. Define Discrete Random Variable

    A random variable whose set of possible values either in finite or countably infinite is called discrete random variable.

19. Define Continuous random variable

    A random variable X is said to be continuous if it takes all possible values between certain limits say from real number ‘a’ to real number ‘b’.

    If X is a continuous random variable for any x_1 and x_2.

    $P(x_1 \leq X \leq x_2) = P(x_1 < X \leq x_2) = P(x_1 \leq X < x_2) = P(x_1 < X < x_2)$

20. List the properties of Probability distribution function.
    1. The distribution function $F_x(x)$ is bounded between zero and one.
    2. The distribution function $F_x(x)$ is monotone non decreasing function of x.

       $F_x(x_1) \leq F_x(x_2)$

21. List the properties of Probability density function.
    1. $f(x) \geq 0$
    2. $\int_{-\infty}^{\infty} (\;) \; = 1$
    3. $(\; \leq \; \leq \;) = \int (\;)$

16 MARK QUESTIONS

1. Explain the following terms (i) Random variable (ii) Gaussian process

2. Define and explain the following :

   (i)Gaussian noise and Gaussian distribution

   (ii)Thermal Noise

   (iii)Shot Noise

   What type of PDF does the Gaussian noise follow?

3. X is uniformly distributed as given below find E(X), $E[X^2]$, E[cosX] and $E[(X-mx)^2]$

4. State and Prove the properties of Gaussian Process.

5. (i) Explain the following terms mean, correlation, covariance, ergodicity.
   (ii) Give the properties of the auto correlation function.

6. (i) An AWGN of power spectral density 1uW is fed through a filter with frequency response
   $H(f) = 1/2$; $|f| < 40$ kHz
   0 ; elsewhere. Calculate the noise power at the output of the filter.
   (ii)Write a note on stationary processes and its classifications.

7. Derive the equation for finding the probability density function of a one to one differential function of a given random variable.

8. (i)Explain about Transmission of random process through a Linear Time Invariant (LTI) filter.

   (ii)Find the autocorrelation of a sequence $x(t) = A\cos(2\pi f_c(t+\theta))$ where A and fc are constant

and is a random variable that is uniformly distributed over the interval $[-\pi\ \pi]$.

9. (i) Define autocorrelation. Discuss the properties of autocorrelation function.
   (ii) Consider the Random processes X(t) & Y(t) have zero mean and they are individually stationary. Consider the sum random process Z(t) = X(t) +Y(t). Determine the power spectral density of Z(t) .

10. State and prove the properties of power spectral density.