



SRI VIDYA COLLEGE OF ENGINEERING & TECHNOLOGY
VIRUDHUNAGAR



QUESTION BANK WITH ANSWER

DEPARTMENT: CIVIL

SEMESTER: 06

**SUBJECT CODE /NAME: CE 6601/DESIGN OF REINFORCED CONCRETE AND BRICK
MASONRY STRUCTURES**

YEAR: IV

UNIT IV – YIELD LINE THEORY

PART - A (2 marks)

1. List the assumptions made in yield line analysis of slabs.

(May/Jun-2013) (Nov/Dec-2011) (May/Jun-2012)

- The reinforcing bars are fully yielded across the yield lines at failure.
- The yield lines divide the slab into various segments who in turn behave classically.
- The entire deformation take place only in the yield lines and the individual segments of the slab are plane segments in the collapse condition.
- The bending and twisting moments have the maximum values and are uniformly distributed along the yield lines.
- The yield lines are straight at in intersection of individual inclined segments.

2. Define yield line theory.

(May/Jun-2013)

The yield line theory is largely based upon the yield lines that develop in any reinforced concrete slab (rectangular, circular, square or any other geometrical shape in plan) before its final collapse. This stage reaches under loads approaching collapse load or ultimate load that the slab can carry.

The collapse loads, movements and shears can be calculated from the crack pattern developed in slab, under idealized support conditions and only uniformly distributed loads.

3. What are the characteristic features of yield lines?

(Nov/Dec-2011)(May/Jun-2012)

- Yield lines end at the supporting edges of the slab
- Yield lines are straight
- A yield line or yield line produced passes through the intersection of the axes of rotation of adjacent slab
- Axes of rotation generally lie along lines of supports and pass over any columns.

4. What is meant by yield lines?

The failure of reinforced concrete slabs of different shapes such as square, rectangular, circular with different types of edge conditions is preceded by a characteristic pattern of cracks, which are generally referred to as yield lines.

5. State the principle of virtual work.**(Nov/Dec-2013)** If a

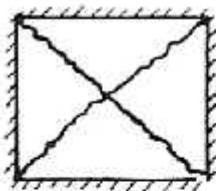
deformable structure in equilibrium under the action of a system of external forces is subjected to a virtual deformation compatible with its condition of support, the work done by these forces on the displacements associated with the virtual deformation is equal to the work done by the internal stresses on the strains associated with this deformation.

6. What is meant by an orthotropically reinforced slab?**(Nov/Dec-2012)**

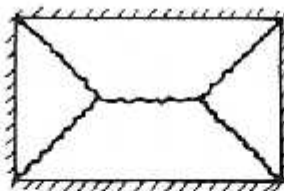
If the reinforcement in the two directions is not the same, it is said to be orthotropically reinforced slab.

7. What is meant by an isotropically reinforced slab?**(Nov/Dec-2012)**

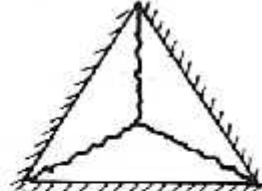
The ultimate moment of resistance in an isotropically reinforced slab, in any direction, is the same.

8. Draw the typical yield line pattern for different slabs. **(Nov/Dec-2012) (Nov/Dec-2013)**

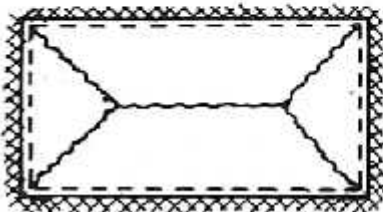
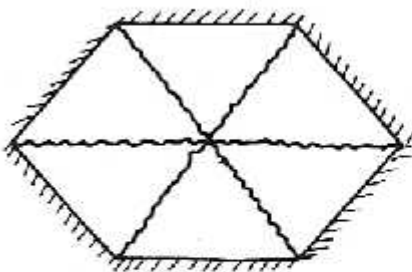
Square slab



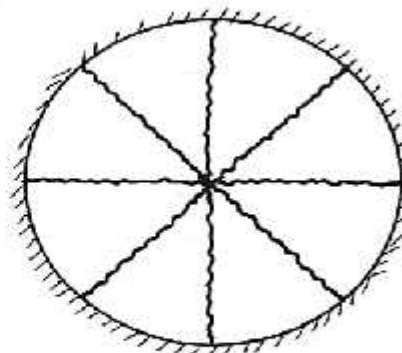
Rectangular slab



Triangular slab

Rectangular slab
(fixed supports)Triangular slab
(adjacent side supports)

Hexagonal slab



Circular slab

Typical yield line patterns in reinforced concrete slabs.

9. What are the two methods of determining the ultimate load capacity of reinforced concrete slabs?

- Virtual work method
- Equilibrium method

10. What is the direction of yield line in one way slab?

In one way slab, the direction of yield line is perpendicular to the direction of steel.

11. What is the direction of yield line in two way slab?

In two way slab, the direction of yield line is not perpendicular to the direction of steel.

12. What is the concept of yield line method?

In the yield line method, the computation of ultimate load is based on the pattern of yield lines that are developed in the slabs under conditions approaching collapse.

13. Who innovated yield line theory?

This method was innovated by Ingerslav (1923) and was greatly extended and advanced by Johanssen.

14. What is a yield line?

A yield line is defined as a line in the plane of the slab across which reinforcing bars have yielded and about which excessive deformation under constant limit moment continues to yield leading to failure.

15. Define static indeterminacy of a structure.

If the conditions of statics i.e., $H=0$, $V=0$ and $M=0$ alone are not sufficient to find either external reactions or internal forces in a structure, the structure is called a statically indeterminate structure.

16. Define: Unit load method.

The external load is removed and the unit load is applied at the point, where the deflection or rotation is to found.

17. What is the absolute maximum bending moment due to a moving udl longer than the span of a simply supported beam?

When a simply supported beam is subjected to a moving udl longer than the span, the absolute maximum bending moment occurs when the whole span is loaded. $M_{max} = wl / 8$

18. State the location of maximum shear force in a simple beam with any kind of loading.

In a simple beam with any kind of load, the maximum positive shear force occurs at the left hand support and maximum negative shear force occurs at right hand support.

19. What is meant by maximum shear force diagram?

Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions. When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram. This diagram yields the 'design shear' for each

20. What do you understand by the term reversal of stresses?

In certain long trusses the web members can develop either tension or compression depending upon the position of live loads. This tendency to change the nature of stresses is called reversal of stresses.

21. What is the moment at a hinged end of a simple beam?

Moment at the hinged ends of a simple beam is zero.

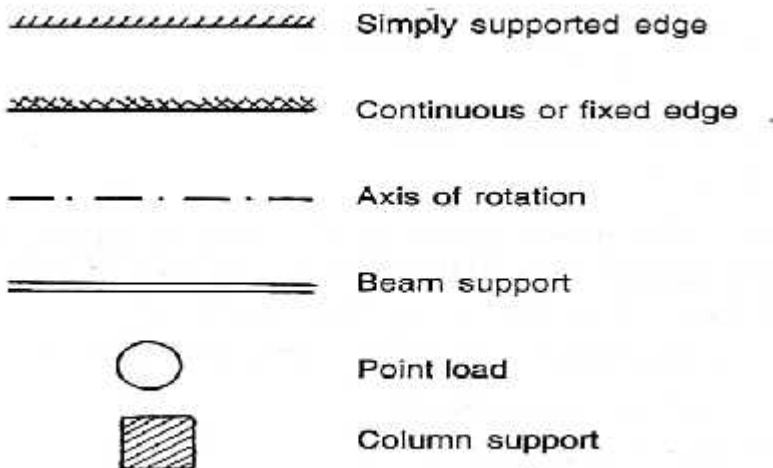
22. Define similitude.

Similitude means similarity between two objects namely the model and the prototype with regard to their physical characteristics:

- Geometric similitude is similarity of form
- Kinematic similitude is similarity of motion Dynamic and/or mechanical similitude is similarity of masses

23. Define: Trussed Beam.

A beam strengthened by providing ties and struts is known as Trussed Beams.

24. Draw the notations for yield lines and supports.

Notations for yield lines and supports.

PART-B (16 MARKS)

1. Using virtual work method, obtain the expression for ultimate moment an isotropically reinforced square slab simply supported and subjected to a uniformly distributed load. (AUC May/Jun-2012)

Isotropically reinforced square slab simply supported and supporting uniformly distributed load : The principle of the virtual work method is to equate the internal work done due to rotation of yield lines to the external work done due to the loads having a virtual displacement.

$$\text{External work done} = \Sigma(W \cdot \delta)$$

where W = Loads

δ = Virtual displacement

$$\text{Internal work done} = \Sigma(M\theta) = \Sigma(m \cdot L \cdot \theta)$$

where m = ultimate moment per unit length of yield line

L = length of yield line

Referring to Fig. 7.12.

The square slab is isotropically reinforced, the ultimate moment along the yield line is also m and the total work done in yield line ac is given by :

$$\Sigma(M\theta)_{ac} = \Sigma m L \theta = m \sqrt{2} \cdot L \cdot \frac{\sqrt{2}}{L} = 4 m$$

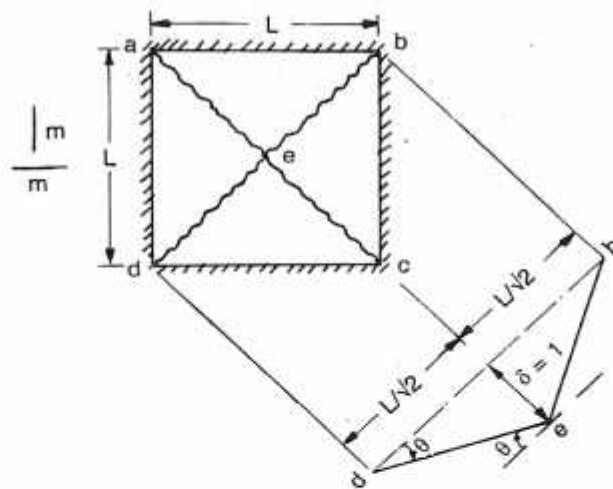


Fig. 7.12. Yield line pattern in a square slab (simply supported).

The work done in yield line bd is the same as in ac .

$$\text{Total internal work done} = \Sigma(M\theta) = 8 m$$

For a virtual displacement of $\delta = 1$ at e , the centre of gravity of each of the triangular elements deflects by $1/3$

$$\therefore \Sigma(W\delta) = 1/3 w \cdot L^2$$

where w = uniformly distributed load on slab. By equating :

$$\Sigma(M \cdot \theta) = \Sigma(W \cdot \delta)$$

We have :

$$m = \left(\frac{wL^2}{24} \right)$$

2. Using virtual work method, obtain the expression for ultimate moment an isotropically reinforced square slab fixed on all edges and subjected to a uniformly distributed load.

(AUC Nov/Dec-2013) (AUC May/Jun-2012)

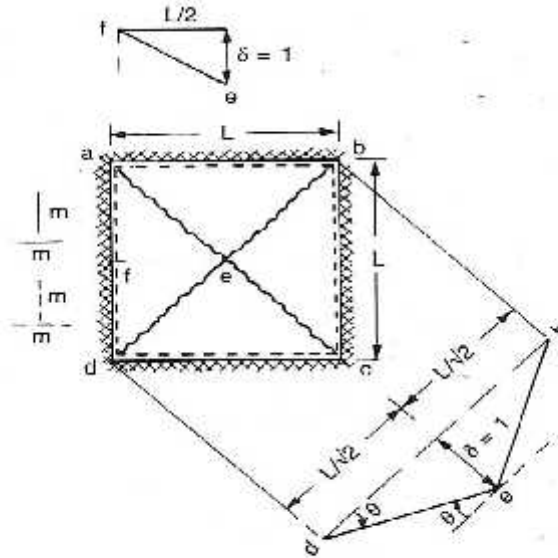


Fig. 7.13. Yield line pattern in a square slab (fixed).

Isotropically reinforced square slab fixed on all edges and subjected to a uniformly distributed load

Referring to Fig. 7.13, since the edges are fixed negative yield lines will form along the edges.

Internal work done along the positive yield lines ac and db is given by :

$$\Sigma (M \cdot \theta) = 8 m \text{ (Refer previous problem)}$$

Internal work done along the negative yield lines ab , bc , cd , and de is given by :

$$\Sigma (M \cdot \theta) = 4 [m \cdot L (2/L)] = 8 m$$

$$\therefore \text{Total internal work done} = \Sigma (M \cdot \theta) = 16 m$$

$$\text{External work done} = \Sigma (W \cdot \delta) = 1/3 w \cdot L^2$$

Equating internal to external work done :

$$\Sigma (M \cdot \theta) = \Sigma (W \cdot \delta)$$

$$16 m = 1/3 w L^2$$

$$m = \left(\frac{wL^2}{48} \right)$$

3. Using virtual work method, obtain the expression for ultimate moment an isotropically reinforced triangular slab simply supported on adjacent edges and subjected to a uniformly distributed load.
(AUC Nov/Dec-2012)

Triangular slab simply supported on adjacent edges and subjected to uniformly distributed load and isotropically reinforced

Referring to Fig. 7.14, the triangular slab acb is simply supported at ac and cb . The yield line formed is cd . Unit displacement is given for point d . Since slab is isotropically reinforced $m_x = m_y = m$.

For element A , $\theta_{Ax} = 1/de = 1/x \tan \phi$ and $\theta_{Ay} = 0$

$$\therefore (M_x \cdot \theta_x + M_y \cdot \theta_y)_A = x \cdot m \cdot \theta_x = m \cot \phi$$

For element B , $\theta_{Bx} = 1/df = 1/(x \tan \psi - x \tan \phi)$

and $\theta_{By} = 1/gd = 1/(x - y \cot \psi)$

$$\begin{aligned} \therefore (M_x \cdot \theta_x + M_y \cdot \theta_y)_B &= m \left[\frac{1}{\tan \psi - \tan \phi} + \frac{1}{\cot \phi - \cot \psi} \right] \\ &= m \left[\frac{1 + \tan \psi \cdot \tan \phi}{\tan \psi - \tan \phi} \right] = m \cot (\psi - \phi) \end{aligned}$$

Thus $\Sigma (M \cdot \theta) = m [\cot (\psi - \phi) + \cot \phi]$

and $\Sigma (W \cdot \delta) = 1/6 w \alpha L^2 \cdot \sin \psi$

Equating $\Sigma (M \cdot \theta) = \Sigma (W \cdot \delta)$

We have :

$$m = \frac{w \cdot \alpha \cdot L^2 \sin \psi}{6[\cot (\psi - \phi) + \cot \phi]}$$

$$m = 1/6 w \alpha L^2 \sin \phi \cdot \sin (\psi - \phi)$$

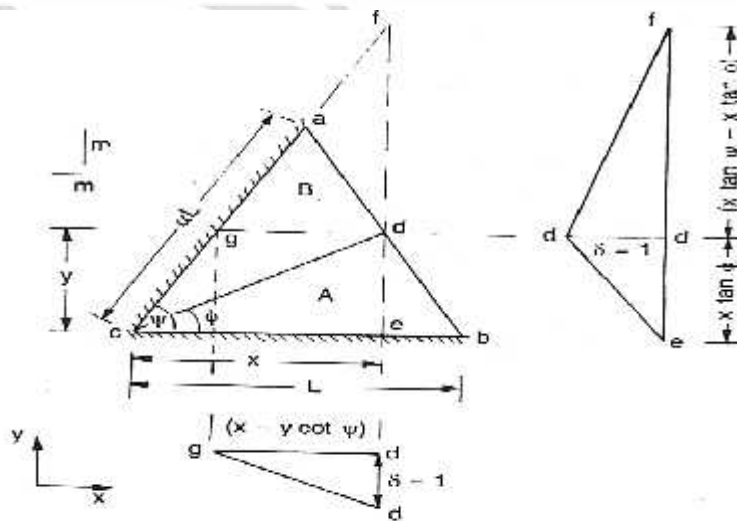


Fig. 7.14. Triangular-slab simply supported on adjacent edges.

For a maximum value of m , $\frac{dm}{d\phi} = 0$

$$\therefore \cos \phi \cdot \sin (\psi - \phi) = \sin \phi \cos (\psi - \phi)$$

$$\tan \phi = \tan (\psi - \phi)$$

$$\phi = (1/2)\psi$$

Hence the yield line bisects the angle opposite the free edge.

Substituting the value of ϕ we have the final value given by :

$$M = \left\{ 1/6 w \alpha L^2 \cdot \sin^2 \left(\frac{\psi}{2} \right) \right\}$$

In a right angled triangle $\psi = 90^\circ$

$$\text{Then } m = (1/6 w \cdot \alpha \cdot L^2)$$

4. Using virtual work method, obtain the expression for ultimate moment an orthotropically reinforced rectangular slab simply supported and subjected to a uniformly distributed load.

Orthotropically reinforced rectangular slab, simply supported along its edges and subjected to a uniformly distributed load of w /Unit area

Referring to Fig. 7.15, the rectangular slab $abcd$ is simply supported at the edges. The yield line pattern assumed is given by ae , de , bf , cf and ef . m and μm are the yield moments along the x and y axis respectively. In the yield line pattern shown ' βL ' is an unknown dimension. The yield line ef is given a virtual displacement of unity.

For element A , $\theta_x = (2/\alpha \cdot L)$, $\theta_y = 0$, $M_x = mL$

$$(M_x \cdot \theta_x + M_y \cdot \theta_y)_A = (2m/\alpha)$$

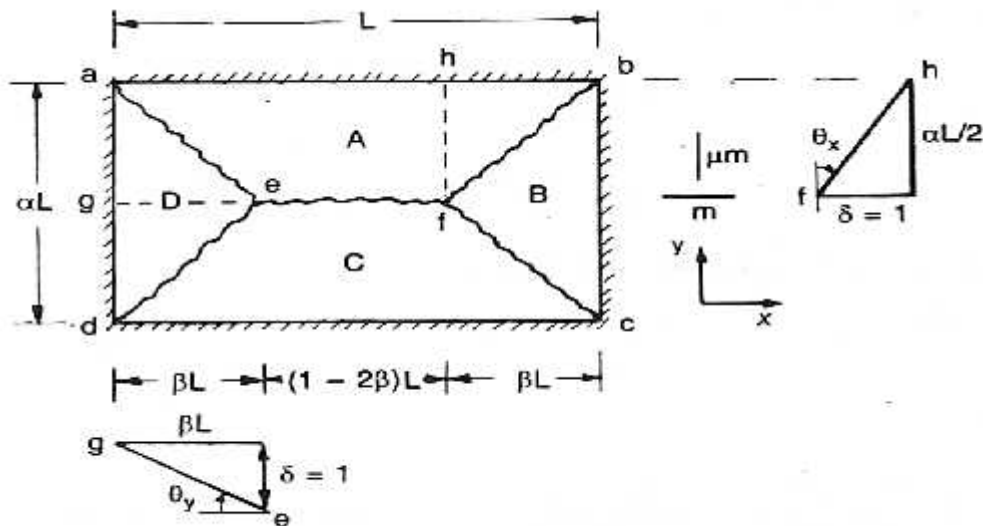


Fig. 7.15. Rectangular slab simply supported at the edges.

For element D , $\theta_x = 0$, $\theta_y = (1/\beta L)$, $M_y = (\alpha L \mu m)$

$$\therefore (M_x \cdot \theta_x + M_y \cdot \theta_y)_D = (\alpha \mu m / \beta)$$

Since element A and C and B and D are similar :

$$\Sigma(M \cdot \theta) = 2 \left(\frac{2m}{\alpha} + \frac{\alpha \mu m}{\beta} \right)$$

The external work done is given by :

$$\Sigma(W \cdot \delta) = w L^2 \left[\frac{2\beta\alpha}{3} + \frac{\alpha(1-2\beta)}{2} \right]$$

Equating $\Sigma(M \cdot \theta) = \Sigma(W \cdot \delta)$ we get :

$$m = \frac{1}{12} \cdot \alpha^2 L^2 \left[\frac{3\beta - 2 \cdot \beta^2}{2\beta + \mu\alpha^2} \right]$$

If the work equation is of the form :

$$m = w \left(\frac{f_1(x_1, x_2)}{f_2(x_1, x_2)} \right)$$

For a maximum value of m :

$$\left(\frac{\delta m}{\delta x} \right) = 0$$

This is obtained for the condition :

$$\frac{f_1(x_1, x_2)}{f_2(x_1, x_2)} = \frac{\frac{\delta}{\delta x} [f_1(x_1, x_2)]}{\frac{\delta}{\delta x} [f_2(x_1, x_2)]}$$

Using this criterion for a maximum value of m :

$$\frac{\delta m}{\delta \beta} = 0$$

Hence we have

$$\left(\frac{3\beta - 2\beta^2}{2\beta + \mu\alpha^2} \right) = \left(\frac{3 - 4\beta}{2} \right)$$

Cross multiplying we get the quadratic as :

$$4\beta^2 + 4\mu\alpha^2\beta - 3\mu\alpha^2 = 0$$

The positive root of this quadratic is :

$$\beta = 1/2 [\sqrt{(3\mu\alpha^2 + \mu^2\alpha^4)} - \mu\alpha^2]$$

Substituting the value of β in the equation for m , we have :

$$m = \frac{w\alpha^2 L^2}{24} [\sqrt{(3 + \mu \cdot \alpha^2)} - \alpha\sqrt{\mu}]^2$$

5. Using virtual work method, obtain the expression for ultimate moment an orthotropically reinforced rectangular slab simply supported and subjected to a uniformly distributed load.

Isotropically reinforced circular slab, simply supported all round and uniformly loaded all round and uniformly loaded

Referring to Fig. 7.16, a circular slab of radius ' r ' is simply supported at the edges and supports a uniformly distributed load of w /unit area. In circular slabs, the failure will take place by the formation of an infinite number of positive yield lines running radially from the centre to the circumference, resulting in the formation of a flat cone at collapse.

For unit displacement at the centre of slab

$$\text{External work done} = \Sigma (W \cdot \delta) = (\pi \cdot r^2 \cdot w/3)$$

For a central displacement of unity :

$$\angle AOB = \phi = 1/r$$

$$\therefore \text{Length } OA = (r_A/\phi) = r_A \cdot r$$

Change of slope in the tangential direction at A , per unit length of arc is equal to the angle between the two normals unit length of arc apart at A and is given by $(1/r_A \cdot r)$. Total change of slope in one complete revolution by :

$$\Sigma\theta = (2\pi r_A \times 1/r_A \cdot r) = 2\pi/r$$

Internal work done in rotation at yield lines = $\Sigma (mL\theta)$ since all the yield lines are of equal length work done :

$$= mL\Sigma\theta = m \cdot r \cdot (2\pi/r) = 2\pi m$$

Equating internal work to external work done we have :

$$(1/3) \cdot \pi \cdot r^2 w = 2 \pi m$$

$$m = \left(\frac{wr^2}{6} \right)$$

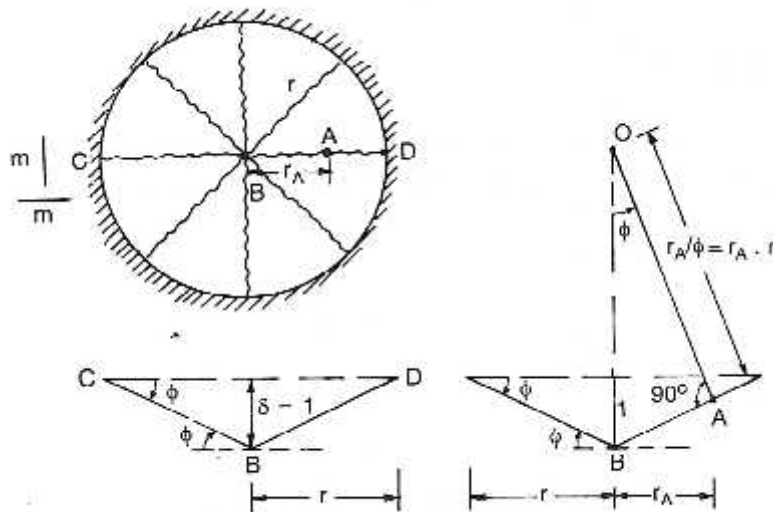


Fig. 7.16. Circular slab simply supported at the edges.

6. Design a simply supported square slab of 4.5m side length to support a service live load of 4kN/m^2 . adopt M20 grade concrete and Fe415 steel. Assume load factor according to IS: 456-2000. (AUC Nov/Dec-2011)

1. Data

Square slab simply supported at edges

Side length = $L = 4.5\text{ m}$

Live load = $q = 4\text{ kN/m}^2$

$f_{ck} = 20\text{ N/mm}^2$

$f_y = 415\text{ N/mm}^2$

2. Depth of slab

For simply supported slab using Fe-415 HYSD bars, according to clause 24.1 of IS : 456-2000 code :

(Span/overall depth) = $(35 \times 0.8) = 28$

\therefore Overall depth = $D = (\text{span}/28) = (4500/28) = 160\text{ mm}$

Adopt overall depth = $D = 160\text{ mm}$ and effective depth = $d = 135\text{ mm}$

3. Loads

Self weight of slab = $(0.16 \times 25) = 4.00\text{ kN/m}^2$

Live load = 4.00 kN/m^2

Floor finish = 1.00 kN/m^2

Total service load = $w = 9.00\text{ kN/m}^2$

\therefore Design ultimate load = $w_u = (1.5 \times 9) = 13.5\text{ kN/m}^2$

4. *Ultimate moments and shear forces*

The yield or ultimate moment capacity of a simply supported square slab is given by the relation

$$m = M_u = (w_u L^2/24) = (13.5 \times 4.5^2)/24 \\ = 11.39 \text{ kN.m/m}$$

$$\text{Ultimate shear } V_u = (0.5 w_u L) = (0.5 \times 13.5 \times 4.5) \\ = 30.375 \text{ kN/m}$$

5. *Limiting Moment capacity of the slab*

$$M_{u, \text{lim}} = 0.138 f_{ck} b d^2 \\ = (0.138 \times 20 \times 10^3 \times 135^2) 10^{-6} \\ = 50.3 \text{ kN.m}$$

Since $M_u < M_{u, \text{lim}}$, the section is under reinforced.

6. *Reinforcement*

$$M_u = 0.87 f_y A_{st} d \left[1 - \left(\frac{A_{st} f_y}{b d f_{ck}} \right) \right]$$

$$(11.39 \times 10^6) = (0.87 \times 415 A_{st} \times 135) \left[1 - \frac{415 A_{st}}{(10^3 \times 135 \times 20)} \right]$$

Solving $A_{st} = 241 \text{ mm}^2$

Adopt 10 mm diameter bars at 300 mm centres ($A_{st} = 262 \text{ mm}^2$)

7. *Check for shear stress*

$$\tau_v = \left(\frac{V_u}{b d} \right) = \left(\frac{30.375 \times 10^3}{1000 \times 135} \right) = 0.225 \text{ N/mm}^2$$

$$p_t = \left(\frac{100 A_{st}}{b_w d} \right) = \left(\frac{100 \times 262}{1000 \times 135} \right) = 0.194$$

Refer Table 19 of IS : 456-2000 and read out the permissible shear stress as :

$$(k_s \cdot \tau_c) = (1.28 \times 0.31) = 0.39 \text{ N/mm}^2$$

Since $(k_s \tau_c) > \tau_v$, the shear stresses are within safe permissible limits

7. Design a rectangular slab 5m by 4m in size and simply supported at the edges to support a service load (live) of 4 kN/m^2 . assume co efficient of orthotropy as 0.7. adopt M20 concrete and Fe415 steel bars. (AUC May/Jun-2013, 2012) (AUC Nov/Dec-2013, 2011)

1. *Data*

$$\begin{aligned} L &= 5 \text{ m} & \mu &= 0.7 \\ \alpha L &= 4 \text{ m} & f_{ck} &= 20 \text{ N/mm}^2 \\ \alpha &= 0.8 & f_y &= 415 \text{ N/mm}^2 \end{aligned}$$

2. *Depth of slab*

$$\text{Overall depth} = (\text{span}/28) = (4000/28) = 143 \text{ mm}$$

$$\text{Adopt overall depth} = D = 150 \text{ mm}$$

$$\text{Effective depth} = d = 125 \text{ mm}$$

3. *Loads*

$$\text{Self weight of slab} = (0.15 \times 25) = 3.75 \text{ kN/m}^2$$

$$\text{Live load} = 4.00$$

$$\text{Floor finish} = 1.25$$

$$\text{Total service load} = w = 9.00 \text{ kN/m}^2$$

$$\text{Design ultimate load} = w_u = (1.5 \times 9) = 13.5 \text{ kN/m}^2$$

4. *Ultimate moments and shear forces*

$$\begin{aligned} M_u = m &= \left(\frac{w_u \alpha^2 L^2}{24} \right) \sqrt{[(3 + \mu \alpha^2) - \alpha \sqrt{\mu}]^2} \\ &= \left(\frac{13.5 \times 16}{24} \right) \sqrt{[(3 + 0.7 \times 0.64) - 0.8 \sqrt{0.7}]^2} \\ &= 12.68 \text{ kN.m/m} \end{aligned}$$

$$V_u = (0.5 w_u L) = (0.5 \times 13.5 \times 4) = 27 \text{ kN/m}$$

5. *Limiting Moment capacity of the slab*

$$\begin{aligned} M_{u, \text{lim}} &= 0.138 f_{ck} b d^2 \\ &= (0.138 \times 20 \times 10^3 \times 135^2) \\ &= 50.3 \text{ kN.m} \end{aligned}$$

Since $M_u < M_{u, \text{lim}}$, the section is under reinforced.

6. Reinforcement

$$M_u \text{ (short span)} = 0.87 f_y A_{st} d \left[1 - \left(\frac{A_{st} f_y}{b d f_{ck}} \right) \right]$$

$$(12.68 \times 10^6) = (0.87 \times 415 A_{st} \times 125) \left[1 - \frac{415 A_{st}}{(10^3 \times 125 \times 20)} \right]$$

Solving $A_{st} = 295 \text{ mm}^2$

Adopt 10 mm diameter bars at 250 mm centres ($A_{st} = 262 \text{ mm}^2$) in the short span direction.

$$\begin{aligned} A_{st} \text{ (long span)} &= \mu A_{st} \\ &= (0.7 \times 250) = 175 \text{ mm}^2 \end{aligned}$$

$$A_{st} \text{ (minimum)} = (0.0012 \times 10^3 \times 150) = 180 \text{ mm}^2$$

Provide 10 mm bars at 300 mm centres ($A_{st} = 262 \text{ mm}^2$)

7. Check for shear stress

$$\tau_v = \left(\frac{V_u}{b d} \right) = \left(\frac{27 \times 10^3}{10 \times 125} \right) = 0.216$$

$$p_t = \left(\frac{100 A_{st}}{b_w d} \right) = \left(\frac{100 \times 315}{10^3 \times 125} \right) = 0.252$$

Permissible shear stress from Table 19 of IS : 456 is :

$$(k_s \cdot \tau_c) = (1.30 \times 0.36) = 0.468 \text{ N/mm}^2$$

Since $(k_s \tau_c) > \tau_v$, the shear stresses are within safe permissible limits.

8. A hexagonal slab of side length 4m is simply supported at the edges and it is isotropically reinforced with 12mm diameter bars at 150mm centers both ways, at an average effective depth of 120mm. the overall depth of the slab is 150mm. calculate the ultimate load capacity of the slab and also the safe permissible live load if M20 grade concrete and Fe415 bars.

1. Data

Hexagonal slab, simply supported at edges having side length = $L = 4 \text{ m}$

12 mm diameter bars provided at 150 mm centres.

$$\therefore A_{st} = \left(\frac{1000 \times 113}{150} \right) = 753 \text{ mm}^2/\text{m}$$

$$D = 150 \text{ mm} \quad d = 120 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2$$

2. Yield or ultimate moment

$$\begin{aligned}
 m = M_u &= 0.87 f_y A_{st} d \left[1 - \left(\frac{A_{st} f_y}{b d f_{ck}} \right) \right] \\
 &= (0.87 \times 415 \times 753 \times 120) \left[1 - \frac{(753 \times 415)}{(10^3 \times 120 \times 20)} \right] \\
 &= 28.38 \times 10^6 \text{ N.mm} \\
 &= 28.38 \text{ kN.m}
 \end{aligned}$$

3. Ultimate load on slab

$$w_u = \left(\frac{8 m}{L^2} \right) = \left(\frac{8 \times 28.38}{4^2} \right) = 14.19 \text{ kN/m}^2$$

4. Service live load

$$\text{Total service load} = \left(\frac{14.19}{1.5} \right) = 9.46 \text{ kN/m}^2$$

$$\text{Self weight of slab} = (0.15 \times 25) = 3.75 \text{ kN/m}^2$$

∴ Safe permissible live load is given by :

$$q = (9.46 - 3.75) = 5.71 \text{ kN/m}^2$$

9. A right angle triangular slab is simply supported at the adjacent edges AB and BC. the side AB=4M and BC=3m and CA=5m.the slab is isotropically reinforced with 10mm diameter bars at 100mm centers both ways at an average effective depth of 120mm.the overall depth of the slab is 150mm.if M20 concrete and Fe415 steel bars are used, estimate the permissible service live load on the slab.

1. Data

$$\begin{array}{ll}
 AB = 4 \text{ m} & f_{ck} = 0.7 \\
 BC = 3 \text{ m} & f_y = 415 \text{ N/mm}^2 \\
 CA = 5 \text{ m} & d = 120 \text{ mm} \\
 L = 4 \text{ m} & D = 150 \text{ mm} \\
 \alpha L = 3 \text{ m} &
 \end{array}$$

Reinforcements provided (10 mm diameter) at 100 mm centres both ways.

$$A_{st} = \left(\frac{1000 \times 78.5}{100} \right) = 785 \text{ mm}^2/\text{m}$$

2. *Yield or ultimate moment*

$$\begin{aligned} m = M_u &= 0.87 f_y A_{st} d \left[1 - \left(\frac{A_{st} f_y}{b d f_{ck}} \right) \right] \\ &= (0.87 \times 415 \times 785 \times 120) \left[1 - \frac{(785 \times 415)}{(1000 \times 120 \times 20)} \right] \\ &= 29 \times 10^6 \text{ N.mm} \\ &= 29 \text{ kN.m} \end{aligned}$$

3. *Ultimate load on slab*

$$w_u = \left(\frac{6 M_u}{\alpha L^2} \right) = \left(\frac{6 \times 29}{0.75 \times 16} \right) = 14.5 \text{ kN/m}^2$$

4. *Service live load*

$$\text{Total service load} = (14.5/1.5) = 9.66 \text{ kN/m}^2$$

$$\text{Dead load of slab} = (0.15 \times 25) = 3.75 \text{ kN/m}^2$$

$$\text{Service live load} = (9.66 - 3.75) = 5.91 \text{ kN/m}^2$$