QUESTION BANK

DEPARTMENT: CIVIL **SEMESTER: VI**

SUBJECT CODE / Name: CE 6602 / STRUCTURAL ANALYSIS-II

UNIT 5 - SPACE AND CABLE STRUCTURES

PART - A (2 marks)

1. **Give any two examples of beams curved in plan.** **(AUC Apr/May 2011)**

   Curved beams are found in the following structures.

   - Beams in a bridge negotiating a curve
   - Ring beams supporting a water tank
   - Beams supporting corner lintels
   - Beams in ramps

2. **What is the nature of forces in the cables?** **(AUC Apr/May 2011)**

   Cables of cable structures have only tension and no compression or bending.

3. **Define tension coefficient. For what type of structures tension coefficient method is employed?** **(AUC Nov/Dec 2011)**

   The tension coefficient for a member of a truss is defined as the pull or tension in the member divided by its length, i. e. the force in the member per unit length.

4. **What are the components of forces acting on the beams curved in plan and show the sign conventions of these forces?** **(AUC Nov/Dec 2011)**

   Beams curved in plan will have the following forces developed in them:

   - Bending moments
   - Shear forces
   - Torsional moments

5. **Define a space frame and what is the nature of joint provided in the space trusses?** **(AUC May/June 2012)**

   A space frame is a structure built up of hinged bars in space. It is three dimensional generalization of a truss.

   Socket joint is provided in the space trusses.

6. **What are the types of stiffening girders?** **(AUC May/June 2012)**

   - Suspension bridges with three hinged stiffening girders
   - Suspension bridges with two hinged stiffening girders

7. **What are the methods available for the analysis of space trusses?** **(AUC May/June 2013)**

   Tension co-efficient method is available for the analysis of space trusses.

8. **What is the need for cable structures?** **(AUC May/June 2013)**

   - The main load bearing member.
   - Flexible throughout.
   - It can take only direct tension and cannot take any bending moment.

9. **What are cable structures?**

Long span structures subjected to tension and uses suspension cables for supports. Examples of cable structures are suspension bridges, cable stayed roof.

10. **What is the true shape of cable structures?**

Cable structures especially the cable of a suspension bridge is in the form of a catenary. Catenary is the shape assumed by a string / cable freely suspended between two points.

11. **Mention the different types of cable structures.**

Cable structures are mainly of two types:
(a) Cable over a guide pulley
(b) Cable over a saddle

12. **Briefly explain cable over a guide pulley.**

Cable over a guide pulley has the following properties:

- Tension in the suspension cable = Tension in the anchor cable
- The supporting tower will be subjected to vertical pressure and bending due to net horizontal cable tension.

13. **Briefly explain cable over saddle.**

Cable over saddle has the following properties:

- Horizontal component of tension in the suspension cable = Horizontal component of tension in the anchor cable
- The supporting tower will be subjected to only vertical pressure due to cable tension.

14. **What are the main functions of stiffening girders in suspension bridges?**

Stiffening girders have the following functions.

- They help in keeping the cables in shape
- They resist part of shear force and bending moment due to live loads.

15. **Differentiate between plane truss and space truss.**

Plane truss:

- All members lie in one plane
- All joints are assumed to be hinged.

Space truss:

- This is a three dimensional truss
- All joints are assumed to be ball and socketed.

16. **What are the significant features of circular beams on equally spaced supports?**

- Slope on either side of any support will be zero.
- Torsional moment on every support will be zero

17. **Give the expression for calculating equivalent UDL on a girder.**

The tension developed in the cable is given by

$$T = \sqrt{H^2 + V^2}$$

Where, H = horizontal component and V = vertical component.

18. **Define tension co-efficient.**

The tension co-efficient for a member of a truss is defined as the pull or tension in that member divided by its length.

19. **What are cables made of?**

Cables can be of mild steel, high strength steel, stainless steel, or polyester fibres. Structural cables are made of a series of small strands twisted or bound together to form a much larger cable.

Steel cables are either spiral strand, where circular rods are twisted together or locked coil strand, where individual interlocking steel strands form the cable (often with a spiral strand core).

Spiral strand is slightly weaker than locked coil strand. Steel spiral strand cables have a Young's modulus, E of 150 ± 10 kN/mm² and come in sizes from 3 to 90 mm diameter. Spiral strand suffers from construction stretch, where the strands compact when the cable is loaded.

20. Give the types of significant cable structures

Linear structures:

- Suspension bridges
- Draped cables
- Cable-stayed beams or trusses
- Cable trusses
- Straight tensioned cables

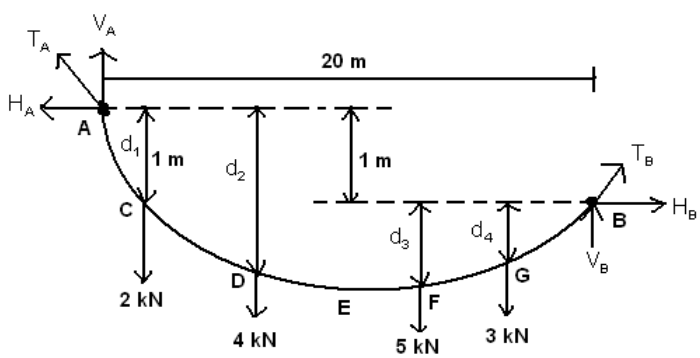
Three-dimensional structures:

- Bi-cycle roof
- 3D cable trusses
- Tensegrity structures
- Tensairity structures

PART - B (16 marks)

1. A suspension cable is supported at two point "A" and "B", "A" being one metre above "B". the distance AB being 20 m. the cable is subjected to 4 loads of 2 kN, 4 kN, 5 kN and 3 kN at distances of 4 m, 8 m, 12 m and 16 m respectively from "A". Find the maximum tension in the cable, if the dip of the cable at point of application of first loads is 1 m with respect to level at A. find also the length of the cable. (AUC Apr/May 2011)

Solution:



Step 1: Reactions:

$$\sum V = 0$$

$$V_A + V_B = 14$$

$$\sum M @ B = 0$$

$$(V_A x 20) - (H x 1) - (2 x 16) - (4 x 12) - (5 x 8) - (3 x 4) = 0$$

$$20 V_A - H - 132 = 0$$

$$V_A = 0.05H + 6.6 \text{ .... (1)}$$

$$\sum H = 0$$
$$H_A = H_B$$
$$\sum M \text{ @ } C = 0$$
$$(V_A \text{ x } 4) - (H \text{ x } 1) = 0$$
$$V_A = 0.25H \quad .... (2)$$
sub. (2) in (1),
$$0.25H = 0.05H + 6.6$$
$$H = 33kN$$
$$(1) \Rightarrow \quad V_A = 8.25 \text{ kN}$$
$$V_B = 5.75 \text{ kN}$$

Step 2: Maximum Tension in the cable:

$$T_A = \sqrt{V_A^2 + H^2} = \sqrt{8.25^2 + 33^2} = 34.02\,kN$$
$$T_B = \sqrt{V_B^2 + H^2} = \sqrt{5.75^2 + 33^2} = 33.49\,kN$$

Maximum Tension in the cable, $T_{max} = 34.09\,kN$.

Step 3: Length of the cable:

Here, $d_1 = 1m$

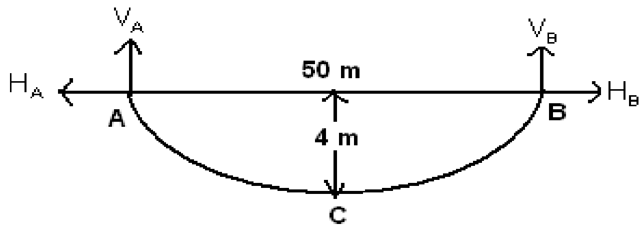
Equating moments about D to zero,
$$(8.25 \text{ x } 8) - (33 \text{ x } d_2) = 0$$
$$d_2 = 2\,m$$
Equating moments about D to zero,
$$(-5.75 \text{ x } 8) + (33 \text{ x } d_3) = 0$$
$$d_3 = 1.39\,m$$
Equating moments about D to zero,
$$(-5.75 \text{ x } 4) + (33 \text{ x } d_4) = 0$$
$$d_4 = 0.69\,m$$

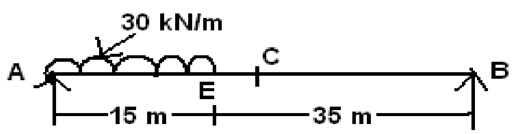
$$AC = \sqrt{4^2 + 1^2} = 4.12 \text{ m}$$
$$CD = \sqrt{4^2 + 2^2} = 4.47 \text{ m}$$
$$FG = \sqrt{4^2 + 1.39^2} = 4.23 \text{ m}$$
$$GB = \sqrt{4^2 + 0.69^2} = 4.06 \text{ m}$$

Length of the cable, $L = AC + CD + FG + BG + DF$
$$= 4.12 + 4.47 + 4.23 + 4.06 + 4$$
$$L = 20.88\,m$$

2. **A suspension bridge has a span 50 m with a 15 m wide runway. It is subjected to a load of 30 kN/m including self weight. The bridge is supported by a pair of cables having a central dip of 4 m. find the cross sectional area of the cable necessary if the maximum permissible stress in the cable materials is not to exceed 600 MPa. (AUC Nov/Dec 2011)**

Solution:





Step 1: Reactions:

$$\sum V = 0$$

$$V_A + V_B = 450$$

$$\sum M @ A = 0$$

$$-(V_B x 50) + \left(\frac{30 \text{ x } 15^2}{2}\right) = 0$$

$$V_B = 67.5 \text{ kN}$$

$$V_A = 382.5 \text{ kN}$$

$$\sum H = 0$$

$$H_A = H_B$$

$$\sum M @ C = 0$$

$$(V_A \text{ x } 25) - (H \text{ x } 4) - (30 \text{ x } 15 \text{ x} (7.5+10)) = 0$$

$$H = 421.87 \text{ kN}$$

Step 2: Maximum Tension in the cable:

$$T_A = \sqrt{V_A^2 + H^2} = \sqrt{382.5^2 + 421.87^2} = 569.46 \text{ kN}$$

$$T_B = \sqrt{V_B^2 + H^2} = \sqrt{67.5^2 + 421.87^2} = 427.24 \text{ kN}$$

Maximum Tension in the cable, $T_{max} = 569.46$ kN.

Step 3: Area:

$$T_{max} = \sigma . A$$

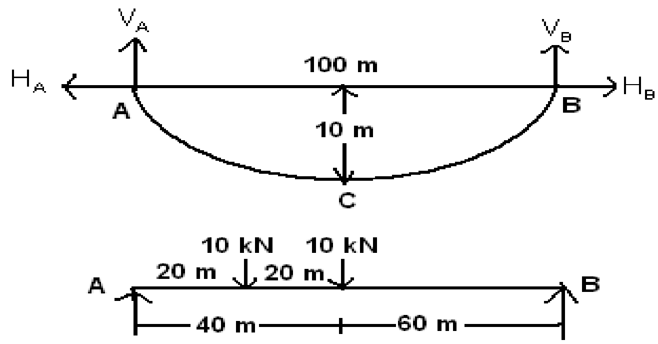
$$A = \frac{T_{max}}{\sigma} = \frac{569.46 \text{ x } 10^3}{600}$$

Area, $A = 949.1 \text{ mm}^2$.

3. A three hinged stiffening girder of a suspension bridge of 100 m span subjected to two point loads 10 kN each placed at 20 m and 40 m respectively from the left hand hinge. Determine the bending moment and shear force in the girder at section 30 m from each end. Also determine the maximum tension in the cable which has a central dip of 10 m.

(AUC May/June 2012)

Solution:



Step 1: Reactions:

$$\sum V = 0$$

$$V_A + V_B = 20$$

$$\sum M @ B = 0$$

$$(V_A \text{ x } 100) - (10 \text{ x } 80) - (10 \text{ x } 60) = 0$$

$$V_A = 14 \text{ kN}$$

$$V_B = 6 \text{ kN}$$

$$\sum H = 0$$

$$H_A = H_B$$

$$\sum M @ C = 0$$

$$(V_A \text{ x } 50) - (H \text{ x } 10) - (10 \text{ x } 30) - (10 \text{ x } 10) = 0$$

$$H = 30 \text{ kN}$$

Step 2: Shear force:

SF at 30 m from left hand hinge.

$$V_{30} = V_A - 10 - H \tan\theta$$

here,

$$\tan\theta = \frac{4d}{\ell^2}(\ell - 2x) = \frac{4 \text{ x} 10}{100^2}(100 - (2 \text{ x } 30))$$

$$\tan\theta = 0.16$$

$$V_{30} = 14 - 10 - (30 \text{ x } 0.16)$$

$$V_{30} = -0.8 \text{ kN}$$

SF at 30 m from right hand hinge.

$$V_{30} = V_B - H \tan\theta$$
$$= 6 - (30 \times 0.16)$$
$$V_{30} = 1.2 \text{ kN}$$

Step 3: Bending Moment:

BM at 30 m from left hand hinge.

$$BM_{30} = V_A \times 30 - H \times y - 10 \times 10$$

here, y at 30 m from each end,

$$y = \frac{4d}{\ell^2} \times X(\ell - X^2) = \frac{4 \times 10}{100^2} \times 30(100 - 30)$$
$$y = 8.4 \text{ m}$$
$$BM_{30} = (14 \times 30) - (30 \times 8.4) - 100 = 68 \text{ kNm.}$$

BM at 30 m from right hand hinge.

$$BM_{30} = -V_B \times 30 + H \times y$$
$$= -(6 \times 30) + (30 \times 8.4)$$
$$BM_{30} = 72 \text{ kNm.}$$

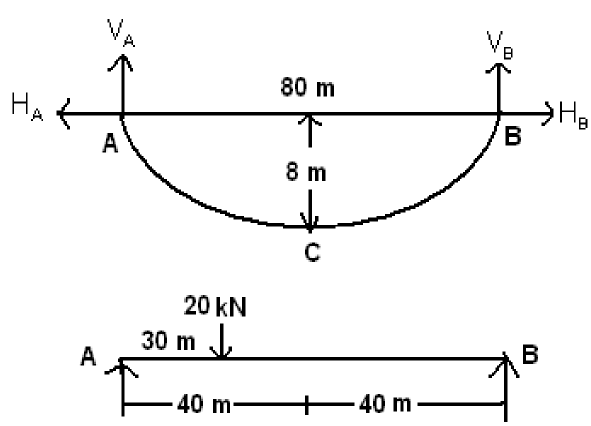
Step 4: Maximum Tension in the cable:

$$T_A = \sqrt{V_A^2 + H^2} = \sqrt{14^2 + 30^2} = 33.11 \text{kN}$$
$$T_B = \sqrt{V_B^2 + H^2} = \sqrt{6^2 + 30^2} = 30.59 \text{kN}$$

Maximum Tension in the cable, $T_{max} = 33.11$ kN.

4. A suspension bridge cable of span 80 m and central dip 8 m is suspended from the same level at two towers. The bridge cable is stiffened by a three hinged stiffening girder which carries a single concentrated load of 20 kN at a point of 30 m from one end. Sketch the SFD for the girder. (AUC May/June 2013)

Solution:



Step 1: Reactions:

$\sum V = 0$

$V_A + V_B = 20$

$\sum M @ B = 0$

$(V_A \text{ x } 80) - (20 \text{ x } 50) = 0$

$V_A = 12.5 \text{ kN}$

$V_B = 7.5 \text{ kN}$

$\sum H = 0$

$H_A = H_B$

$\sum M @ C = 0$

$(V_A \text{ x } 40) - (20 \text{ x } 10) - (H \text{ x } 8) = 0$

$H = 37.5 \text{ kN}$

Step 2: Shear force:

SF at 40 m from left hand hinge.

$V_{40} = V_A - 20 - H \tan\theta$

here,

$$\tan\theta = \frac{4d}{\ell^2}(\ell - 2x) = \frac{4 \text{ x } 8}{80^2}(80 - (2 \text{ x } 40))$$

$\tan\theta = 0$

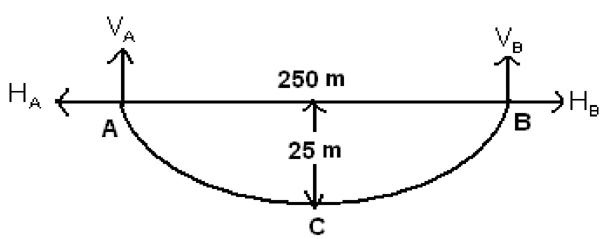
$V_{40} = 12.5 - 20 - (37.5 \text{ x } 0)$

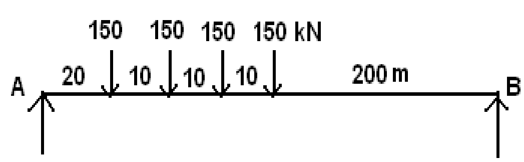
$V_{40} = -7.5 \text{ kN}$

5. A suspension bridge 0f 250 m span has two nos. of three hinged stiffening girders supported by cables with a central dip of 25 m. if 4 point loads of 300 kN each are placed at the centre line of the roadway at 20, 30, 40 and 50 m from left hand hinge. Find the shear force and bending moment in each girder at 62.5 m from each end. Calculate also the maximum tension in the cable.

Solution:

The load system is shared equally by the two girders and cables. Take the loads as 150 kN each.





Step 1: Reactions:

$$\sum V = 0$$

$$V_A + V_B = 600$$

$$\sum M @ B = 0$$

$$(V_A \text{ x } 250) - (150 \text{ x } 230) - (150 \text{ x } 220) - (150 \text{ x } 210) - (150 \text{ x } 200) = 0$$

$$V_A = 516 \text{ kN}$$

$$V_B = 84 \text{ kN}$$

$$\sum H = 0$$

$$H_A = H_B$$

$$\sum M @ C = 0$$

$$(V_A \text{ x } 125) - (H \text{ x } 25) - (150 \text{ x } 105) - (150 \text{ x } 95) - (150 \text{ x } 85) - (150 \text{ x } 75) = 0$$

$$H = 420 \text{ kN}$$

Step 2: Shear force:

SF at 62.5 m from left hand hinge.

$$V_{62.5} = V_A - 150 - 150 - 150 - 150 - H \tan\theta$$

here,

$$\tan\theta = \frac{4d}{\ell^2}(\ell - 2x) = \frac{4 \text{ x } 25}{250^2}(250 - (2 \text{ x } 62.5))$$

$$\tan\theta = 0.2$$

$$V_{62.5} = 516 - 150 - 150 - 150 - 150 - (420 \text{ x } 0.2)$$

$$V_{62.5} = -168 \text{ kN}$$

SF at 62.5 m from right hand hinge.

$$V_{62.5} = V_B - H \tan\theta$$

$$= 84 - (420 \text{ x } 0.2)$$

$$V_{62.5} = 0$$

Step 3: Bending Moment:

BM at 62.5 m from left hand hinge.

$$BM_{62.5} = V_A \text{ x } 62.5 - (150 \text{ x } 42.5) - (150 \text{ x } 32.5) - (150 \text{ x } 22.5) - (150 \text{ x } 12.5) - H \text{ x } y$$

here, y at 62.5 m from each end,

$$y = \frac{4d}{\ell^2} \text{ x } X(\ell - X^2) = \frac{4 \text{ x } 25}{250^2} \text{ x } 62.5(250 - 62.5)$$

$$y = 18.75 \text{ m}$$

$$BM_{62.5} = (516 \text{ x } 62.5) - (150 \text{ x } 42.5) - (150 \text{ x } 32.5) - (150 \text{ x } 22.5) - (150 \text{ x } 12.5) - (420 \text{ x } 18.75)$$

$$BM_{62.5} = 7875 \text{ kNm.}$$

BM at 62.5 m from right hand hinge.

$$BM_{62.5} = -V_B x\, 62.5 + H \text{ x } y$$

$$= -(84 \text{ x } 62.5) + (420 \text{ x } 18.75)$$

$$BM_{62.5} = 2625 \text{ kNm.}$$

Step 4: Maximum Tension in the cable:

Bending moment for the cable,

$$Hd = \frac{w\ell^2}{8}$$

$$w = \frac{H \text{ x } d \text{ x } 8}{\ell^2} = \frac{420 \text{ x } 25 \text{ x } 8}{250^2} = 1.344 \text{ kN/m}$$

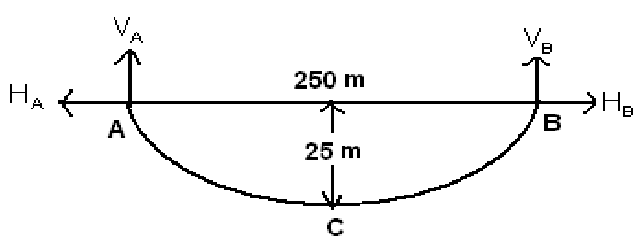
$$V_A = V_B = \frac{w\ell}{2} = \frac{1.344 \text{ x } 250}{2} = 168 \text{ kN}$$

$$T_{max} = \sqrt{V_A^2 + H^2} = \sqrt{168^2 + 420^2} = 452.35 \text{ kN}$$

Maximum Tension in the cable, $T_{max} = 452.35$ kN.

6. A suspension bridge is of 160 m span. The cable of the bridge has a dip of 12 m. the cable is stiffened by a three hinged girder with hinges at either end and at centre. The dead load of the girder is 15 kN/m. find the greatest positive and negative bending moments in the girder when a single concentrated load of 340 kN passes through it. Also find the maximum tension in the cable.

Solution:



Step1: Bending Moment :

The uniformly distributed dead load will not cause any bending moment in the stiffening girder. The live load is a single concentrated moving load.

$$\text{Max.} + \text{ve BM} = 0.096\, W\ell = 0.096 \text{ x } 340 \text{ x } 160$$

$$= 5222.4 \text{ kNm.}$$

This will occur at $0.211\ell = 0.211 \text{ x } 160$

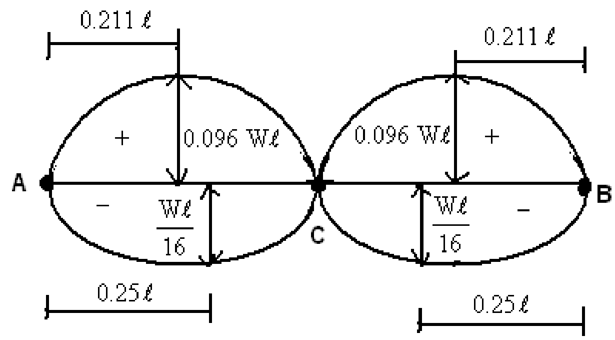
$$= 33.76 \text{ m from either end.}$$

$$\text{Max.} - \text{ve BM} = -\frac{W\ell}{16} = -\frac{340 \text{ x } 160}{16}$$

$$= -3400 \text{ kNm.}$$

This will occur at $0.25\ell = 0.25 \text{ x } 160$

$$= 40 \text{ m from either end.}$$



Step 2 : Maximum tension in the cable :

Dead load of the girder (transmitted to the cable directly)

$$p_d = 15 \text{ kN/m}$$

Equivalent udl transmitted to the cable due to the moving concentrated load,

$$p_\ell = \frac{2 \text{ x } 340}{160} = 4.25 \text{ kN/m}$$

Total load transmitted to the cable, $p = p_d + p_\ell = 15 + 4.25 = 19.25 \text{ kN/m}$

$$\text{Vertical reaction, } V = \frac{p\ell}{2} = \frac{19.25 \text{ x } 160}{2} = 1540 \text{ kN}$$

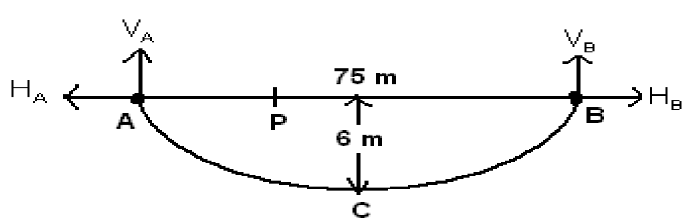
$$\text{Horizontal pull, } H = \frac{p\ell^2}{8d} = \frac{19.25 \text{ x } 160^2}{8 \text{ x } 12} = 5133.2 \text{ kN}$$

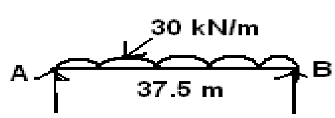
$$\text{Maximum tension, } T_{max} = \sqrt{V_A^{\ 2} + H^2} = \sqrt{1540^2 + 5133.3^2}$$

$$T_{max} = 5359.3 \text{ kN.}$$

7. A suspension cable of 75 m horizontal span and central dip 6 m has a stiffening girder hinged at both ends. The dead load transmitted to the cable including its own weight is 1500 kN. The girder carries a live load of 30 kN/m uniformly distributed over the left half of the span. Assuming the girder to be rigid, calculate the shear force and bending moment in the girder at 20 m from left support. Also calculate the maximum tension in the cable.

Solution:





$\ell = 75 \text{m}; \ d = 6 \text{m}; \ DL = 1500 \text{ kN}; \ LL = 30 \text{ kN/m}$

Since the girder is rigid, the live load is transmitted to the cable as an udl whatever the position of the load.

Horizontal force due to live load, $H_\ell = \frac{P\ell}{8d} = \frac{(30 \text{ x } 37.5) \text{ x } 75}{8 \text{ x } 6} = 1757.8 \text{ kN}$

Horizontal force due to dead load, $H_d = \frac{P\ell}{8d} = \frac{1500 \text{ x } 75}{8 \text{ x } 6} = 2343.8 \text{ kN}$

Total horizontal force, $H = H_\ell + H_d = 1757.8 + 2343.8 = 4101.6 \text{ kN}$

$$V_A = V_B = \frac{\text{Total load}}{2} = \frac{W_\ell + W_d}{2}$$

$$= \frac{(30 \text{ x } 37.5) + 1500}{2} = 1312.5 \text{ kN}$$

Maximum tension in the cable:

$$T_{max} = \sqrt{H^2 + V^2} = \sqrt{4101.6^2 + 1312.5^2}$$

$$T_{max} = 4306.5 \text{ kN}$$

Dip at $x = 20\text{m}$:

$$y = \frac{4d}{\ell^2} \text{ x } X(\ell - X^2) = \frac{4 \text{ x } 6}{75^2} \text{ x } 20(75 - 20) = 4.69\text{m}$$

$$\tan\theta = \frac{4d}{\ell^2}(\ell - 2x) = \frac{4 \text{ x } 6}{75^2} \text{ x } (75 - 2 \text{ x } 20) = 0.149$$

To find V_A and V_B:

$$V_A + V_B = 1125$$

Equating moments about A to zero

$$(V_B \text{ x } 75) - (30 \text{ x } 37.5 \text{ x } 18.75) = 0$$

$$V_B = 281.25 \text{ kN}$$

$$V_A = 843.75 \text{ kN}$$

Bending Moment at P:

$$BM_{20} = V_A \text{ x } 20 - H_\ell \text{ x } y - \frac{w\ell^2}{2}$$

$$= (843.75 \text{ x } 20) - (1757.8 \text{ x } 4.69) - \frac{30 \text{ x } 20^2}{2}$$

$$BM_{20} = 2630.92 \text{ kNm.}$$

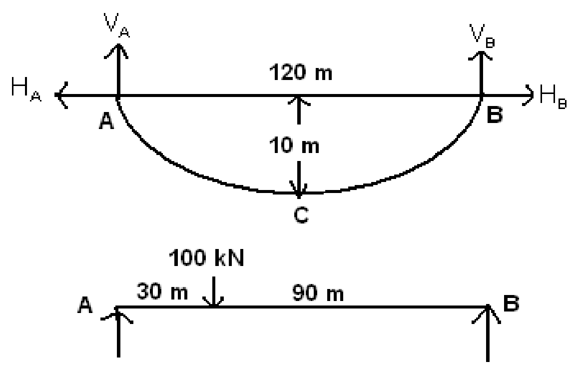
Shear force at P:

$$SF_{20} = V_A - H_\ell \text{ x } \tan\theta - w\ell = 843.75 - (1757.8 \text{ x } 0.149) - (30 \text{ x } 20)$$

$$SF_{20} = -18.16 \text{ kN.}$$

8. A suspension cable has a span of 120 m and a central dip of 10 m and is suspended from the same level at both towers. The bridge is stiffened by a stiffening girder hinged at the end supports. The girder carries a single concentrated load of 100 kN at a point 30 m from left end. Assuming equal tension in the suspension hangers. Calculate the horizontal tension in the cable and the maximum positive bending moment.

Solution:



Step 1: Reactions:

$$\sum V = 0$$

$$V_A + V_B = 100$$

$$\sum M @ A = 0$$

$$(100 \times 30) - (V_B \times 120) = 0$$

$$V_B = 25 \text{kN}$$

$$V_A = 75 \text{kN}$$

$$\sum H = 0$$

$$H_A = H_B$$

$$\sum M @ C = 0$$

$$-(V_B \times 60) + (H \times 10) = 0$$

$$H = 150 \text{kN}$$

Step 2: Maximum Tension in the cable:

Bending moment for the cable,

$$w = \frac{100}{\ell} = \frac{100}{120} = 0.83 \text{ kN/m}$$

$$V_A = V_B = \frac{w\ell}{2} = \frac{0.83 \times 120}{2} = 50 \text{kN}$$

$$T_{max} = \sqrt{V_A^2 + H^2} = \sqrt{50^2 + 150^2} = 158.1 \text{kN}$$

Maximum Tension in the cable, $T_{max} = 158.1 \text{kN}$.

$\text{Step 3: Maximum positive Bending Moment:}$

Maximum positive Bending moment will occur at under the point load.

$$BM_{30} = V_A \times 30 - H \times y$$

here, y at 30m from left end,

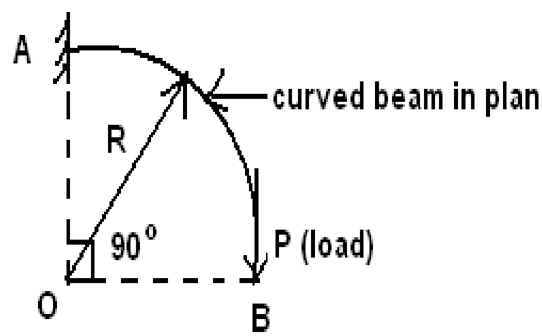
$$y = \frac{4d}{\ell^2} \times X(\ell - X^2) = \frac{4 \times 10}{120^2} \times 30(120 - 30)$$

$$y = 7.5\,m$$

$$BM_{30} = (75 \times 30) - (150 \times 7.5)$$

$$BM_{30} = 1125\,kNm.$$

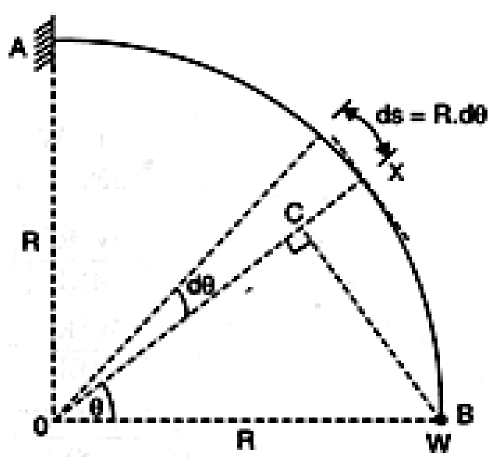
9. A quarter circular beam of radius 'R' curved in plan is fixed at A and free at B as shown in figure. It carries a vertical load P at its free end. Determine the deflection at free end and draw the bending moment and torsional moment diagrams. Assume flexural rigidity (EI) = torsional rigidity (GJ). (227) (AUC May/June 2012)



Solution:

The given cantilever is a statically determinate structure. Consider any point X on the beam at an angle θ from OB.

$$CX = R\,(1 - \cos\theta)$$



Step 1: Shear force:

SF at the section X, $F_o = W$

F_o is independent of θ and uniform throughout.

Step 2: Bending Moment:

BM at the section X, $M_\theta = -W(CB)$

$M_\theta = -W \cdot R\sin\theta$

At $\theta = 0$, $M_B = 0$

At $\theta = \frac{\pi}{2}$, $M_A = -WR$

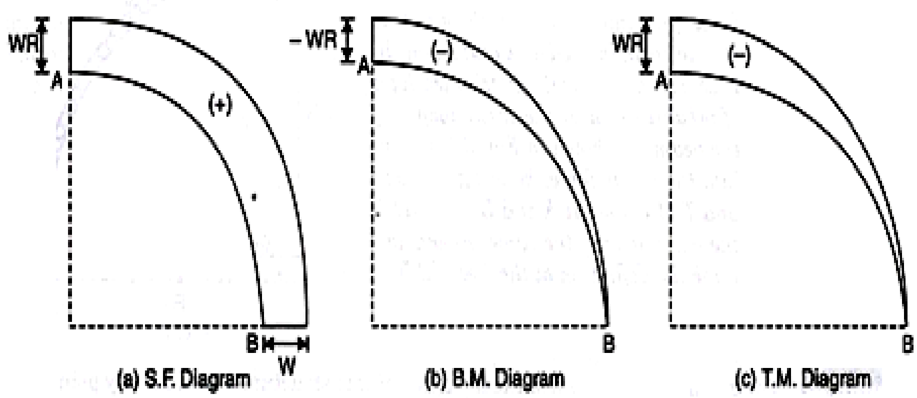
Step 3: Twisting Moment:

Twisting moment at the section X, $T_\theta = -W(CX)$

$T_\theta = -WR(1-\cos\theta)$

At $\theta = 0$, $T_B = -WR(1-\cos\theta) = 0$

At $\theta = \frac{\pi}{2}$, $T_A = -WR\left(1-\cos\frac{\pi}{2}\right) = -WR$



Step 4: Deflection at the free end B:

Method of strain energy is used to find the deflection at the free end B.

Strain energy, $U = \int \frac{M_\theta^2\, ds}{2\,EI} + \int \frac{T_\theta^2\, ds}{2\,GJ}$

$$= \frac{1}{2EI}\int_0^{\pi/2} (-WR\sin\theta)^2 R\,d\theta + \frac{1}{2GJ}\int_0^{\pi/2} [-WR(1-\cos\theta)]^2 R\,d\theta$$

$$= \frac{1}{2EI}\int_0^{\pi/2} (W^2R^2\sin^2\theta)R\,d\theta + \frac{1}{2GJ}\int_0^{\pi/2} [W^2R^2(1+\cos^2\theta-2\cos\theta)R\,d\theta$$

$$= \frac{1}{2EI}\,W^2R^3\int_0^{\pi/2}\left(\frac{1-\cos 2\theta}{2}\right)d\theta + \frac{1}{2GJ}\,x\,W^2R^3\int_0^{\pi/2}\left(1+\frac{1+\cos 2\theta}{2}-2\cos\theta\right)d\theta$$

$$=\frac{W^2R^3}{4EI}\int_0^{\pi/2} 1-\cos 2\theta \quad d\theta + \frac{W^2R^3}{4GJ}\; x \int_0^{\pi/2} 2+1+\cos 2\theta - 4\cos\theta \quad d\theta$$

$$=\frac{W^2R^3}{4EI}\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\pi/2} + \frac{W^2R^3}{4GJ}\; x \left[3\theta+\frac{\sin 2\theta}{2}-4\sin\theta\right]_0^{\pi/2}$$

$$=\frac{W^2R^3}{4EI}\left[\frac{\pi}{2}\right] + \frac{W^2R^3}{4GJ}\left[\frac{3\pi}{2}-4\right]$$

$$U = \frac{\pi W^2R^3}{8EI} + \frac{W^2R^3}{8GJ}(3\pi-8)$$

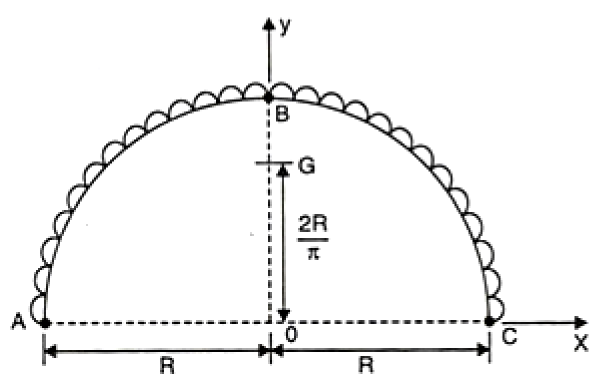
$$\delta_B = \frac{dU}{dW}$$

$$\delta_B = \frac{\pi WR^3}{4EI} + \frac{WR^3}{4GJ}(3\pi-8)$$

10. A semicircular beam of radius 'R' in plan is subjected to udl and simply supported by three columns spaced equally. Derive the expression for bending moment and torsional moment at x be a point on the beam making an angle a' with axis passing through the base of the circle. (AUC Apr/May 2011) (AUC May/June 2013) (AUC Nov/Dec 2011)

Solution:

The curved beam given is shown in Fig. 6.10. The XX and YY axes are as shown and ZZ is the vertical axis, The unknown reactions are V_A, V_B and V_C. The end supported do not exert any moment reaction. There are three equations of static equilibrium, (*i.e.*) $\Sigma M_{xx} = 0$, $\Sigma M_{yy} = 0$ and $\Sigma F_{zz} = 0$. Hence the structure is externally determinate.



Total load on the beam = $w \times \pi R$

Vertical support reactions:

The structure, loading and support conditions are symmetric with respect to the YY axis

Hence $V_A = V_C$

Taking moments of all forces about a tangent at point B,

$$2(V_A \times R) - \pi wR\left(R - \frac{2R}{\pi}\right) = 0$$

$$V_A = \frac{1}{2R} \times \pi\, wR\left(\frac{\pi R - 2R}{\pi}\right)$$

$$= \frac{w}{2}\, R\,(\pi - 2)$$

$$V_A = \frac{(\pi - 2)}{2}\, wR$$

$$V_B = \text{Total load} - 2\, V_A$$

$$= \pi\, wR - 2x\,\frac{(\pi - 2)}{2}\, wR$$

$$= wR\,[\pi - (\pi - 2)]$$

$$V_B = 2wR$$

Bending moment and twisting moment :

Consider a section X located at an angle θ with OA

Take a segment $Rd\,\phi$ at an angle ϕ from x :

Bending moment, M_θ at x :

$$M_\theta = V_A \times (AN) - \int_0^\theta wR\, d\phi\, R \sin\phi$$

Using equation 6.9

$$M_\theta = \frac{(\pi - 2)}{2} wR\, R \sin\theta - \int_0^\theta wR\, d\phi\, R\sin\phi$$

$$= \frac{(\pi - 2)}{2} wR^2 \sin\theta - \int_0^\theta wR^2 \sin\phi\, d\phi$$

$$= \frac{(\pi - 2)}{2} wR^2 \sin\theta - wR^2 (1 - \cos\theta)$$

$$M_\theta = wR^2\left[\frac{(\pi - 2)}{2}\sin\theta - (1 - \cos\theta)\right]$$

At $\theta = 0$,

$$M_A = wR^2\left[\frac{(\pi - 2)}{2}\sin 0 - (1 - \cos 0)\right]$$

$$M_A = 0 \text{ (hinged end)}$$

At $\theta = \frac{\pi}{2}$,

$$M_B = wR^2\left[\frac{(\pi - 2)}{2}\sin\frac{\pi}{2} - \left(1 - \cos\frac{\pi}{2}\right)\right]$$

$$= wR^2\left[\frac{(\pi - 2)}{2} \times 1 - (1 - 0)\right]$$

$$= wR^2\left[\frac{(\pi - 2)}{2} - 1\right]$$

$$= -0.429\, wR^2$$

$$M_B = -0.429\, wR^2 \qquad \text{...(6.12)}$$

Therefore moment at B = $0.429\, wR^2$ (Hogging)

Maximum bending moment might occur between A and B (and also between B and C by symmetry

For M_θ to be maximum

$$\frac{dM_\theta}{d\theta} = 0$$

$$\frac{d}{d\theta}\left(wR^2\left[\frac{(\pi-2)}{2}\sin\theta - (1-\cos\theta)\right]\right) = 0$$

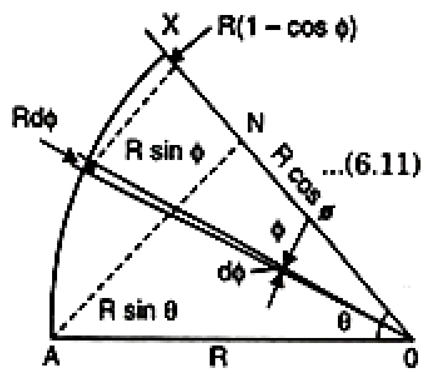
$$\frac{(\pi-2)}{2}\cos\theta - (0+\sin\theta) = 0$$

$$\frac{(\pi-2)}{2}\cos\theta - \sin\theta = 0$$

$$\frac{(\pi-2)}{2} = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{(\pi-2)}{2}$$

For M_{max}, $\theta = 29°43'$



Therefore, M_{max} $= wR^2\left[\frac{(\pi-2)}{2}\sin 29°\,43' - (1-\cos 29°\,43')\right]$

$= wR^2\,[0.2830 - 0.1315]$

$M_{max} = 0.1515\,wR^2$ sagging ...(6.14)

Maximum sagging moment = $0.1515\,wR^2$ and this will occur at $\theta = 29°43'$ from OA (OC)

Point of contraflexure :

At the point of contraflexure, $M_\theta = 0$

$$wR^2\left[\frac{(\pi-2)}{2}\sin\theta - (1-\cos\theta)\right] = 0$$

$$\frac{(\pi-2)}{2}\sin\theta - 1 + \cos = 0$$

$$\frac{(1-\cos\theta)}{\sin\theta} = \frac{(\pi-2)}{2}$$

$\theta = 59°27'$, by trial and error

Therefore the point of contraflexure occurs at 59°27′ from OA (and from OC)

The bending moment diagram is as shown in Fig. 6.12

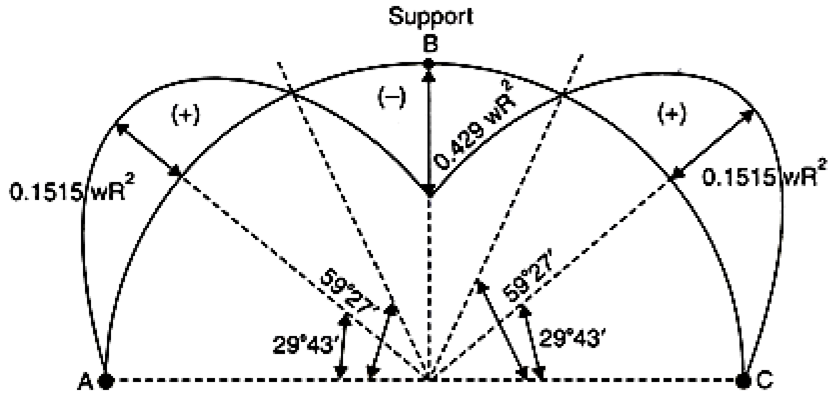


Fig. 6.12 B.M. Diagram

Twisting moment, T_θ at X :

$$T_\theta = -V_A (XN) + \int_0^\theta wR\, d\phi\, (1 - \cos\phi)$$

$$= -\frac{(\pi-2)}{2} wR\, R\,(1-\cos\theta) + \int_0^\theta wR^2 (1-\cos\phi)\, d\phi$$

$$= -\frac{(\pi-2)}{2} wR^2 (1-\cos\theta) + wR^2(\theta - \sin\theta)$$

$$T_\theta = wR^2\left[-\frac{(\pi-2)}{2}(1-\cos\theta) + \theta - \sin\theta\right]$$

when $\theta = 0$, $\quad T_A = wR^2\left[-\frac{(\pi-2)}{2}(1-1) + 0 - 0\right]$

when $\theta = \frac{\pi}{2}$, $\quad T_B = wR^2\left[-\frac{(\pi-2)}{2}(1-0) + \frac{\pi}{2} - 1\right]$

$$= wR^2\left[-\frac{(\pi-2)}{2} + \frac{(\pi-2)}{2}\right] = 0$$

For maximum value of Torsional moment,

$$\frac{dT_\theta}{d\theta} = 0$$

i.e. $\quad \frac{d}{d\theta}\left[wR^2\left(-\frac{(\pi-2)}{2}(1-\cos\theta) + \theta - \sin\theta\right)\right] = 0$

$$wR^2\left[-\frac{(\pi-2)}{2}(\sin\theta) + 1 - \cos\theta\right] = 0$$

$$1 - \cos\theta = \frac{(\pi-2)}{2}\sin\theta$$

$$\frac{1-\cos\theta}{\sin\theta} = \frac{(\pi-2)}{2}$$

$$\theta = 59°27'$$

$$T_{max} = wR^2\left[-\frac{(\pi-2)}{2}(1 - \cos 59°27') + 59°27' \times \frac{\pi}{180} - \sin 59°27'\right]$$

$$= wR^2\,[-0.2807 + 1.0376 - 0.8612]$$

$$= -0.1043\, wR^2$$

$$T_{max} = -0.1043\, wR^2 \qquad \ldots(6.15)$$

The twisting moment diagram is as shown in Fig. 6.13.

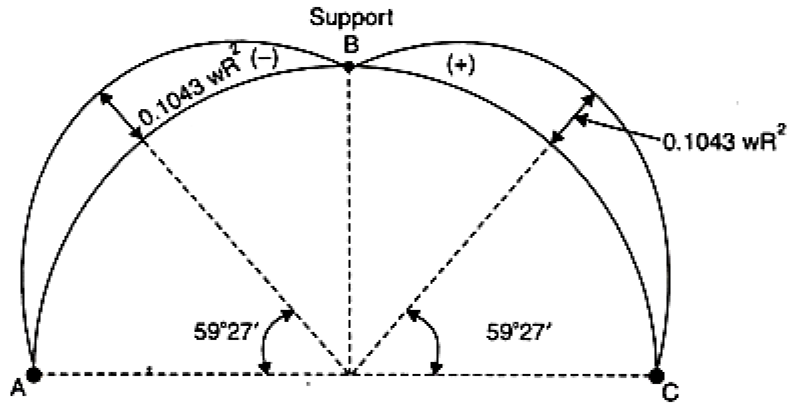


Fig. 6.13 T.M. Diagram

It is observed that the twisting moment is maximum at the section where the bending moment is zero.