QUESTION BANK

CE6702 – PRESTRESSED CONCRETE STRUCTURES

UNIT 4 – COMPOSITE BEAMS AND CONTINUOUS BEAMS

PART – A (2 marks)

1. **Define propped construction. (AUC May/June 2013, Nov/Dec 2013)**

   The dead load stress developed in the precast prestressed units can be minimized by propping them while casting the concrete in situ. This method of construction is termed as propped construction.

2. **How to achieve compositeness between precast and cast in situ part and show the sketches? (AUC May/June 2013, Nov/Dec 2013)**

   The composite action between the two components is achieved by roughening the surface of the prestressed unit on to which the concrete is cast in situ, thus giving a better frictional resistance or by stirrups protruding from the prestressed unit into the added concrete or by castellations on the surface of the prestressed unit adjoining the concrete which is cast in situ.

3. **What is meant by composite construction of prestressed and in situ concrete? (AUC Nov/Dec & Apr/May 2011)**

   In a composite construction, precast prestressed members are used in conjunction with the concrete cast in situ, so that the members behave as monolithic unit under service loads. The high strength prestressed units are used in the tension zone while the concrete, which is the cast in situ of relatively lower compressive strength is used in the compression zone of the composite members.

4. **How deflections in composite members are computed? (AUC Nov/Dec 2011)**

   In the case of composite members, deflections are computed by taking into account the different stages of loading as well as the differences in the modulus of elasticity of concrete in the precast prestressed unit and the in situ cast element.

5. **What do you mean by unpropped construction? (AUC Nov/Dec 2012)**

   If the precast units are not propped while placing the in situ concrete, stresses are developed in the unit due to the self weight of the member and the dead weight of the in situ concrete. This method of construction is referred to as unpropped construction.

6. **What are the forces considered in the calculation of deflection of prestressed concrete beams? (AUC Apr/May 2010)**

   - Prestressing force
   - Self weight of the beam
   - Dead load of the concrete
   - Live load acting on the concrete

7. What are the roles played by shear connectors in composite construction?

(AUC Apr/May 2010)

It is generally assumed that the natural bond at the interface contributes a part of the required shear resistance depending upon the strength of the in situ cast concrete and the roughness of the precast element. Any extra shear resistance over and above this should be provided by shear connectors.

8. What are the advantages in using precast prestressed units?

(AUC Apr/May 2011, Nov/Dec 2010 & 2012)

- Saving in the cost of steel in a composite member compared with a reinforced or prestressed concrete member.
- Sizes of precast prestressed units can be reduced due to the effect of composite
- action. Low ratio of size of the precast unit to that of the whole composite member.
- Composite members are ideally suited for construction bridge decks without the disruption of normal traffic.

9. Name the loadings to be considered for computing initial Deflection.

(AUC Nov/Dec 2010)

- Prestress
- Self weight of the beam
- Weight of the in situ cast concrete

10. How do you compute the shrinkage and resultant stresses in composite member?

(AUC Nov/Dec 2012)

The magnitude of differential shrinkage is influenced by the composition of concrete and the environmental conditions to which the composite member is exposed. In the absence of exact data, a general value of 100 micro strains is provided for computing shrinkage stresses.

11. Distinguish between propped and unpropped construction methods.

(AUC Nov/Dec 2012)

S.No	Propped construction	Unpropped construction
1	The dead load stress developed in the precast prestressed units can be minimized by propping them while casting the concrete in situ. This method of construction is termed as propped construction.	If the precast units are not propped while placing them in situ concrete, stresses are developed in the unit due to the self weight of the member and the dead weight of the in situ concrete. This method of construction is referred to as unpropped construction.
2	If the pretensioned beam supports the weight of the slab while casting.	If the slab is externally supported while casting.

12. What are the assumptions made in stresses developed due to differential shrinkage?

- The shrinkage is uniform over the in situ part of the section.
- Effect of creep and increase in modulus of elasticity with age and the component of shrinkage, which is common to both the units are negligible.

13. Name the loadings to be considered for computing deflection if the beam is propped section.

- Prestress
- Self weight of the beam
- Dead weight of the in situ cast concrete
- Live load of the in situ cast concrete

14. Name the loadings to be considered for computing deflection if the beam is unpropped section.

- Prestress
- Self weight of the beam
- Live load of the in situ cast concrete

15. Sketch the typical cross section of precast prestressed concrete beam.

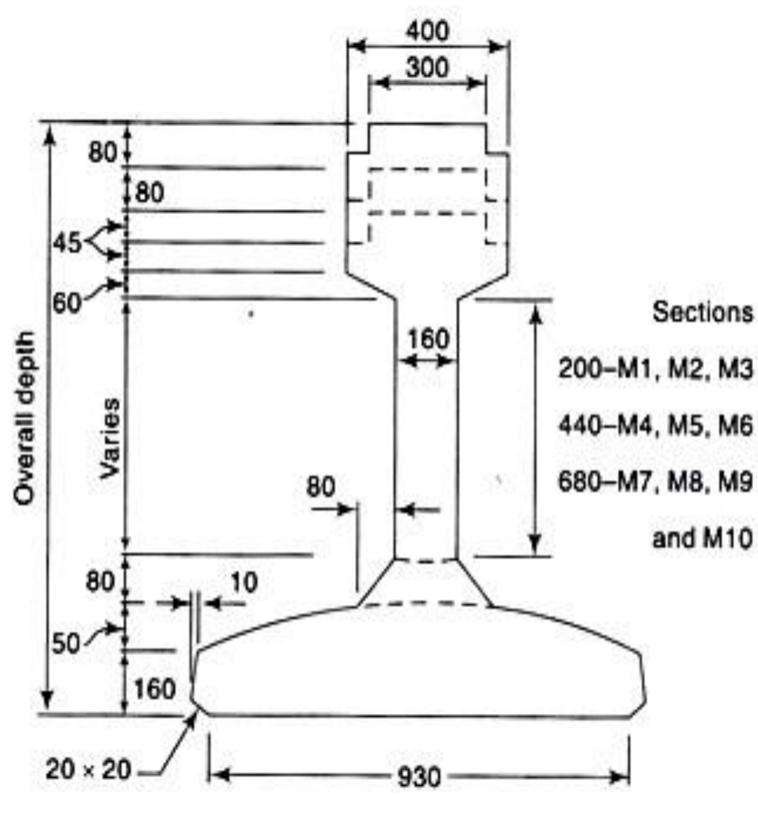

Fig. *Cross-section of Standard C and C.A. Beams*

PART – B (16 marks)

1. **Design a precast prestressed inverted T – section to be used in a composite slab of total depth 600 mm and width 300 mm. the composite slab is required to support an imposed load of 16 kN/m^2 over a span of 14 m. the compressive stress in concrete at transfer and the tensile stress under working loads may be assumed to be 20 and 1 N/mm^2 respectively. The loss ratio is 0.85. Determine the prestressing force required for the section. (AUC May/June 2013, Nov/Dec 2013)**

Solution:

Load due to self-weight of the precast beam and *in situ* concrete,

$$g = (0.3 \times 0.6 \times 24) = 4.32 \text{ kN/m}$$

Corresponding moment, $M = (0.125 \times 4.32 \times 14^2) = 105$ kN m

moment due to live loads, $M' = (0.125 \times 0.3 \times 1 \times 16 \times 14^2) = 118$ kN m

Section modulus, $$Z'_b = \left[\frac{(300 \times 600^2)}{6}\right] = 18 \times 10^6 \text{ mm}^3$$

The required section modulus of the precast prestressed inverted T is given by

$$Z_b \geq \left[\frac{18 \times 10^6 (105 \times 10^6 - 0)}{18 \times 10^6 (0.85 \times 20 + 1) - 118 \times 10^6}\right] \geq 9.2 \times 10^6 \text{ mm}^3$$

An inverted T with the following dimensions and section properties will provide the required section modulus.

Thickness and width of the top flange = 100 mm and 150 mm, respectively

Thickness and width of the bottom flange = 100 mm and 300 mm, respectively

2. Explain the term shrinkage stresses in composite beams.(AUC May/June 2013)

In composite members using precast prestressed units *in situ* cast concrete, a considerable proportion of the total shrinkage will have already taken place in the precast prestressed beam before the casting and hardening of the *in situ* concrete. Due to the high water/cement ratios used in the *in situ* concrete, there will be considerable shrinkage of this part in the composite section. Consequently, the differential shrinkage between the precast and *in situ* cast units results in stresses in both. The magnitude of differential shrinkage is influenced by the composition of concrete and the environmental conditions to which the composite member is exposed. In the absence of exact data, a general value of 100 micro strains is provided for in the British code BS: 8110[6] for computing shrinkage stresses.

A reasonable estimation of stresses developed due to differential shrinkage may be made using the following assumptions:

1. the shrinkage is uniform over the *in situ* part of the section, and
2. effect of creep and increase in modulus of elasticity with age and the component of shrinkage, which is common to both the units, is negligible.

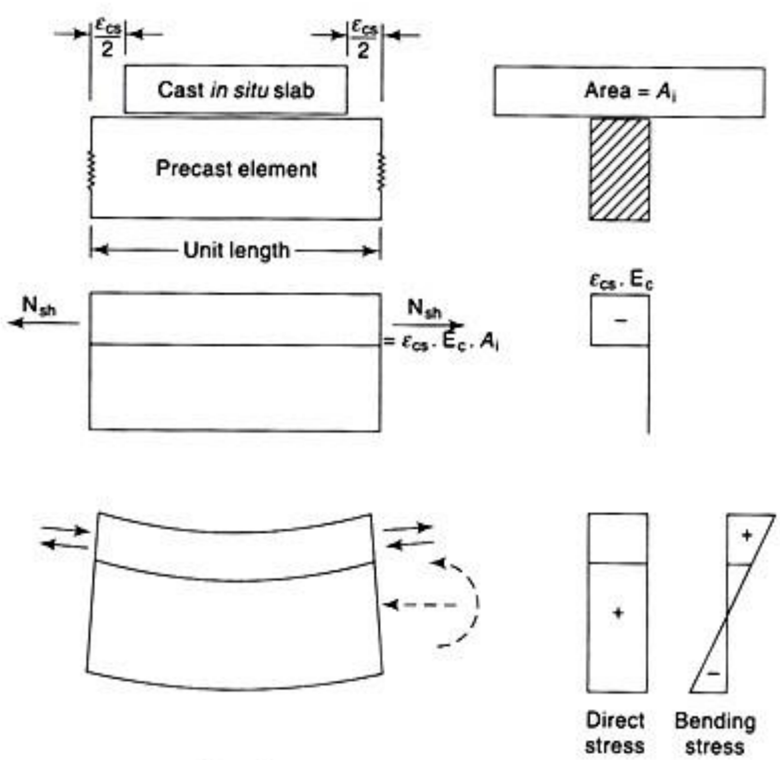

Fig. *Stresses due to Differential Shrinkage*

The method of computing the stresses is illustrated in Fig. 14.5 in which the *in situ* cast slab is first allowed to undergo the full amount of differential shrinkage ε_{cs}. Tensile forces of intensity N_{sh} are then applied to each end acting at the centroid of the cast *in situ* slab so that the slab is restored to the length of the precast element. Consequently the uniform tensile stress induced in the *in situ* concrete is $\varepsilon_{cs}E_c$ and the magnitude of the tensile force is computed as

$$N_{sh} = \varepsilon_{cs}E_cA_i,$$

where A_i = area of the *in situ* concrete section

E_c = modulus of elasticity of the *in situ* concrete

The composite member is in a state of internal equilibrium without any external forces acting on it. Hence the tensile force must be balanced by the application of a compressive force of equal magnitude along the same line. The compressive force applied at the centroid of the cast *in situ* slab is equivalent to a direct compressive force acting at the centroid of the composite section together with a bending moment which will induce direct and bending stresses in the composite section. These stresses are superposed on the existing tensile stresses in the cast *in situ* slab to compute the final stresses.

3. **A precast pretensioned beam of rectangular section has a breadth of 100 mm and a depth of 200 mm. the beam with an effective span of 5 m is prestressed by tendons with their centroids coinciding with the bottom kern. The initial force in the tendons is 150 kN. The loss of prestress may be assumed to be 15 percent. The beam is incorporated in a composite T – beam by casting a top flange of breadth 400 mm and thickness 40 mm. if the composite beam supports a live load of 8 kN/m^2. Calculate the resultant stresses developed in the precast and insitu concrete assuming the pretensioned beam as:**
   (a) Unpropped, (b) propped during the casting of the slab. Assume the same modulus of elasticity for concrete in precast beam and insitu cast slab.
   (AUC Nov/Dec 2011)
   Solution:

Section properties of the pre-tensioned beam

$$A = (100 \times 200) = 20000 \text{ mm}^2$$

$$Z = \frac{(100 \times 200^2)}{6} = 667 \times 10^3 \text{ mm}^3$$

Initial prestressing force, $P = 150$ kN

$$\text{Stresses due to prestressing force} = \left(\frac{2P}{A}\right) = \left[\frac{(2 \times 150 \times 10^3)}{(20000)}\right]$$

$= 15$ N/mm^2 at the bottom and zero at the top fibre respectively.

$$\text{Effective prestress after losses} = (0.85 \times 15) = 12.8 \text{ N/mm}^2$$

$$\text{Self-weight of the precast beam} = (0.1 \times 0.2 \times 24 \times 10^3) = 480 \text{ N/m}$$

$$\text{Self-weight moment} = (0.125 \times 480 \times 5^2) = 1500 \text{ Nm}$$

$$\text{Stresses at top and bottom fibre} = \frac{(1500000)}{(667 \times 10^3)} = \pm 2.25 \text{ N/mm}^2$$

Self-weight of *in situ* cast slab = $(0.04 \times 0.4 \times 24 \times 10^3) = 384$ Nm

Moment due to slab-weight = $(0.125 \times 384 \times 5^2) = 1200$ Nm

Stresses due to slab-weight in the precast section = $\dfrac{(1200000)}{(667 \times 10^3)} = \pm 1.8$ N/mm^2

Section properties of the composite section

Distance of the centroid from the top fibre = 87 mm

Second moment of area, $I = (1948 \times 10^5)$ mm^4

Second moduli, $Z_t = (225 \times 10^4)$ mm^3

$Z_b = (128 \times 10^4)$ mm^3

Live load on the composite section = $(0.4 \times 1 \times 8000) = 3200$ N/m

Maximum live-load moment = $(0.125 \times 3200 \times 5^2) = 10000$ Nm

Live load stresses in the composite section

$$\text{At top} = \left(\frac{10^7}{225} \times 10^4\right) = 4.45 \text{ N/mm}^2 \text{ (compression)}$$

$$\text{At bottom} = \left(\frac{10^7}{128} \times 10^4\right) = 7.85 \text{ N/mm}^2 \text{ (tension)}$$

If the pre-tensioned beam is propped, the self-weight of the slab acts on the composite section.

Moment due to slab-weight = 1200 Nm

Stresses due to this moment in the composite section

$$\text{At top} = \left(\frac{1200000}{225 \times 10^4}\right) = 0.53 \text{ N/mm}^2 \text{ (compression)}$$

$$\text{At bottom} = \left(\frac{1200000}{128 \times 10^4}\right) = 0.94 \text{ N/mm}^2 \text{ (tension)}$$

The distribution of stresses for the various stages of loading for the propped and unpropped construction is shown in Fig.

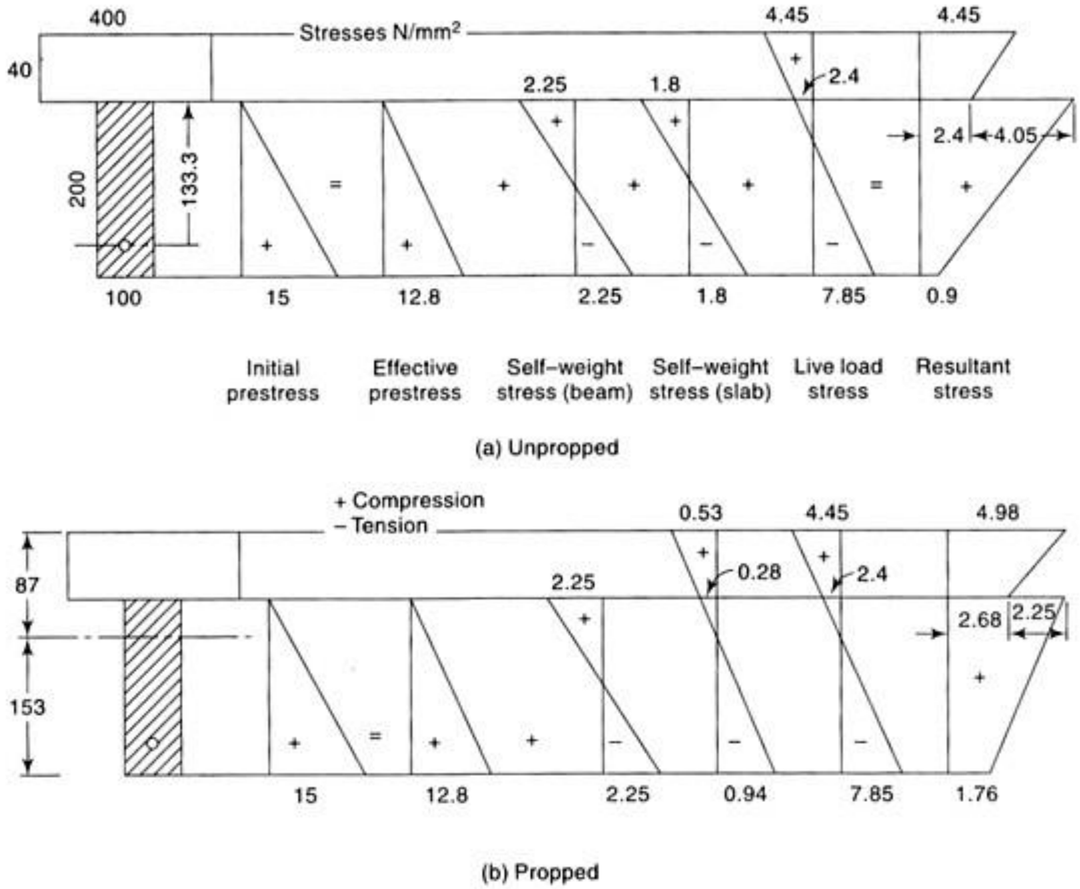

Fig. *Stresses in Precast Pre-tensioned Beam and Cast in situ Slab*

4. Discuss in detail about the factors which influence flexural strength and shear strength of composite prestressed section. (AUC Nov/Dec 2011)

Flexural strength of composite section:

The ultimate strength of composite prestressed sections in flexure is governed by the same principles used for ordinary prestressed sections discussed in Chapter 7. In the case of composite sections, the percentage of tensioned reinforcement is less than that in most simple beams, so that the section is invariably under-reinforced. The compression zone generally consists entirely of *in situ* concrete of lower compressive strength, and the value of the cube strength of concrete to be used in flexural strength equations will obviously be that of *in situ* cast concrete. However, if the compression zone contains a part of the precast element, the average compressive strength computed by considering the cross-sectional areas of *in situ* and precast concrete is used in the computations of compressive force. The following examples illustrate the method of estimating the ultimate flexural strength of composite sections.

Shear strength of composite section:

The support sections of composite members where web shear cracks are likely to develop should be checked under service loads for safety against cracking in shear. As outlined in Sec. 8.1, the principal tension developed in the webs of precast elements in the composite section is computed using the values of the shear and bending stresses in the section. If the principal tensile stress exceeds the design tensile strength of concrete, suitable reinforcements are to be designed according to the elastic design principles.

The ultimate shear strength of composite sections with web-shear or flexure-shear cracks is computed using the empirical expressions suggested in the British, American and Indian standard codes presented in Sec. 8.2.

If the shear at the section under design ultimate loads exceeds the shear strength, suitable shear reinforcements are designed according the design code provisions outlined in Sec. 8.3. The composite action of the integral unit is mainly dependent upon an effective shear connection at the contact surface between the precast and *in situ* cast elements.

3. 2.5 N/mm^2, when minimum vertical ties are provided and the contact surface is roughened to an amplitude of 5 mm.

4. When shear stress exceeds 2.5 N/mm^2, then shear friction reinforcement is to be designed and the required area of reinforcements is given by

$$A_{vf} = \left(\frac{V_u}{\phi f_y \mu}\right)$$

where f_y = characteristic tensile strength of tie reinforcement

ϕ = capacity reduction factor having a value of 0.85 for shear computations

μ = coefficient of friction having the following values:

(a) Concrete placed monolithically = 1.4 λ

(b) Concrete placed against hardened concrete with surface intentionally roughened = 1.0 λ

(c) Concrete placed against hardened concrete not intentionally roughened = 0.6 λ

(d) Concrete anchored to rolled structural steel by headed studs or by reinforcing bars = 0.7 λ

where λ = 1.0 for normal density concrete

= 0.85 for "sand low density" concrete

= 0.75 for "all low density" concrete.

The ties consisting of single bars, multiple-leg-stirrups or vertical legs of welded wire-fabric should have a spacing not exceeding four times the least dimension of the supported element, nor 600 mm, whichever is less.

5. A precast PSC beam of rectangular section has a breadth of 100 mm and a depth of 200 mm. The beam with an effective span of 5 m is prestressed by tendons with their centroids coinciding with bottom kern. The initial force in the tendon is 150 kN. The loss ratio = 0.85. The beam is incorporated in a composite T beam by casting a top flange of breadth 400 mm and thickness 40 mm. If the composite beam supports a live load of 8 kN/m^2. Calculate the resultant stresses developed in the precast and insitu cast concrete assuming the pretensioned beam as unpropped during the casting of the slab.
$E_{precastpsc}$ = 35 kN/mm^2, $E_{insitu\ con}$ = 28 kN/mm^2. (AUC Nov/Dec 2012)

Solution:

Ratio of modulus of elasticity = $\left(\frac{35}{28}\right) = 1.25$

Properties of equivalent composite section

Area of *in situ* slab = $(400 \times 40) = 16000\ \text{mm}^2$

Area of prestressed beam = $(200 \times 100) = 20000\ \text{mm}^2$

The centroid of the equivalent composite section is determined by taking moments about an axis passing through the soffit of the beam.

If y = distance of the centroid from the soffit,

$$(16 + 1.25 \times 20)\ 10^3 \times y = (16 \times 10^3 \times 220) + (1.25 \times 20 \times 10^3 \times 100)$$

$$y = 146 \text{ mm}$$

Second moment of area of the equivalent composite section is given by,

$$I_c = \left(\frac{400 \times 40^3}{12} + 16 \times 10^3 \times 74^2\right) + 1.25\left(\frac{100 \times 200^3}{12} + 20 \times 10^3 \times 46^2\right)$$

$$= 226 \times 10^6 \text{ mm}^4.$$

Live Load Moment = 10^7 N mm

Stresses developed in cast in situ slab

$$\text{At the top of slab} = \frac{(10^7 \times 94)}{(226 \times 10^6)} = +4.15 \text{ N/mm}^2$$

$$\text{At bottom of slab} = \frac{(10^7 \times 54)}{(226 \times 10^6)} = +2.2 \text{ N/mm}^2$$

Stresses *developed in the pre-tensioned beam*

$$\text{At top} = \left[\frac{(10^7 \times 54)}{(226 \times 10^6)}\right] \times 1.25 = +2.75 \text{ N/mm}^2$$

$$\text{At bottom} = \left[\frac{(10^7 \times 146)}{(226 \times 10^6)}\right] \times 1.25 = -8.1 \text{ N/mm}^2$$

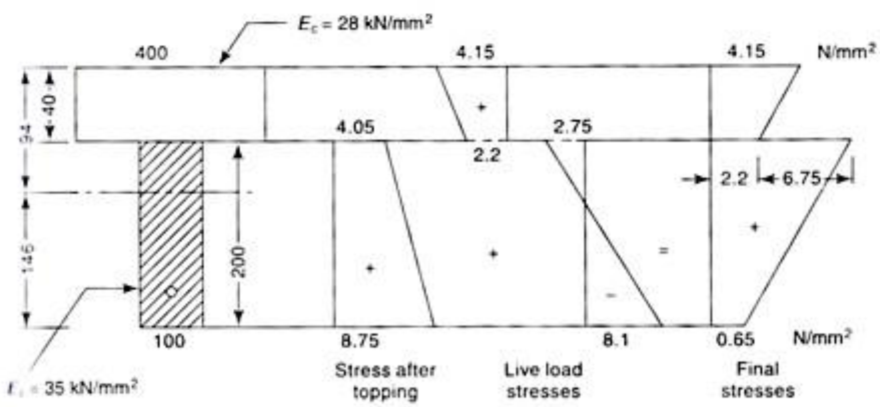

Fig. *Stress Distribution*

6. Write step by step design procedure for composite construction. (AUC Nov/Dec 2013)

The dimensioning of composite sections involves determining the required size of the composite section using a standard precast prestressed beam of known section properties in order to support the required design service loads. Alternatively, it may become necessary to determine the section modulus of the precast prestressed section for a composite slab of given depth. In either case, formulae relating the section moduli of the precast prestressed and composite section, loading on the member, permissible stresses in the concrete and loss ratio, may be developed by considering various stages of loading

The critical stress condition generally occurs at the soffit of the precast prestressed element under minimum and maximum moments. Hence, at the stage of transfer, when the minimum moment (self-weight of precast beam) is acting on the precast prestressed beam, the stress condition is,

$$\left(f_{\mathrm{inf}} - \frac{M_{\mathrm{min}}}{Z_{\mathrm{b}}} \right) \le f_{\mathrm{ct}}$$

If Z'_{b} = section modulus of the bottom fibre of the composite section

M = moment acting on the precast part of a composite section during construction

M' = moment acting on the composite section which is generally due to imposed loads

η = loss ratio

We then have the stress condition at the soffit of the composite section,

$$\left(\eta f_{\mathrm{inf}} - \frac{M}{Z_{\mathrm{b}}} - \frac{M'}{Z'_{\mathrm{b}}} \right) \ge f_{\mathrm{tw}}$$

By eliminating the prestress, f_{inf}, from Eqs 14.1 and 14.2, the required section modulus of the composite section is given by

$$Z'_{\mathrm{b}} \ge \left[\frac{Z_{\mathrm{b}} M'}{Z_{\mathrm{b}}(\eta f_{\mathrm{ct}} - f_{\mathrm{tw}}) - (M - \eta M_{\mathrm{min}})} \right]$$

The prestress required at the bottom and top fibres of the precast prestressed beam is computed using the following equations:

$$f_{\mathrm{inf}} \ge \left[\frac{f_{\mathrm{tw}}}{\eta} + \frac{M}{\eta Z_{\mathrm{b}}} + \frac{M'}{\eta Z'_{\mathrm{b}}} \right]$$

$$f_{\mathrm{sup}} \ge \left[f_{\mathrm{tt}} - \frac{M_{\mathrm{min}}}{Z_{\mathrm{t}}} \right]$$

The prestressing force and the corresponding eccentricity are directly obtained by Eqs 12.9 and 12.10.

If it is required to determine the section modulus of the precast prestressed section in a composite slab of given depth, Eq. 14.3 is arranged in an alternative form of the type,

$$Z_{\mathrm{b}} \ge \left[\frac{Z'_{\mathrm{b}}(M - \eta M_{\mathrm{min}})}{Z'_{\mathrm{b}}(\eta f_{\mathrm{ct}} - f_{\mathrm{tw}}) - M'} \right]$$

The required prestress is calculated using Eqs 14.4 and 14.5 and the stresses developed in the *in situ* and prestressed components are checked under working loads.

7. Explain the advantages of using precast prestressed elements along with in-situ concrete. **(AUC Apr/May 2010)**

The advantages in using precast prestressed units in association with the *in situ* concrete are:

1. appreciable saving in the cost of steel in a composite member compared with a reinforced or prestressed concrete member,
2. sizes of precast prestressed units can be reduced due to the effect of composite action,
3. low ratio of size of the precast unit to that of the whole composite member,
4. in many cases, precast prestressed units serve as supports and dispense with the form work for placement of *in situ* concrete,
5. composite members are ideally suited for constructing bridge decks without the disruption of normal traffic,
6. efficient utilization of material in a composite section in which the low and medium strength concrete of *in situ* construction resists compressive forces while the high-strength prestressed units resist the tensile forces,
7. the precast prestressed units which require skilled labour and workmanship can be cast in a factory or casting yard and conveyed to the site of construction,
8. combination of light weight concrete for the cast *in situ* slab results in reduced dead loads, leading to economy in the overall costs.

8. Explain different types of composite construction with sketches.
(AUC Apr/May 2010, 2011, Nov/Dec 2012)

The most common type of composite construction consists of a number of precast prestressed inverted T-beams, placed side by side and connected by a continuous top slab of *in situ* concrete. This type of construction is widely used in the construction of bridge decks. Transverse prestressing is also used to develop monolithic action in the lateral direction. The dead-weight of the deck can be considerably reduced by using voids or light-weight longitudinal cores in the space between the precast prestressed units. For large-span composite bridge decks of spans exceeding 30 m, the commonly used precast prestressed concrete units consist of I, unsymmetrical T or box sections. The concrete cast *in situ* forms the deck slab, interconnecting the precast units.

The precast prestressed I and T-beams have been standardised by the Cement and Concrete Association[3] for use in the construction of bridge decks of span varying from 7 to 36 m. Standard I and T units are extensively used as highway bridge beams.

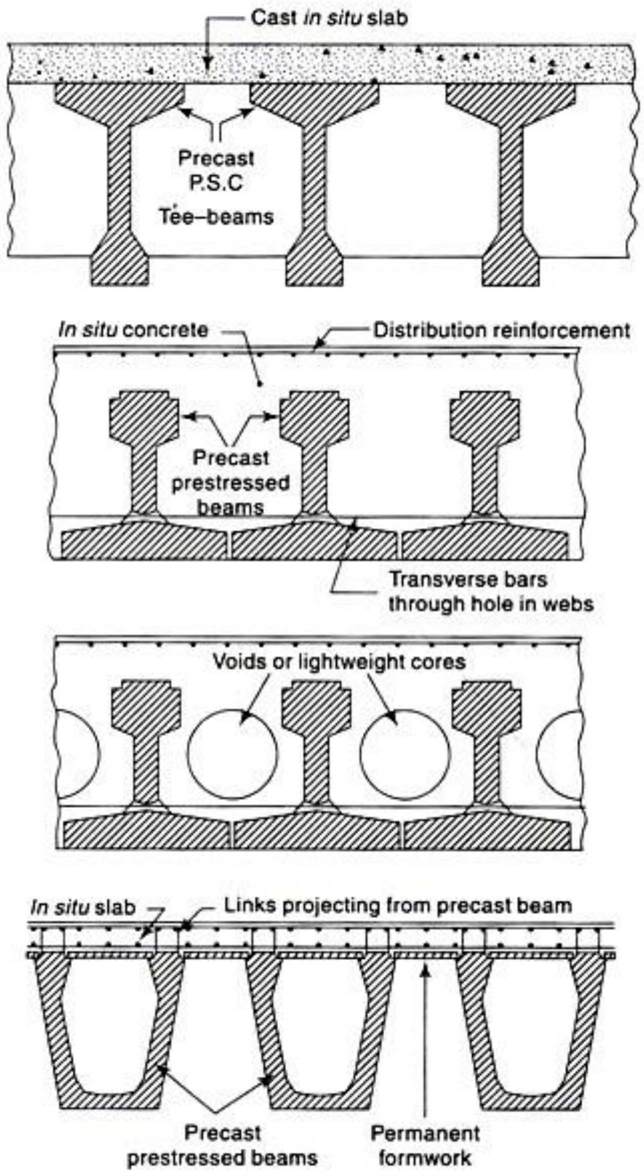

Fig. *Composite Bridge Decks with Precast Prestressed Elements*

The use of prestressed concrete tie beam in a reinforced concrete truss considerably reduces the cross-sectional dimensions of the bottom chord member, which is subjected to high degree of tension in the case of large span trusses. Reinforced and prestressed concrete trusses are generally used for spans ranging from 18 to 36 m and this form of construction is ideally suited for industrial structures . The dead load stress developed in the precast prestressed units can be minimised by propping them while casting the concrete *in situ*. This method of construction is termed as *propped construction*. If the precast units are not propped while placing the *in situ* concrete, stresses are developed in the unit due to the self-weight of the member and the dead weight of the *in situ* concrete. This method of construction is referred to as *unpropped construction*.

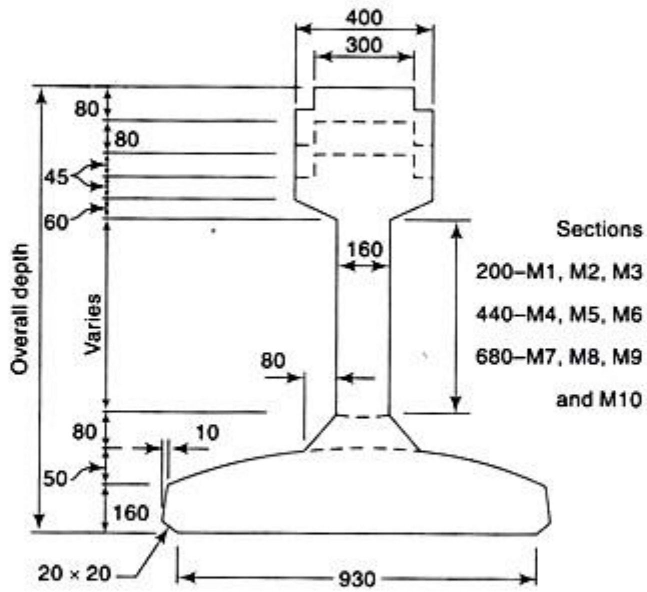

Fig. *Cross-section of Standard C and C.A. Beams*

9. A composite beam of rectangular section is made of inverted T-beam having a slab thickness of 150 mm and width of 1000 mm. the rib size in 150 mm x 850 mm. The in situ concrete slab has E_C = 30kN/m^2 and the thickness of cast in situ slab is 1000 mm. If the differential shrinkage in 100 x 10^{-6} units, estimate the shrinkage stress developed in the precast and cast in situ units. (AUC Apr/May 2011)

Solution:

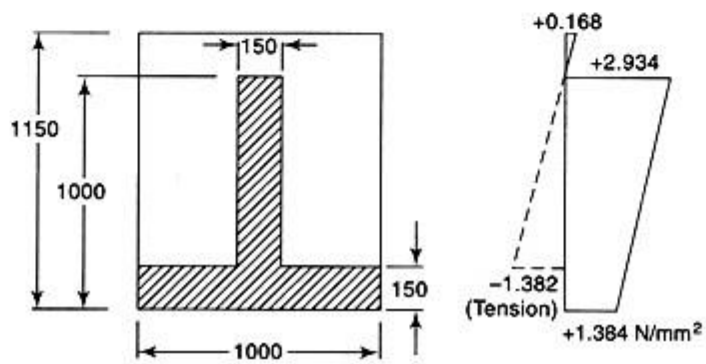

Fig. *Differential Shrinkage Stresses*

Area of *in situ* concrete, $A_i = 872500 \text{ mm}^2$

Area of the composite section, $A_c = 1150 \times 10^3 \text{ mm}^2$

Uniform tensile stress induced in the cast *in situ* slab

$$= \varepsilon_c E_{cs} = [(100 \times 10^{-6})(30 \times 10^3)] = 3.0 \text{ N/mm}^2$$

Force, $N_{sh} = \varepsilon_c E_c A_i = [(100 \times 10^{-6})\ (30 \times 10^3)(872500)] = 2617500$ N

The centroid of the composite section is located 575 mm from the top fibre.

Eccentricity of the compressive force, N_{sh}, from the centroid of the composite section

$$= (575 - 500) = 75 \text{ mm}$$

$$\therefore \quad \text{Moment} = (2617500) \times 75 = 196.3 \times 10^6 \text{ N mm}$$

Second moment of area of the composite section = (126739×10^6) mm^4

Section moduli for the various fibres

Pre-tensioned beam: Top fibre = (298×10^6) mm^3

Bottom fibre = (220×10^6) mm^3

In situ slab: Top fibre = (220×10^6) mm^3

Bottom fibre = (298×10^6) mm^3

Direct compressive stress in the composite section

$$= \left(\frac{2617500}{1150 \times 10^3}\right) = 2.276 \text{ N/mm}^2$$

Bending stress

$$\text{Top fibre} = \left[\frac{(196.3 \times 10^6)}{(220 \times 10^6)}\right] = 0.892 \text{ N/mm}^2$$

$$\text{Bottom fibre} = \left[\frac{(196.3 \times 10^6)}{(220 \times 10^6)}\right] = 0.892 \text{ N/mm}^2$$

$$\text{Junction} = \left[\frac{(196.3 \times 10^6)}{(298 \times 10^6)}\right] = 0.658 \text{ N/mm}^2$$

Differential shrinkage stresses

(a) In the precast pre-tensioned beam (compression + ve),

At the top of beam = $(2.276 + 0.658) = 2.934$ N/mm^2

At the bottom of beam = $(2.276 - 0.892) = 1.384$ N/mm^2

(b) In *in situ* cast slab

At the top of slab = $(2.276 + 0.892 - 3.00) = 0.168$ N/mm^2

At the bottom of slab = $(2.276 - 0.658 - 3.00) = -1.382$ N/mm^2

10. **A composite T - beam is made up of a pretensioned rib 100 mm wide and 200 mm deep and cast in situ slab 400 mm wide and 40 mm thick having a modulus of elasticity of 28kN/mm^2. If the differential shrinkage is 100 x 10^{-6} units, estimate the shrinkage stress developed in the precast and cast in situ units. (AUC Nov/Dec 2010) Solution:**

Differential shrinkage, $\varepsilon_{cs} = 100 \times 10^{-6}$

Area of *in situ* concrete, $A_i = (400 \times 40) = 16000 \text{ mm}^2$

Uniform tensile stress induced in the cast *in situ* slab $= \varepsilon_{cs} E_c$

$$= (100 \times 10^{-6})(28 \times 10^3) = 2.8 \text{ N/mm}^2$$

Force, $N_{sh} = \varepsilon_{cs} E_c A_i = (100 \times 10^{-6})(28 \times 10^3)(16 \times 10^3) = (44.8 \times 10^3)\text{N}$

The centroid of the composite section is located 87 mm from the top fibre. Eccentricity of the compressive force, N_{sh}, from the centroid of the composite section

$$= (87 - 20) = 67 \text{ mm}$$

Moment $= (44.8 \times 10^3) \times 67 = 3 \times 10^6$ N mm

Second moment of area of the composite section $= (1948 \times 10^5) \text{ mm}^4$

Section moduli for the various fibres:

Top fibre, $Z_t = (225 \times 10^4) \text{ mm}^3$

Bottom fibre, $Z_b = (128 \times 10^4) \text{ mm}^3$

Junction, $Z_j = (414 \times 10^4) \text{ mm}^3$

$$\text{Direct compressive stress} = \left[\frac{(44.8 \times 10^3)}{(36 \times 10^3)}\right] = 1.24 \text{ N/mm}^2$$

Bending stress:

$$\text{Top fibre} = \frac{(3 \times 10^6)}{(225 \times 10^4)} = 1.33 \text{ N/mm}^2$$

$$\text{Bottom fibre} = \frac{(3 \times 10^6)}{(128 \times 10^4)} = 2.34 \text{ N/mm}^2$$

$$\text{Junction} = \frac{(3 \times 10^6)}{(414 \times 10^4)} = 0.72 \text{ N/mm}^2$$

Differential shrinkage stresses:

(a) In precast pre-tensioned beam (+ compression – tension)

At the top of beam $= (1.24 + 0.72) = 1.96 \text{ N/mm}^2$

At the bottom of beam $= (1.24 - 2.34) = -1.10 \text{ N/mm}^2$

(b) In *in situ* cast slab,

$$\text{At the top of slab} = (1.24 + 1.33 - 2.8) = -0.23 \text{ N/mm}^2$$

$$\text{At the bottom of slab (junction)} = (1.24 + 0.72 - 2.8) = -0.84 \text{ N/mm}^2$$

The resultant shrinkage stress distribution is shown in Fig.

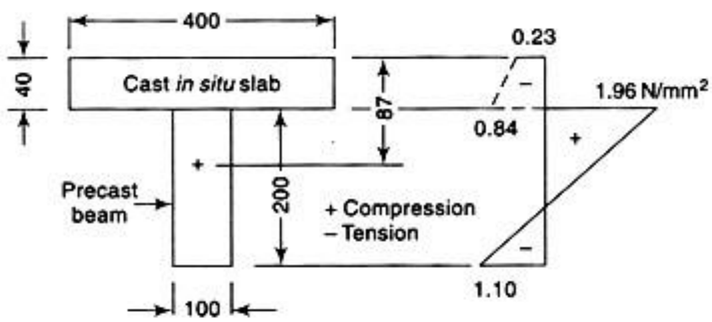

Fig. *Stresses due to Differential Shrinkage*

11. Explain the precast prestressed concrete stresses at serviceability limit state.

(AUC Nov/Dec 2012)

The maximum permissible stresses in the precast prestressed concrete and the *in situ* cast concrete are mainly governed by the compressive strength of concrete in the respective elements. In general, the permissible stresses in a precast prestressed concrete are governed by the normal rules used for prestressed concrete as detailed in Table 2.1. However, certain exceptions are made regarding high stresses developed at the interface of the precast and *in situ* cast elements. The British code BS: 8110 provides for a higher value of compressive stress, equal to $0.5 f_{cu}$, which is 50 per cent higher than the normally allowable value in prestressed elements.

The higher value of compressive stress is permissible only in composite sections with the stipulation that the failure of the section is due to excessive elongation of steel. This requirement is to safeguard against the explosive compressive failure of the concrete at the limit state of collapse. The permissible flexural tensile stress in the *in situ* concrete at the contact surface with the prestressed element, as prescribed in the British code BS: 8110 varies from 3.2 to 5.0 N/mm² corresponding to various grades of concrete shown in the following table.

Grade of in situ concrete	Allowable flexural tensile stress N/mm²
25	3.2
30	3.6
40	4.4
50	5.0

12. Design a composite slab for the bridge deck using a standard invested T-section. The top flange is 250 mm wide and 100 mm thick. The bottom flange is 500 mm wide and 250 mm thick. The web thickness is 100 mm and the overall depth of the inverted T-section is 655 mm. The bridge deck has to support a characteristic imposed load of 50 kN/m^2 over an effective span of 12 m. Grade 40 concrete is specified for the precast pretensioned T section with a compressive strength at transfer of 36 N/mm^2. Concrete of grade 30 is used for the insitu part. Determine the minimum prestress necessary and check for safety under serviceability limit state. Section properties: Area = 180500 mm^2, position of centroid = 220 mm from the soffit.I = 81.1 x 10^8 mm^4, Z_t = 18.7 x 10^6 mm^3, Z_b = 37 x 10^6 mm^3. Loss ratio = 0.8, M_{min} = 0. (AUC Nov/Dec 2012)

Solution:

The overall depth of the composite slab is estimated as = (655 + 95) = 750 mm

Load due to self-weight of the precast beam and the *in situ* concrete is given by,

$$g = 0.75 \times 0.5 \times 24 = 9 \text{ kN/m}$$

Corresponding moment, $M = (0.125 \times 9 \times 12^2) = 162$ kN m

Live load moment, $M' = (0.125 \times 0.5 \times 50 \times 12^2) = 450$ kN m

Permissible compressive stress in concrete at transfer according to the code is

$$0.5 f_{ci} = (0.5 \times 36) = 18 \text{ N/mm}^2$$

Permissible tensile stress in concrete under service loads, $f_{tw} = -1$ N/mm^2.

The minimum section modulus required for the composite section is given by

$$Z_b' \geq \left[\frac{37 \times 10^6 \times 450 \times 10^6}{37 \times 10^6(0.8 \times 18 + 1) - 10^6(162 - 0)}\right] \geq 41 \times 10^6 \text{ mm}^3$$

If h = total depth of composite slab

$$\left(\frac{bh^2}{6}\right) = 41 \times 10^6 \qquad \text{where } b = 500 \text{ mm}$$

$$h = 705 \text{ mm}$$

The total depth of 750 mm is adequate.

$$Z_b' = \frac{500 \times 750^2}{6} = 46.87 \times 10^6 \text{ mm}^3$$

The minimum prestress required is obtained as,

$$f_{inf} = -\frac{1}{0.8} + \frac{162 \times 10^6}{0.8 \times 37 \times 10^6} + \frac{450 \times 10^6}{0.8 \times 46.87 \times 10^6}$$

$$= 16.2 \text{ N/mm}^2$$

$$f_{sup} = (-1 - 0) = -1 \text{ N/mm}^2$$

The minimum prestressing force P is computed as,

$$P = A\frac{(f_{inf} Z_b + f_{sup} Z_t)}{(Z_t + Z_b)}$$

$$= \left[\frac{180500 \times 10^6 (16.2 \times 37 - 1 \times 18.7)}{(55.7 \times 10^6 \times 10^3)}\right]$$

$$P = 1881 \text{ kN}$$

The eccentricity of the prestressing force is,

$$e = \frac{Z_t Z_b (f_{inf} - f_{sup})}{A\,(f_{inf} Z_b + f_{sup} Z_t)}$$

$$= \frac{18.7 \times 37 \times 10^{12} (16.2 + 1)}{180500 \times 10^6 (16.2 \times 37 - 1 \times 8.7)} = 113 \text{ mm}$$

A suitable system of tendons is arranged with straight, parallel wires distributed in the bottom and top flanges so as to achieve the required eccentricity.

The stresses due to prestress and the loads developed in the precast and *in situ* cast concrete are shown in Fig.

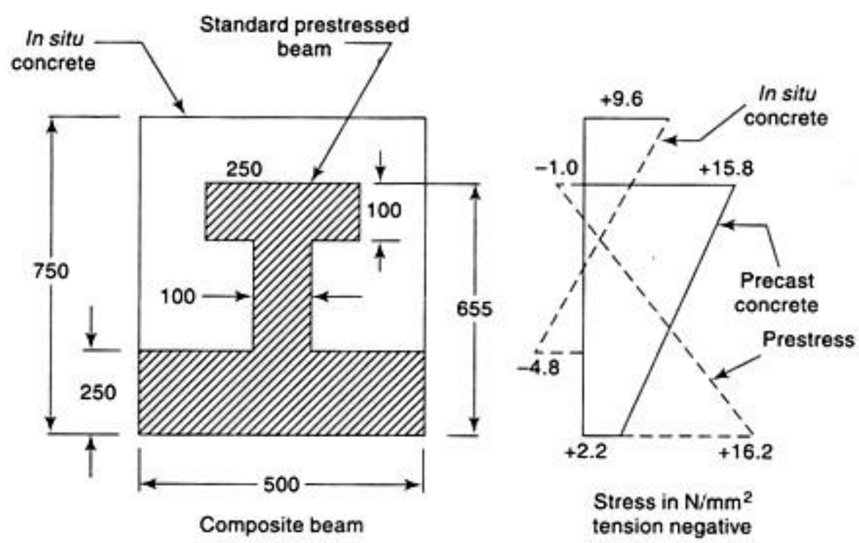

Fig. *Design of Composite Deck Slab Using a Standard Prestressed Beam*