



QUESTION BANK

DEPARTMENT: CIVIL

SEMESTER: VI

SUBJECT CODE / Name: CE 6602 / STRUCTURAL ANALYSIS - II

UNIT 4 - PLASTIC ANALYSIS OF STRUCTURES

PART - A (2 marks)

1. **What is shape factor?** (AUC Apr/May 2011) (AUC Nov/Dec 2011)
The shape factor is defined as the ratio of the plastic moment of a section to the yield moment of the section.
2. **State upper bound theorem.** (AUC Apr/May 2011) (AUC May/June 2013)
Upper bound theorem states that "A load computed on the basis of an assumed mechanism is always greater than or equal to the true ultimate load".
3. **Define plastic modulus.** (AUC Nov/Dec 2011)
The plastic modulus of a section is the first moment of the area above and below the equal area axis. It is the resisting modulus of a fully plasticized section.
$$Z_p = \frac{A}{2}(y_1 + y_2)$$
4. **What are meant by load factor and collapse load?** (AUC Nov/Dec 2011 & May/June 2012)
Load factor:
Load factor is defined as the ratio of collapse load to working load.
$$\text{Load factor, } \lambda = \frac{\text{collapse load}}{\text{working load}} = \frac{W_c}{W}$$

Collapse load:
The load that causes the (n + 1) the hinge to form a mechanism is called collapse load where n is the degree of statically indeterminacy. Once the structure becomes a mechanism.
5. **Define plastic hinge with an example.** (AUC May/June 2012 & 2013)
When a section attains full plastic moment M_p , it acts as hinge which is called a plastic hinge. It is defined as the yielded zone due to bending at which large rotations can occur with a constant value of plastic moment M_p .
6. **What is difference between plastic hinge and mechanical hinge?**
Plastic hinges modify the behavior of structures in the same way as mechanical hinges. The only difference is that plastic hinges permit rotation with a constant resisting moment equal to the plastic moment M_p . At mechanical hinges, the resisting moment is equal to zero.
7. **List out the assumptions made for plastic analysis.**
The assumptions for plastic analysis are:
 - Plane transverse sections remain plane and normal to the longitudinal axis before and after bending.

- Effect of shear is neglected.
- The material is homogeneous and isotropic both in the elastic and plastic state.
- Modulus of elasticity has the same value both in tension and compression.
- There is no resultant axial force in the beam.
- The cross-section of the beam is symmetrical about an axis through its centroid and parallel to the plane of bending.

8. List out the shape factors for the following sections.

- Rectangular section, $S = 1.5$
- Triangular section, $S = 2.346$
- Circular section, $S = 1.697$
- Diamond section, $S = 2$

9. Mention the section having maximum shape factor.

The section having maximum shape factor is a triangular section, $S = 2.345$.

10. State lower bound theory.

Lower bound theory states that the collapse load is determined by assuming suitable moment distribution diagram. The moment distribution diagram is drawn in such a way that the conditions of equilibrium are satisfied.

11. What are the different types of mechanisms?

The different types of mechanisms are:

- Beam mechanism
- Column mechanism
- Panel or sway mechanism
- Cable mechanism
- Combined or composite mechanism

12. Mention the types of frames.

Frames are broadly of two types:

- Symmetric frames
- Un-symmetric frames

13. What are symmetric frames and how they analyzed?

Symmetric frames are frames having the same support conditions, lengths and loading conditions on the columns and beams of the frame. Symmetric frames can be analyzed by:

- Beam mechanism
- Column mechanism

14. What are unsymmetrical frames and how are they analyzed?

Un-symmetric frames have different support conditions, lengths and loading conditions on its columns and beams. These frames can be analyzed by:

- Beam mechanism
- Column mechanism
- Panel or sway mechanism
- Combined mechanism

15. How is the shape factor of a hollow circular section related to the shape factor of a ordinary circular section?

The shape factor of a hollow circular section = A factor $K \times$ shape factor of ordinary circular section. SF of hollow circular section = SF of circular section $\times \{(1 - c^3)/(1 - c^4)\}$

16. Give the governing equation for bending.

The governing equation for bending is given by

$$\frac{M}{I} = \frac{\sigma}{y}$$

Where M = Bending moment

I = Moment of inertia

σ = Stress

y = C.G. distance

17. Give the theorems for determining the collapse load.

The two theorems for the determination of collapse load are:

- Static Method [Lower bound Theorem]
- Kinematic Method [Upper bound Theorem]

18. What is a mechanism?

When a n-degree indeterminate structure develops n plastic hinges, it becomes determinate and the formation of an additional hinge will reduce the structure to a mechanism. Once a structure becomes a mechanism, it will collapse.

19. What are the assumptions made in fully plastic moment of a section?

- Plane traverse sections remain plane and normal to the longitudinal axis after bending, the effect of shear being neglected.
- Modulus of elasticity has the same value in tension and compression.
- The material is homogeneous and isotropic in both the elastic and plastic state.
- There is no resultant axial force on the beam. i.e., total compression = total tension.
- The cross-section of the beam is symmetrical about an axis through its centroid parallel to the plane of bending.
- Longitudinal fibres are free to expand and contract without affecting the fibres in the lateral dimension.

20. What are the limitations of load factor concept?

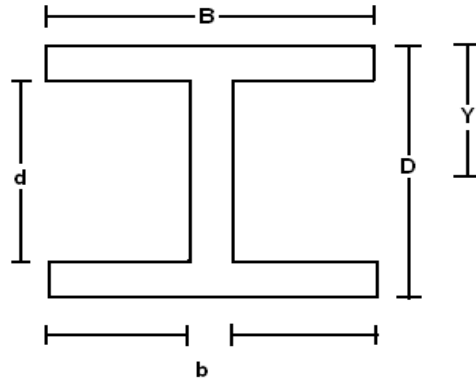
- The analysis procedure does not give us any clue if at a load W_u / load factor the structure behaves well.
- The stresses are within limit, so we have to check the stresses at crucial points by conventional elastic method.
- This is a peculiar and unrealistic assumption.
- The assumption of monotonic increase in loading is a simplistic.

PART - B (16 marks)

1. Derive the shape factor for I section and circular section.

(AUC Apr/May 2011)

I section:



Shape factor, $S = \frac{Z_p}{Z} = \frac{\text{Plastic modulus}}{\text{Elastic modulus}}$

Elastic modulus (Z):

$$Z = \frac{I}{Y}$$

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$Y = \frac{D}{2}$$

$$Z = \frac{\left(\frac{BD^3}{12} - \frac{bd^3}{12} \right)}{\left(\frac{D}{2} \right)} = \left(\frac{BD^3}{12} - \frac{bd^3}{12} \right) \times \frac{2}{D}$$

$$Z = \frac{BD^3 - bd^3}{6D}$$

Plastic modulus (Z_p):

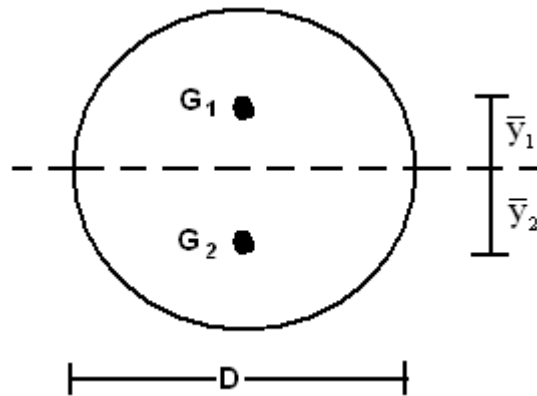
$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$$

$$A = 2(b_1 d_1) + b_2 d_2$$

$$\bar{y}_1 = \bar{y}_2 = \frac{a_1 \bar{y}_1 + a_2 \bar{y}_2}{a_1 + a_2}$$

$$S = \frac{Z_p}{Z} = \frac{\frac{A}{2} (\bar{y}_1 + \bar{y}_2)}{\left(\frac{BD^3 - bd^3}{6D} \right)}$$

Circular Section:



$$\text{Shape factor, } S = \frac{Z_p}{Z} = \frac{\text{Plastic modulus}}{\text{Elastic modulus}}$$

Elastic modulus (Z):

$$Z = \frac{I}{y} = \frac{\left(\frac{\pi D^4}{64} \right)}{\frac{D}{2}}$$

$$Z = \frac{\pi D^3}{32}$$

Plastic modulus (Z_p):

$$Z_p = \frac{A}{2} \bar{y}_1 + \bar{y}_2$$

$$A = \frac{\pi D^2}{4}$$

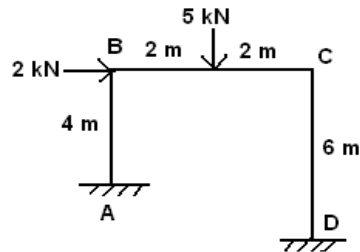
$$\bar{y}_1 = \bar{y}_2 = \frac{4r}{3\pi} = \frac{2D}{3\pi}$$

$$Z_p = \frac{\pi D^2}{4 \times 2} \left(\frac{2D}{3\pi} + \frac{2D}{3\pi} \right) = \frac{\pi D^2}{8} \times \frac{4D}{3\pi} = \frac{D^3}{6}$$

$$S = \frac{Z_p}{Z} = \frac{\left(\frac{D^3}{6} \right)}{\left(\frac{\pi D^3}{32} \right)} = \frac{D^3}{6} \times \frac{32}{\pi D^3} = \frac{32}{6\pi}$$

$$S = 1.697$$

2. Find the fully plastic moment required for the frame shown in figure, if all the members have same value of M_p .
(AUC Apr/May 2011)



Solution:

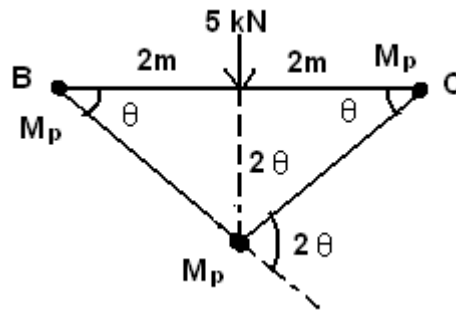
Step 1: Degree of indeterminacy:

$$\begin{aligned}\text{Degree of indeterminacy} &= (\text{No. of closed loops} \times 3) - \text{No. of releases} \\ &= (1 \times 3) - 0 = 3\end{aligned}$$

$$\text{No. of possible plastic hinges} = 5$$

$$\text{No. of independent mechanisms} = 5 - 3 = 2$$

Step 2: Beam Mechanism:



$$\text{EWD} = 5(2\theta) = 10\theta$$

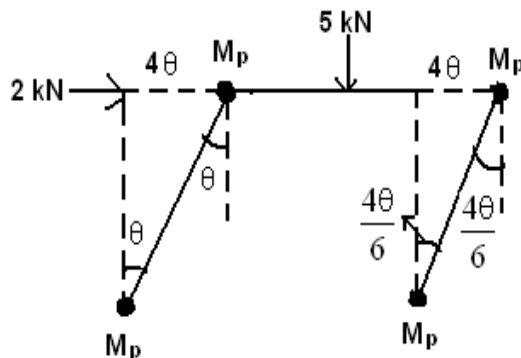
$$\text{IWD} = M_p \theta + 2M_p \theta + M_p \theta = 4M_p \theta$$

$$\text{EWD} = \text{IWD}$$

$$10\theta = 4M_p \theta$$

$$M_p = 2.5 \text{ kN.m}$$

Step 3: Sway Mechanism:



$$EWD = (2 \times 4\theta) = 8\theta$$

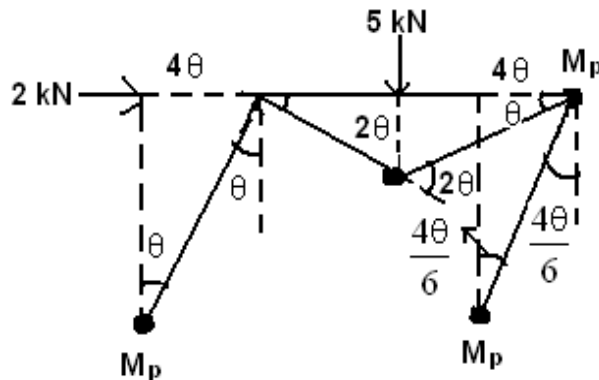
$$IWD = M_p \theta + M_p \theta + M_p \left(\frac{4\theta}{6} \right) + M_p \left(\frac{4\theta}{6} \right) = 3.33 M_p \theta$$

$$EWD = IWD$$

$$8\theta = 3.33 M_p \theta$$

$$M_p = 2.4 \text{ kN.m}$$

Step 4: Combined Mechanism:



$$EWD = (2 \times 4\theta) + (5 \times 2\theta) = 18\theta$$

$$IWD = M_p \theta + M_p (2\theta) + M_p \left(\theta + \frac{4\theta}{6} \right) + M_p \left(\frac{4\theta}{6} \right) = 5.33 M_p \theta$$

$$EWD = IWD$$

$$18\theta = 5.33 M_p \theta$$

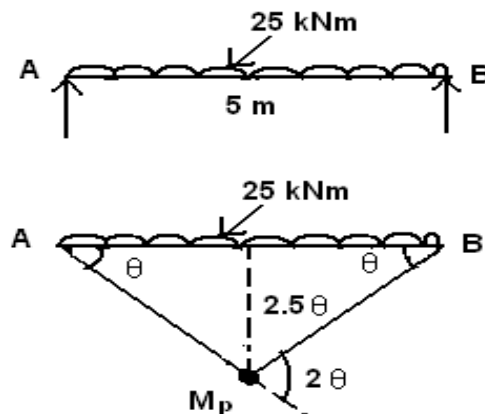
$$M_p = 3.38 \text{ kN.m}$$

The fully plastic moment, $M_p = 3.38 \text{ kNm}$.

3. A simply supported beam of span 5 m is to be designed for an udl of 25 kN/m. Design a suitable I section using plastic theory, assuming yield stress in steel as $f_y = 250 \text{ N/mm}^2$.

(AUC Nov/Dec 2011)

Solution:



$$IWD = 0 + M_p(2\theta) + 0 = 2M_p\theta$$

EWD = Load intensity X area of triangle under the load

$$= 25 \times \left(\frac{1}{2} \times 5 \times 2.5\theta \right)$$

$$= 156.25\theta$$

$$IWD = EWD$$

$$2M_p\theta = 156.25\theta$$

$$M_p = 78.125 \text{ kNm}$$

W.K.T.,

$$M_p = \sigma_y \times Z_p$$

$$Z_p = \frac{M_p}{\sigma_y} = \frac{78.125 \times 10^6}{250} = 3.12 \times 10^5 \text{ mm}^3$$

Assuming the shape factor for I-section as 1.15

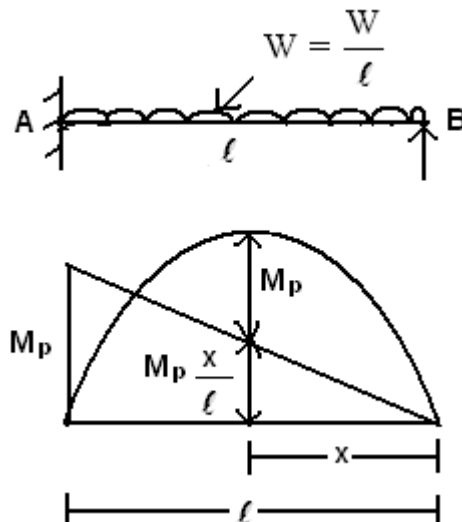
$$S = \frac{Z_p}{Z}$$

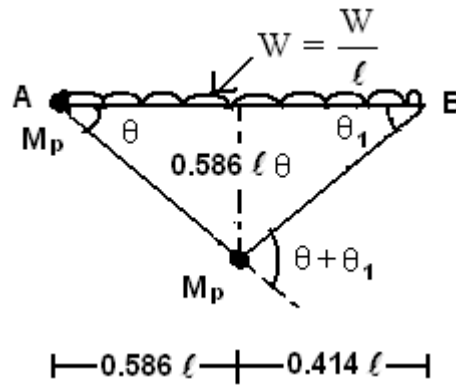
$$Z = \frac{Z_p}{S} = \frac{3.12 \times 10^5}{1.15} = 271.74 \times 10^3 \text{ mm}^3.$$

Adopt ISLB 250 @ 279 N / m (from steel table)

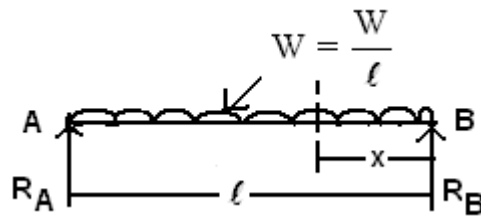
4. Analyse a propped cantilever of length 'L' and subjected to udl of w/m length for the entire span and find the collapse load. (AUC Nov/Dec 2011)

Solution:





Consider the moment at A as redundant and that it reaches M_p . the second hinge will form where the net positive BM is maximum.



$$\sum V = 0$$

$$R_A + R_B = W_c$$

$$R_A = R_B = \frac{W_c}{2}$$

$$M_x = \frac{W_c X}{2} - \frac{W_c X^2}{2\ell}$$

$$M_p + \frac{M_p X}{\ell} = \frac{W_c X}{2} - \frac{W_c X^2}{2\ell}$$

$$M_p \left(1 + \frac{X}{\ell} \right) = \frac{W_c X}{2} \left(1 - \frac{X}{\ell} \right)$$

$$M_p \left(\frac{\ell + X}{\ell} \right) = \frac{W_c X}{2} \left(\frac{\ell - X}{\ell} \right)$$

$$M_p = \frac{W_c X}{2} \left(\frac{\ell - X}{\ell + X} \right) = \frac{W_c}{2} \left(\frac{\ell X - X^2}{\ell + X} \right)$$

For M_p to be maximum, $\frac{dM_p}{dx} = 0$

$$\frac{dM_p}{dx} = \frac{W_c}{2} \left[\frac{(\ell + x)(\ell - 2x) - (\ell x - x^2)(1)}{(\ell + x)^2} \right] = 0$$

$$\begin{aligned}
 (\ell + x)(\ell - 2x) - (\ell x - x^2) &= 0 \\
 \ell^2 - 2\ell x + x\ell - 2x^2 - \ell x + x^2 &= 0 \\
 \ell^2 - 2\ell x - x^2 &= 0 \\
 x^2 + 2\ell x - \ell^2 &= 0 \\
 x &= \frac{-2\ell \pm \sqrt{8\ell^2}}{2} \\
 x &= 0.414\ell
 \end{aligned}$$

Mechanism :

$$0.586\ell\theta = 0.414\ell\theta_1$$

$$\theta_1 = 1.4155\theta$$

$$\theta + \theta_1 = \theta + 1.4155\theta = 2.4155\theta$$

$$EWD = \frac{W_c}{\ell} \times \frac{1}{2} \times \ell \times 0.586\ell\theta = 0.293W_c\ell\theta$$

$$IWD = M_p\theta + M_p(2.4155\theta) + 0 = 3.4155M_p\theta$$

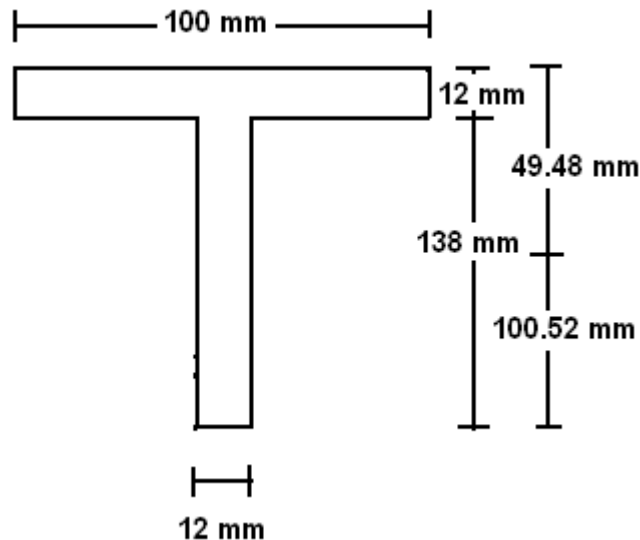
$$EWD = IWD$$

$$0.293W_c\ell\theta = 3.4155M_p\theta$$

$$W_c = \frac{11.66M_p}{\ell}$$

5. Determine the shape factor of a T-section beam of flange dimension 100 x 12 mm and web dimension 138 x 12 mm thick. (AUC May/June 2012)

Solution:



$$\text{Shape factor, } S = \frac{Z_p}{Z} = \frac{\text{Plastic modulus}}{\text{Elastic modulus}}$$

i) Elastic modulus (Z_e):

$$\bar{y}_t = \frac{(100 \times 12 \times 6) + (12 \times 138 \times 81)}{(100 \times 12) + (12 \times 138)} = 49.48 \text{ mm}$$

$$\bar{y}_b = 150 - 49.48 = 100.52 \text{ mm}$$

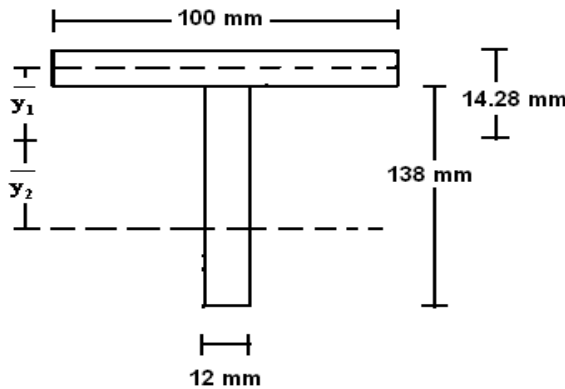
$$I_{xx} = \left[\frac{b_1 d_1^3}{12} + A_1 h_1^2 \right] + \left[\frac{b_2 d_2^3}{12} + A_2 h_2^2 \right]$$

$$= \left[\frac{100 \times 12^3}{12} + (100 \times 12 \times 43.48^2) \right] + \left[\frac{12 \times 138^3}{12} + (10 \times 138 \times 31.52^2) \right]$$

$$I_{xx} = 6.27 \times 10^6 \text{ mm}^4$$

$$Z_e = \frac{I}{y_{\max}} = \frac{6.27 \times 10^6}{100.52} = 62375.65 \text{ mm}^3$$

ii) Plastic modulus :



Equal area axis,

$$\frac{A}{2} = \text{width of the flange} \times h$$

$$\frac{2856}{2} = 100 h$$

$$h = 14.28 \text{ mm (from top)}$$

$$\bar{y}_1 = \frac{(100 \times 12 \times (6 + 2.28)) + (12 \times 135.72 \times 67.86)}{(100 \times 12) + (12 \times 135.72)} = 42.58 \text{ mm}$$

$$\bar{y}_2 = \frac{107.42}{2} = 53.71 \text{ mm}$$

$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2) = \frac{2856}{2} (42.58 + 53.71)$$

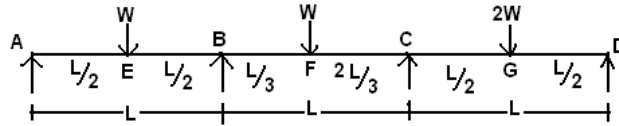
$$Z_p = 137502.12 \text{ mm}^3$$

Shape factor,

$$S = \frac{Z_p}{Z} = \frac{137502.12}{62375.65}$$

$$S = 2.20$$

6. Determine the collapse load 'W' for a three span continuous beam of constant plastic moment ' M_p ' loaded as shown in figure. (AUC May/June 2012)



Solution:

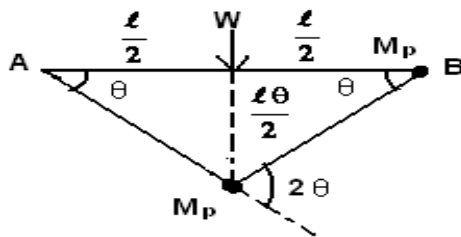
Step 1: Degree of indeterminacy:

$$\text{Degree of indeterminacy} = 4 - 2 = 2$$

$$\text{No. of possible plastic hinges} = 5$$

$$\text{No. of independent mechanisms} = 5 - 2 = 3$$

Step 2: Mechanism (1):



$$EWD = W \times \frac{l\theta}{2} = \frac{Wl\theta}{2}$$

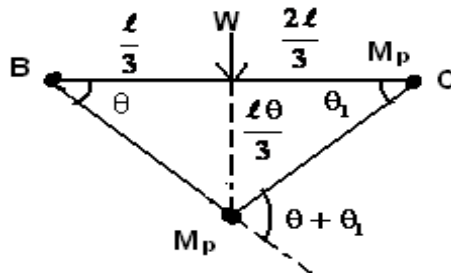
$$IWD = M_p(2\theta) + M_p\theta = 3M_p\theta$$

$$IWD = EWD$$

$$3M_p\theta = \frac{Wl\theta}{2}$$

$$W_c = \frac{6M_p}{l}$$

Step 3: Mechanism (2):



$$\frac{\ell \theta}{3} = \frac{2 \ell \theta_1}{3}$$

$$\theta_1 = \frac{\theta}{2}$$

$$\theta + \theta_1 = \theta + \frac{\theta}{2} = \frac{3\theta}{2}$$

$$\text{EWD} = W \times \frac{\ell \theta}{3} = \frac{W \ell \theta}{3}$$

$$\text{IWD} = M_p \theta + M_p (\theta + \theta_1) + M_p \theta_1$$

$$= M_p \theta + \frac{3M_p \theta}{2} + \frac{M_p \theta}{2}$$

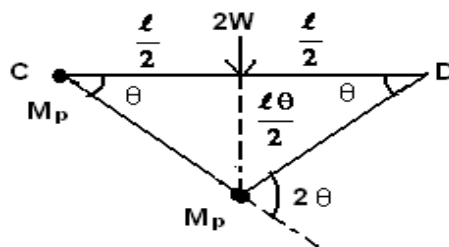
$$= 3M_p \theta$$

$$\text{IWD} = \text{EWD}$$

$$3M_p \theta = \frac{W \ell \theta}{3}$$

$$W_c = \frac{9M_p}{\ell}$$

Step 4: Mechanism (3):



$$\text{EWD} = 2W \times \frac{\ell \theta}{2} = W \ell \theta$$

$$\text{IWD} = M_p \theta + M_p (2\theta) = 3M_p \theta$$

$$\text{IWD} = \text{EWD}$$

$$3M_p \theta = W \ell \theta$$

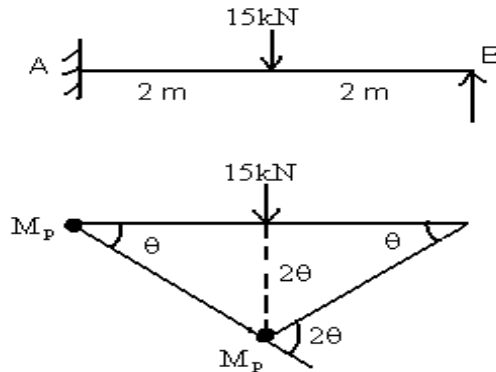
$$W_c = \frac{3M_p}{\ell}$$

The collapse load $W_c = \frac{3M_p}{\ell}$ and the beam will fail.

7. A uniform beam of span 4 m and fully plastic moment M_p is simply supported at one end and rigidly clamped at other end. A concentrated load of 15 kN may be applied anywhere within the span. Find the smallest value of M_p such that collapse would first occur when the load is in its most unfavourable position. (AUC May/June 2013)

Solution:

i) When the load is at centre:



$$\text{Degree of indeterminacy} = 4 - 3 = 1$$

$$\text{No. of possible plastic hinges} = 2$$

$$\text{No. of independent mechanisms} = 2 - 1 = 1$$

$$\text{EWD} = 15 (2\theta) = 30 \theta$$

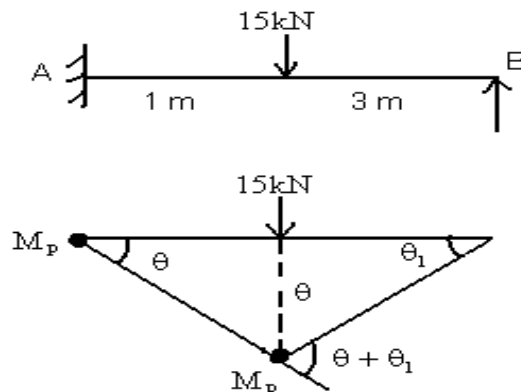
$$\text{IWD} = M_p \theta + M_p (2\theta) = 3M_p \theta$$

$$\text{IWD} = \text{EWD}$$

$$3M_p \theta = 30 \theta$$

$$M_p = 10 \text{ kNm}$$

ii) When the load is at unfavourable position:



$$1 \times \theta = 3 \times \theta_1$$

$$\theta_1 = \frac{\theta}{3}$$

$$EWD = 15\theta$$

$$IWD = M_p \theta + M_p (\theta + \theta_1) = M_p \theta + M_p \left(\theta + \frac{\theta}{3} \right)$$

$$= \frac{7}{3} M_p \theta$$

$$IWD = EWD$$

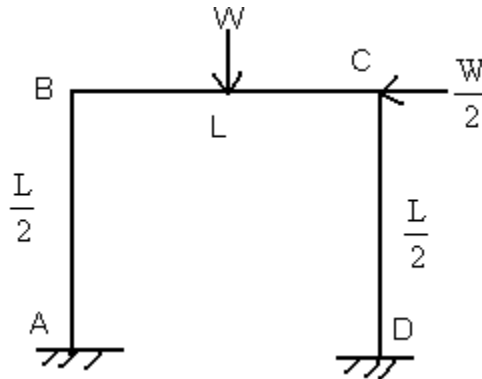
$$\frac{7}{3} M_p \theta = 15\theta$$

$$M_p = 6.43 \text{ kNm}$$

The smallest value of M_p is 6.43 kNm.

8. A rectangular portal frame of span L and $L/2$ is fixed to the ground at both ends and has a uniform section throughout with its fully plastic moment of resistance equal to M_p . It is loaded with a point load W at centre of span as well as a horizontal force $W/2$ at its top right corner. Calculate the value of W at collapse of the frame. (AUC May/June 2013)

Solution:



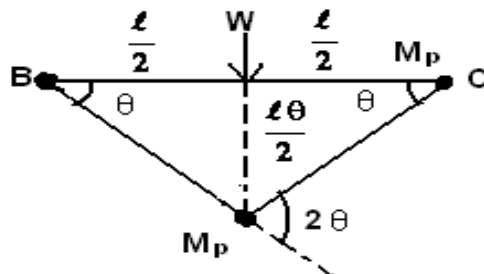
Step 1: Degree of indeterminacy:

$$\begin{aligned} \text{Degree of indeterminacy} &= (\text{No. of closed loops} \times 3) - \text{No. of releases} \\ &= (1 \times 3) - 0 = 3 \end{aligned}$$

$$\text{No. of possible plastic hinges} = 5$$

$$\text{No. of independent mechanisms} = 5 - 3 = 2$$

Step 2: Beam Mechanism:



$$EWD = \frac{W \ell \theta}{2}$$

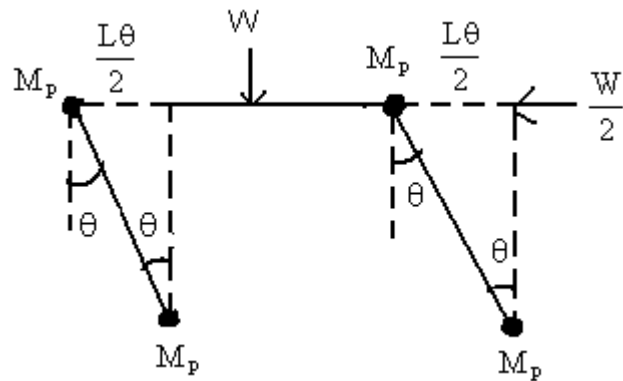
$$IWD = M_p \theta + M_p (2\theta) + M_p \theta = 4M_p \theta$$

$$EWD = IWD$$

$$\frac{W \ell \theta}{2} = 4M_p \theta$$

$$W_c = \frac{8M_p}{\ell}$$

Step 3: Sway Mechanism:



$$EWD = \frac{W \ell \theta}{4}$$

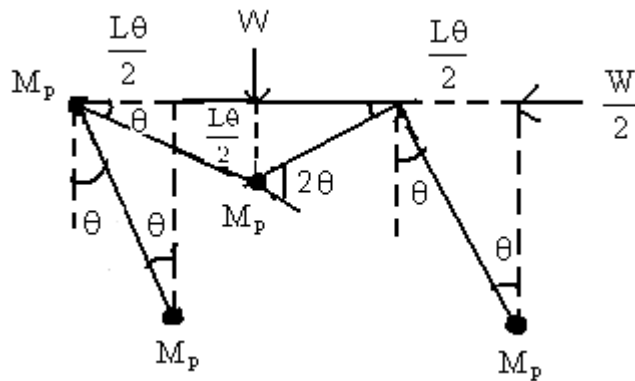
$$IWD = M_p \theta + M_p \theta + M_p \theta + M_p \theta = 4M_p \theta$$

$$EWD = IWD$$

$$\frac{W \ell \theta}{4} = 4M_p \theta$$

$$W_c = \frac{16M_p}{\ell}$$

Step 4: Combined Mechanism:



$$EWD = \left(\frac{W\ell\theta}{2} \right) + \left(\frac{W\ell\theta}{4} \right) = \frac{3W\ell\theta}{4}$$

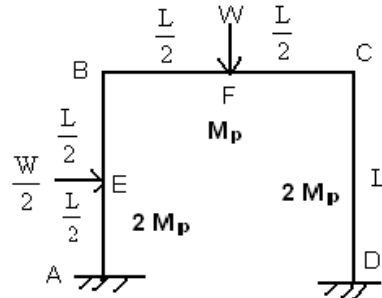
$$IWD = M_p\theta + M_p(2\theta) + M_p(2\theta) + M_p\theta = 6M_p\theta$$

$$EWD = IWD$$

$$\frac{3W\ell\theta}{4} = 6M_p\theta$$

$$W_c = \frac{8M_p}{\ell}$$

9. Find the collapse load for the frame shown in figure.



Solution:

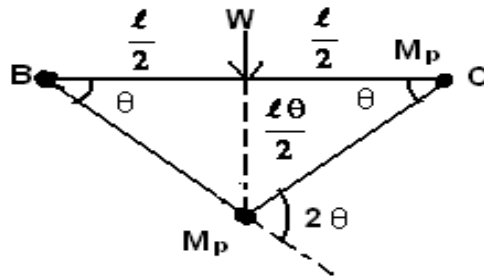
Step 1: Degree of indeterminacy:

$$\begin{aligned} \text{Degree of indeterminacy} &= (\text{No. of closed loops} \times 3) - \text{No. of releases} \\ &= (1 \times 3) - 1 = 2 \end{aligned}$$

$$\text{No. of possible plastic hinges} = 5$$

$$\text{No. of independent mechanisms} = 5 - 2 = 3$$

Step 2: Beam Mechanism:



$$EWD = \frac{W\ell\theta}{2}$$

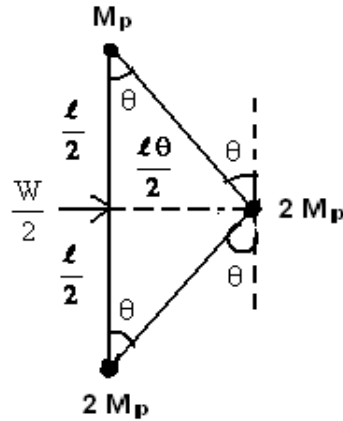
$$IWD = M_p\theta + M_p(2\theta) + M_p\theta = 4M_p\theta$$

$$EWD = IWD$$

$$\frac{W\ell\theta}{2} = 4M_p\theta$$

$$W_c = \frac{8M_p}{\ell}$$

Step 3: Column Mechanism:



$$EWD = \frac{W l \theta}{4}$$

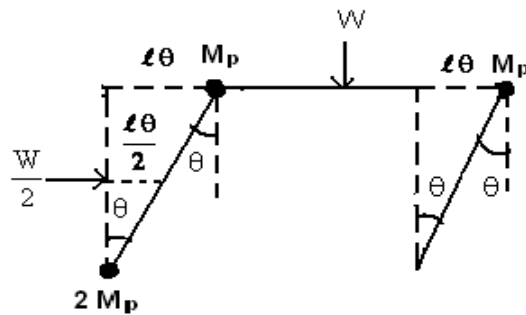
$$IWD = 2M_p \theta + 2M_p (2\theta) + M_p \theta = 7M_p \theta$$

$$EWD = IWD$$

$$\frac{W l \theta}{2} = 7M_p \theta$$

$$W_c = \frac{28M_p}{l}$$

Step 3: Sway Mechanism:



$$EWD = \frac{W l \theta}{4}$$

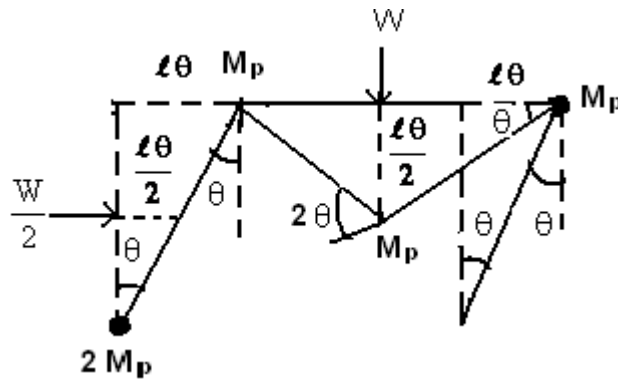
$$IWD = 2M_p \theta + M_p \theta + M_p \theta = 4M_p \theta$$

$$EWD = IWD$$

$$\frac{W l \theta}{4} = 4M_p \theta$$

$$W_c = \frac{16 M_p}{l}$$

Step 4: Combined Mechanism:



$$EWD = \left(\frac{W \ell \theta}{4} \right) + \left(\frac{W \ell \theta}{2} \right) = \frac{3W \ell \theta}{4}$$

$$IWD = 2M_p \theta + M_p(2\theta) + M_p(2\theta) = 6M_p \theta$$

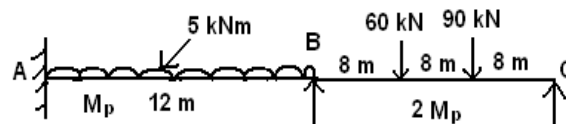
$$EWD = IWD$$

$$\frac{3W \ell \theta}{4} = 6M_p \theta$$

$$W_c = \frac{8M_p}{\ell}$$

$$\text{Hence the collapse load, } W_c = \frac{8M_p}{\ell}$$

10. A continuous beam ABC is loaded as shown in figure. Determine the required M_p if the load factor is 3.2.



Solution:

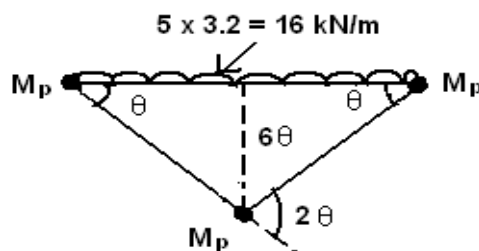
Step 1: Degree of indeterminacy:

$$\text{Degree of indeterminacy} = 5 - 3 = 2$$

$$\text{No. of possible plastic hinges} = 5$$

$$\text{No. of independent mechanisms} = 5 - 2 = 3$$

Step 2: Mechanism (1):



$$EWD = 16 \times \frac{1}{2} \times 12 \times 6\theta$$

$$= 576 \theta$$

$$IWD = M_p\theta + M_p(2\theta) + M_p\theta$$

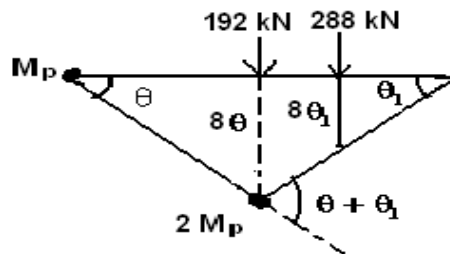
$$= 4M_p\theta$$

$$IWD = EWD$$

$$4M_p\theta = 576 \theta$$

$$M_p = 144 \text{ kNm}$$

Step 3: Mechanism (2):



$$8\theta = 16\theta_1$$

$$\theta_1 = \frac{\theta}{2}$$

$$\theta + \theta_1 = \theta + \frac{\theta}{2} = \frac{3\theta}{2}$$

$$EWD = (192 \times 8\theta) + (288 \times 4\theta) = 2688 \theta$$

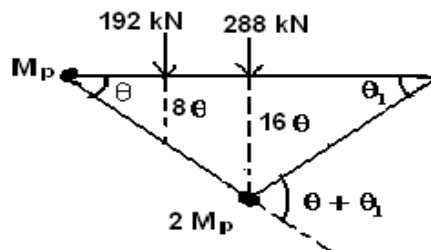
$$IWD = M_p\theta + 2M_p(\theta + \theta_1) = 4M_p\theta$$

$$IWD = EWD$$

$$4M_p\theta = 2688 \theta$$

$$M_p = 672 \text{ kNm}$$

Step 4: Mechanism (3):



$$16\theta = 8\theta_1$$

$$\theta_1 = 2\theta$$

$$\text{EWD} = (192 \times 8\theta) + (288 \times 16\theta) = 6144 \theta$$

$$\text{IWD} = M_p \theta + 2M_p (3\theta) = 7M_p \theta$$

$$\text{IWD} = \text{EWD}$$

$$7M_p \theta = 6144 \theta$$

$$M_p = 877.71 \text{ kNm}$$

The required plastic moment of the beam section shall be $M_p = 877.71 \text{ kNm}$.