QUESTION BANK

DEPARTMENT: CIVIL   **SEMESTER: V**

SUBJECT CODE / Name: CE 2302 / STRUCTURAL ANALYSIS-I

Unit 4 – SLOPE DEFLECTION METHOD

PART - A (2 marks)

1. **What are the assumptions made in slope-deflection method?**

   (AUC Apr/May 2012, Nov/Dec 2013)

   i) Between each pair of the supports the beam section is constant.

   ii) The joint in structure may rotate or deflect as a whole, but the angles between the members meeting at that joint remain the same.

2. **What is the limitation of slope-deflection equations applied in structural analysis?**

   (AUC Apr/May 2012)

   i) It is not easy to account for varying member sections.

   ii) It becomes very cumbersome when the unknown displacements are large in number.

3. **Mention the causes for sway in portal frames.**   **(AUC Nov/Dec 2012, May/June 2014)**

   Because of sway, there will be rotations in the vertical members of a frame. This causes moments in the vertical members. To account for this, besides the equilibrium, one more equation namely shear equation connecting the joint-moments is used.

4. **Explain the use of slope deflection method.**   **(AUC Nov/Dec 2012)**

   i) It can be used to analyze statically determinate and indeterminate beams and frames.

   ii) In this method it is assumed that all deformations are due to bending only.

   iii) In other words deformations due to axial forces are neglected.

   iv) The slope-deflection equations are not that lengthy in comparison.

5. **Compute the rotation at middle support of a two equal span continuous beam fixed at the ends and carrying UDL of 10 kN/m over the entire beam span 5 m. Take EI = 60000 kNm^2.**

   (AUC Apr/May 2011)

   $\theta_B = 0.00155$ mm and

   $\theta_C = 0.0062$ mm

6. **Write down the slope deflection equation for a beam AB fixed at A and B subjected to a settlement δ at B.**   **(AUC Apr/May 2011, May/June 2014)**

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B + \frac{3\delta}{L}\right)$$

7. **Mention two assumptions made in slope deflection method.**   **(AUC Nov/Dec 2010)**
   i)   Between each pair of the supports of the beam is constant.
   ii) The joint in a structure may rotate or deflect as a whole, but the angles between the members meeting at that joint remain the same.

8. **Write down the fundamental equation of slope deflection method. (AUC Nov/Dec 2010, 2013)**

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B + \frac{3\delta}{L}\right)$$

9. **How many slope deflection equations are available for a two span continuous beam?**
   There will be 4 Nos. of slope deflection equations, two for each span.

10. **What is the moment at a hinged end of a simple beam?**
   Moment at the hinged ends of a simple beam is zero.

11. **What are the quantities in terms of which the unknown moments are expressed in slope deflection method?**
   In slope-deflection method, unknown moments are expressed in terms of
     (i)   Slopes (θ) and
     (ii) Deflections (Δ)

12. **The beam shown in figure is to be analyzed by slope deflection method. What are the unknowns and to determine them, what are the conditions used?**
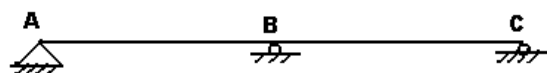
   Unknowns are θ_A, θ_B and θ_C
   Equilibrium equations used:
         (i) $M_{AB} = 0$
         (ii) $M_{BA} + M_{BC} = 0$
         (iii) $M_{CB} = 0$

13. Mention any three reasons due to which sway may occur in portal frames.

Sway in portal frames may occur due to
i)   Unsymmetry in geometry of the frame
ii) Unsymmetry in loading or
iii) Settlement of one end of a frame.

14. Write down the general slope deflection equations and state what each term represents.

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B + \frac{3\delta}{L}\right)$$

Where, M_{AB}, M_{BA} = fixed end moments at A and B due to given loading.

θ_A, θ_B = slopes at A and B.

Δ = Sinking of support A with respect to B.

15. A rigid frame is having totally 10 joints including support joints. Out of slope deflection and moment distribution methods, which method would you prefer for analysis? Why?

Moment distribution method is preferable.

If we use slope-deflection method, there would be 10 (or more) unknown displacements and an equal number of equilibrium equations. In addition, there would be 2 unknown support moments per span and the same number of slope-deflection equations. Solving them is difficult.
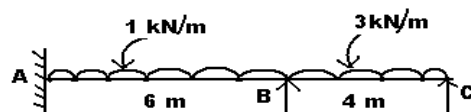
PART – B (16 marks)

1. Analyze the continuous beam ABC shown in figure by slope deflection method and sketch the bending moment diagram. Take EI = constant.   **(AUC Apr/May 2012)**

Fig.

Solution:

step 1: Fixed End Moment :-

$$M_{FAB} = -\frac{\omega l^2}{12} = -\frac{1 \times (6)^2}{12} = -3\ kNm.$$

$$M_{FBA} = \frac{\omega l^2}{12} = \frac{1 \times (6)^2}{12} = 3\ kNm.$$

$$M_{FBC} = -\frac{\omega l^2}{12} = -\frac{3 \times (4)^2}{12} = -4\ kNm$$

$$M_{FCB} = \frac{\omega l^2}{12} = \frac{3 \times (4)^2}{12} = 4\ kNm.$$

Step 2 : SLOPE DEFLECTION EQUATION :-

Here, $\theta_A = 0$ (fixed end) ; $\delta = 0$

Unknowns, θ_B, θ_C.

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\delta}{l}\right]$$

$$= -3 + \frac{2EI}{6}(\theta_B)$$

$$M_{AB} = -3 + 0.33 EI \theta_B \longrightarrow \textcircled{1}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B + \frac{3\delta}{l}\right]$$

$$= 3 + \frac{2EI}{6}(2\theta_B)$$

$$= 3 + \frac{4EI\theta_B}{6}$$

$$M_{BA} = 3 + 0.67 EI \theta_B \longrightarrow \textcircled{2}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C + \frac{3\delta}{l}\right]$$

$$= -4 + \frac{2EI}{4}\left[2\theta_B + \theta_C\right]$$

$$= -4 + \frac{4EI\theta_B}{4} + \frac{2EI\theta_C}{4}$$

$$M_{BC} = -4 + EI\theta_B + 0.5 EI\theta_C \longrightarrow \textcircled{3}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[\theta_B + 2\theta_C + \frac{3\delta}{l}\right]$$

$$= 4 + \frac{2EI\theta_B}{4} + \frac{4EI\theta_C}{4}$$

$$M_{CB} = 4 + 0.5 EI\theta_B + EI\theta_C \longrightarrow \textcircled{4}$$

Step 3 : Joint Equilibrium Equations :-

$$M_{BA} + M_{BC} = 0 \longrightarrow \textcircled{5}$$

$$M_{CB} = 0 \longrightarrow \textcircled{6}$$

$$\textcircled{5} \Rightarrow M_{BA} + M_{BC} = 0$$

$$3 + 0.67 EI \theta_B - 4 + EI\theta_B + 0.5 EI\theta_C = 0$$

$$1.67 EI\theta_B + 0.5 EI\theta_C = 1 \longrightarrow \textcircled{7}$$

$6 \Rightarrow \quad M_{CB} = 0$

$4 + 0.5 EI\theta_B + EI\theta_c = 0$

$\therefore 0.5 EI\theta_B + EI\theta_c = -4 \rightarrow ⑧$

By solving ⑦ & ⑧ we get

$$\theta_B = \frac{2.11}{EI}$$
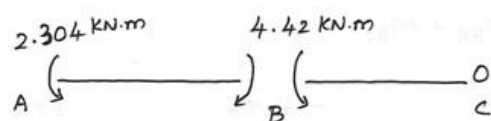
$$\theta_c = \frac{-5.06}{EI}$$

step 4 : Final Moments :-

$$M_{AB} = -3 + 0.33 EI\left(\frac{2.11}{EI}\right) = -2.304 \text{ kNm}.$$

$$M_{BA} = 3 + 0.67 EI\left(\frac{2.11}{EI}\right) = 4.414 \text{ kNm}.$$

$$M_{BC} = -4 + EI\left(\frac{2.11}{EI}\right) + 0.5EI\left(-\frac{5.06}{EI}\right) = -4.42 \text{ kNm}$$

$$M_{CB} = 4 + 0.5 EI\left(\frac{2.11}{EI}\right) + EI\left(-\frac{5.06}{EI}\right)$$
$$= -0.005 \text{ kNm}$$

step 5 : Bending Moment diagram :-

For span AB,

$$\frac{wl^2}{8} = \frac{1 \times (6)^2}{8} = 4.5 \text{ kN.m}$$

For span BC,

$$\frac{wl^2}{8} = \frac{3 \times (4)^2}{8} = 6 \text{ kN.m}$$
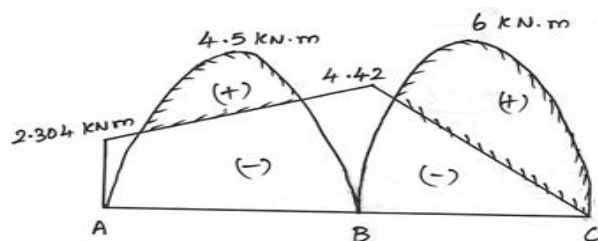
2. **Analyze the portal frame ABCD shown in figure by slope deflection method and draw the bending moment diagram. Take EI = constant.** (AUC Apr/May 2012)

Fig.
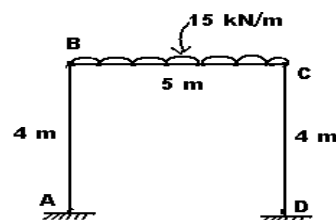
Solution:

Step 1 : Fixed End Moment :-

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = -\frac{\omega l^2}{12} = -\frac{15 \times (5)^2}{12} = -31.25 \text{ kNm}$$

$$M_{FCB} = \frac{\omega l^2}{12} = \frac{15 \times (5)^2}{12} = 31.25 \text{ kNm}$$

Step 2 : Slope Deflection Equation :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= 0 + \frac{2EI}{4}(\theta_B)$$

$$= 0.5 \, EI \, \theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(\theta_A + 2\theta_B)$$

$$= 0 + \frac{2EI(2\theta_B)}{4}$$

$$= EI \, \theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -31.25 + \frac{4EI\theta_B}{5} + \frac{2EI\theta_C}{5}$$

$$= -31.25 + 0.8 \, EI\theta_B + 0.4 \, EI\theta_C$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(\theta_B + 2\theta_c\right)$$

$$= 31.25 + \frac{2EI\theta_B}{5} + \frac{4EI\theta_c}{5}$$

$$= 31.25 + 0.4EI\theta_B + 0.8EI\theta_c$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_c + \theta_D\right)$$

$$= 0 + \frac{4EI\theta_c}{4}$$

$$= EI\theta_c$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(\theta_c + 2\theta_D\right)$$

$$= 0 + \frac{2EI\theta_c}{4}$$

$$= 0.5EI\theta_c$$

Step 3 : Equilibrium Equations :-

$$M_{BA} + M_{BC} = 0 \quad \rightarrow ①$$

$$M_{CB} + M_{CD} = 0 \quad \rightarrow ②$$

$$① \Rightarrow EI\theta_B - 31.25 + 0.8EI\theta_B + 0.4EI\theta_c = 0$$

$$1.8EI\theta_B + 0.4EI\theta_c = 31.25 \quad \rightarrow ③$$

$$② \Rightarrow 31.25 + 0.4EI\theta_B + 0.8EI\theta_c + EI\theta_c = 0$$

$$0.4EI\theta_B + 1.8EI\theta_c = -31.25 \quad \rightarrow ④$$

By solving ③ & ④,

$$\theta_B = \frac{22.32}{EI}$$

$$\theta_c = -\frac{22.32}{EI}$$

Step 4 : Final Moments :-

$$M_{AB} = 0.5\,EI\left(\frac{22.32}{EI}\right)$$

$$= 11.16 \text{ KN m}$$

$$M_{BA} = EI\left(\frac{22.32}{EI}\right)$$

$$= 22.32 \; kNm$$

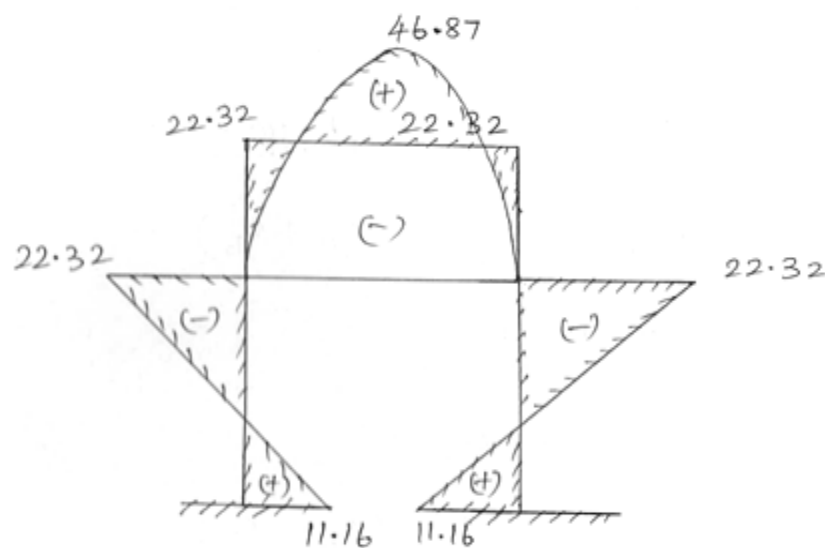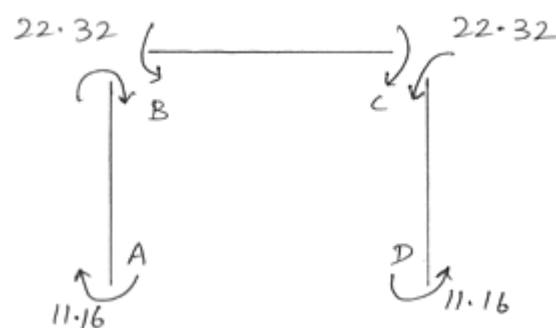$$M_{BC} = -31.25 + 0.8\,EI\left(\frac{22.32}{EI}\right) + 0.4\,EI\left(\frac{-22.32}{EI}\right)$$

$$= -22.32 \; kNm$$

$$M_{CB} = 31.25 + 0.4\,EI\left(\frac{22.32}{EI}\right) + 0.8\,EI\left(\frac{-22.32}{EI}\right)$$

$$= 22.32 \; kNm$$

$$M_{CD} = EI\left(-\frac{22.32}{EI}\right) = -22.32 \; kNm$$

$$M_{DC} = 0.5\,EI\left(-\frac{22.32}{EI}\right) = -11.16 \; kNm$$

3. **A continuous beam ABC consists of spans AB and BC of 5 m length in each. Both ends of the beam are fixed. The span AB carries a point load of 15 kN at its middle point. The span BC carries a point load of 25 kN at its middle point. Find the moments and reactions at the supports. Assume the beam is of uniform section. Use slope deflection method.**

(AUC Nov/Dec 2012, 2013)

Solution:
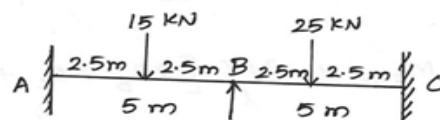
Step 1 : Fixed End Moment :

$$M_{FAB} = -\frac{\omega l}{8} = -\frac{15 \times 5}{8} = -9.37 \, kNm$$

$$M_{FBA} = \frac{\omega l}{8} = \frac{15 \times 5}{8} = 9.37 \, kNm.$$

$$M_{FBC} = -\frac{\omega l}{8} = -\frac{25 \times 5}{8} = -15.62 \, kNm.$$

$$M_{FCB} = \frac{\omega l}{8} = \frac{25 \times 5}{8} = 15.62 \, kNm.$$

Step 2 : SLOPE DEFLECTION EQUATIONS : (θ_B)

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\delta}{l}\right]$$

$$= -9.37 + \frac{2EI\theta_B}{5}$$

$$M_{AB} = -9.37 + 0.4EI\theta_B \rightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B + \frac{3\delta}{l}\right]$$

$$= 9.37 + \frac{4EI\theta_B}{5}$$

$$M_{BA} = 9.37 + 0.8EI\theta_B \rightarrow ②$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_c + \frac{3\delta}{l}\right]$$

$$= -15.62 + \frac{4EI\theta_B}{5}$$

$$M_{BC} = -15.62 + 0.8EI\theta_B \rightarrow ③$$

$$M_{CB} = M_{F'CB} + \frac{2EI}{l}\left[\theta_B + 2\theta_c + \frac{3\delta}{l}\right]$$

$$= 15.62 + \frac{2EI\,\theta_B}{5}$$

$$M_{CB} = 15.62 + 0.4\,EI\,\theta_B \rightarrow ④$$

Step 3 : Joint Equilibrium Equations :

$$M_{BA} + M_{BC} = 0 \rightarrow ⑤$$

$$9.37 + 0.8\,EI\,\theta_B + (-15.62) + 0.8\,EI\,\theta_B = 0$$

$$1.6\,EI\,\theta_B - 6.25 = 0$$

$$1.6\,EI\,\theta_B = 6.25$$

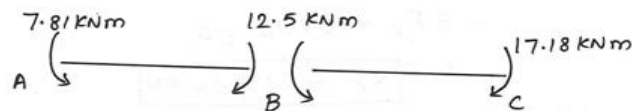$$EI\,\theta_B = \frac{6.25}{1.6} = 3.91$$

$$\theta_B = \frac{3.91}{EI}$$

Step 4 : Final Moments :

$$M_{AB} = -9.37 + 0.4\,EI\left(\frac{3.91}{EI}\right) = -7.81\ kNm.$$

$$M_{BA} = 9.37 + 0.8\,EI\left(\frac{3.91}{EI}\right) = 12.5\ kNm.$$

$$M_{BC} = -15.62 + 0.8\,EI\left(\frac{3.91}{EI}\right) = -12.5\ kNm.$$

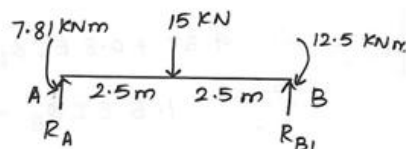$$M_{CB} = 15.62 + 0.4\,EI\left(\frac{3.91}{EI}\right) = 17.18\ kNm.$$

Step 5 : Shear force and Bending Moment :
 Shear force :
    For span AB,

$$\Sigma V = 0$$

$$R_A + R_{B_1} = 15 \rightarrow (i)$$

Taking moment at A,

$$-R_{B_1} \times 5 + 12.5 + (15 \times 2.5) - 7.81 = 0$$

$$-5R_{B_1} + 22.19 = 0$$

$$\boxed{R_{B_1} = 4.44 \, KN}$$

$(i) \Rightarrow R_A + 4.44 = 15$

$$\boxed{R_A = 10.56 \, KN}$$

For span BC,
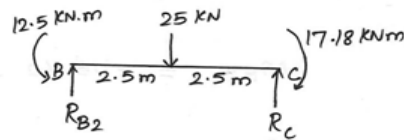
$\Sigma V = 0$

$R_{B_2} + R_c = 25 \rightarrow (ii)$

Taking moment at B,
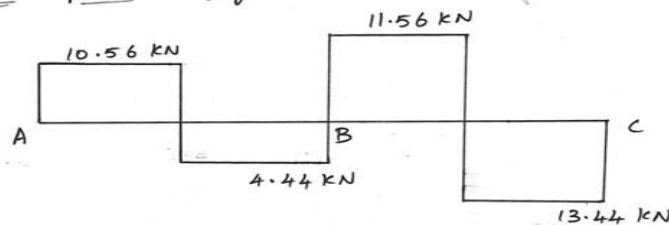
$$-R_c \times 5 + 17.18 + (25 \times 2.5) - 12.5 = 0$$

$$-5R_c + 67.18 = 0$$

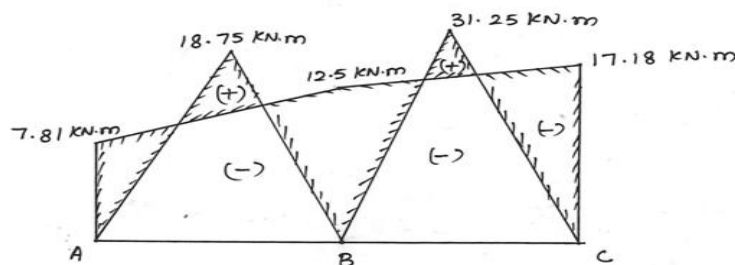$$\boxed{R_c = 13.44 \, KN}$$

$(ii) \Rightarrow \boxed{R_{B_2} = 11.56 \, KN}$
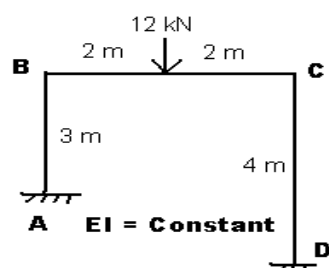
Shear force diagram :

Bending Moment diagram :

For span AB, $= \dfrac{\omega l}{4} = \dfrac{15 \times 5}{4} = 18.75 \, KN.m$

For span BC $= \dfrac{\omega l}{4} = \dfrac{25 \times 5}{4} = 31.25 \, KNm.$

4. Analyse the portal frame shown in figure by slope deflection method.

(AUC Nov/Dec 2012, 2013)

Solution:

Step 1 : Fixed End Moment :-

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = -\frac{wl}{8} = -\frac{12 \times 4}{8} = -6 \, kNm.$$

$$M_{FCB} = \frac{wl}{8} = \frac{12 \times 4}{8} = 6 \, kNm.$$

Step 2 : Slope deflection Equation :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= 0 + \frac{2EI\theta_B}{3}$$

$$= 0.67 \, EI\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(\theta_A + 2\theta_B)$$

$$= 0 + \frac{2EI(2\theta_B)}{3}$$

$$= 1.33 \, EI\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -6 + \frac{4EI\theta_B}{4} + \frac{2EI\theta_C}{4}$$

$$= -6 + EI\theta_B + 0.5 \, EI\theta_C$$

$$M_{CB} = 6 + \frac{2EI\theta_B}{4} + \frac{4EI\theta_C}{4}$$

$$= 6 + 0.5 \, EI\theta_B + EI\theta_C$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}(2\theta_C + \theta_D)$$

$$= 0 + \frac{4EI\theta_C}{4}$$

$$= EI\theta_C$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}(\theta_C + 2\theta_D)$$

$$= 0 + \frac{2EI\theta_C}{4}$$

$$= 0.5 EI\theta_C$$

Step 3 : Equilibrium Equations :-

$$M_{BA} + M_{BC} = 0 \longrightarrow \textcircled{1}$$

$$M_{CB} + M_{CD} = 0 \longrightarrow \textcircled{2}$$

$$\textcircled{1} \Rightarrow 1.33 EI\theta_B - 6 + EI\theta_B + 0.5EI\theta_C = 0$$

$$2.33 EI\theta_B + 0.5 EI\theta_C = 6 \longrightarrow \textcircled{3}$$

$$\textcircled{2} \Rightarrow 6 + 0.5EI\theta_B + EI\theta_C + EI\theta_C = 0$$

$$0.5 EI\theta_B + 2 EI\theta_C = -6 \longrightarrow \textcircled{4}$$

By solving ③ & ④ we get,

$$\theta_B = \frac{3.4}{EI}$$
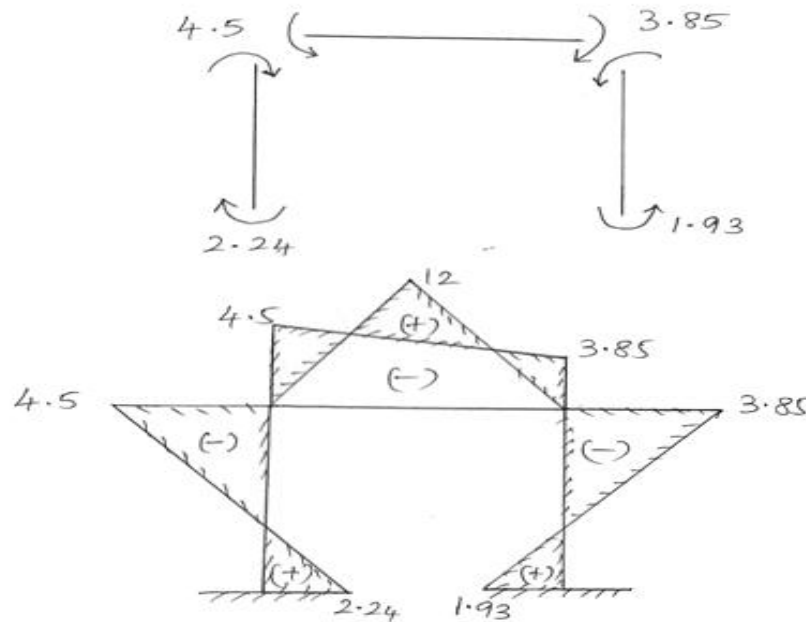
$$\theta_C = -\frac{3.85}{EI}$$

Step 4 : Final Moments :-

$$M_{AB} = 0.67 EI\theta_B = 0.67EI\left(\frac{3.4}{EI}\right) = 2.24 \, kNm.$$

$$M_{BA} = 1.33 EI\left(\frac{3.4}{EI}\right) = 4.5 \, kNm.$$
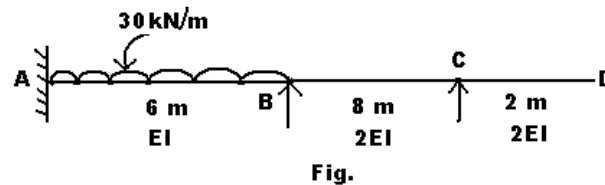
$$M_{BC} = -6 + EI\left(\frac{3.4}{EI}\right) + 0.5 EI\left(-\frac{3.85}{EI}\right) = -4.5 \, kNm.$$

$$M_{CB} = 6 + 0.5 EI\left(+\frac{3.4}{EI}\right) + EI\left(-\frac{3.85}{EI}\right) = 3.85 \, kNm.$$

$$M_{CD} = EI\left(-\frac{3.85}{EI}\right)$$

$$= -3.85 \, KNm$$

$$M_{DC} = 0.5 \, EI\left(-\frac{3.85}{EI}\right) = -1.93 \, KNm$$

5. **Using slope deflection method, analyze the beam shown in figure and draw the BMD.**

(AUC Apr/May 2011)

Fig.

Solution:

Step 1 : Fixed End Moment :-

$$M_{FAB} = -\frac{\omega l^2}{12} = -\frac{30 \times (6)^2}{12} = -90 \, KNm$$

$$M_{FBA} = \frac{\omega l^2}{12} = \frac{30 \times (6)^2}{12} = 90 \, KNm.$$

$$M_{FBC} = M_{FCB} = M_{FCD} = M_{FDC} = 0.$$

Step 2 : Slope Deflection Equation :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B + \frac{3\delta}{l}\right)$$

$$= -90 + \frac{2EI}{6}\left(\theta_B\right)$$

$$M_{AB} = -90 + 0.33\, EI\,\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B + \frac{3\delta}{l}\right)$$

$$= 90 + \frac{4EI\,\theta_B}{6}$$

$$M_{BA} = 90 + 0.67\, EI\,\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{3\delta}{l}\right)$$

$$= 0 + \frac{4EI(2)}{8}\left(2\theta_B\right) + \frac{4EI\,\theta_C}{8}$$

$$M_{BC} = 2EI\,\theta_B + 0.5\, EI\,\theta_C$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(\theta_B + 2\theta_C + \frac{3\delta}{l}\right)$$

$$= 0 + \frac{4EI\,\theta_B}{8} + \frac{8EI\,\theta_C}{8}$$

$$M_{CB} = 0.5\, EI\,\theta_B + EI\,\theta_C$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D + \frac{3\delta}{l}\right)$$

$$= 0 + \frac{8EI\,\theta_C}{2} + \frac{4EI\,\theta_D}{2}$$

$$M_{CD} = 4EI\,\theta_C + 2EI\,\theta_D$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(\theta_C + 2\theta_D + \frac{3\delta}{l}\right)$$

$$= 0 + \frac{4EI\,\theta_C}{2} + \frac{8EI\,\theta_D}{2}$$

$$M_{DC} = 2EI\,\theta_C + 4EI\,\theta_D$$

Step 3 : Equilibrium Equations :-

$$M_{BA} + M_{BC} = 0 \quad \longrightarrow ①$$

$$M_{CB} + M_{CD} = 0 \quad \longrightarrow ②$$

$$M_{DC} = 0 \quad \longrightarrow ③$$

$$M_{BA} + M_{BC} = 0$$

$$90 + 0.67 EI\theta_B + EI\theta_B + 0.5 EI\theta_c = 0$$

$$1.67 EI\theta_B + 0.5 EI\theta_c = -90 \longrightarrow ④$$

$$M_{CB} + M_{CD} = 0$$

$$0.5 EI\theta_B + EI\theta_c + 4EI\theta_c + 2EI\theta_D = 0$$

$$0.5 EI\theta_B + 5EI\theta_c + 2EI\theta_D = 0 \longrightarrow ⑤$$

$$M_{DC} = 0$$

$$2EI\theta_c + 4EI\theta_D = 0 \longrightarrow ⑥$$

$$④ + ⑥ \Rightarrow$$

$$1.67EI\theta_B + 2.5 EI\theta_c + 4EI\theta_D = -90 \longrightarrow ⑦$$

By solving eqn ④, ⑤ & ⑥ we get,

$$\theta_B = -\frac{56.33}{EI} \ ; \quad \theta_c = \frac{7.04}{EI} \ ; \quad \theta_D = -\frac{3.52}{EI}$$

Step 4 : Final Moments :

$$M_{AB} = -90 + 0.33 EI\left(-\frac{56.33}{EI}\right) = -108.5 \ kNm$$

$$M_{BA} = 90 + 0.66 EI\left(-\frac{56.33}{EI}\right) = 52.82 \ kNm$$

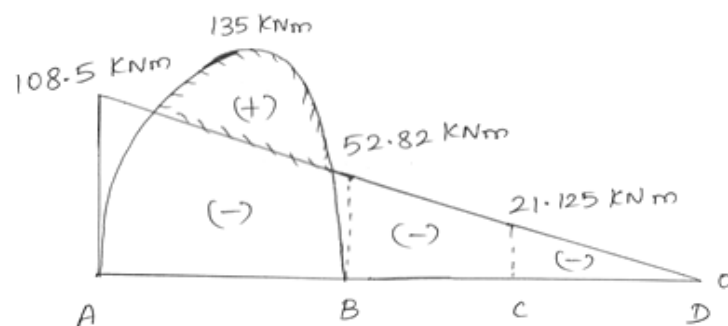$$M_{BC} = EI\left(-\frac{56.33}{EI}\right) + 0.5 EI\left(\frac{7.04}{EI}\right) = -52.82 kNm$$

$$M_{CB} = EI\left(\frac{7.04}{EI}\right) + 0.5\left(-\frac{56.33}{EI}\right) = -21.125 \ kNm$$

$$M_{CD} = 4 EI\left(\frac{7.04}{EI}\right) + 2 EI\left(-\frac{3.52}{EI}\right) = 21.125 \ kNm$$

$$M_{DC} = 4 EI\left(-\frac{3.52}{EI}\right) + 2 EI\left(\frac{7.04}{EI}\right) = 0$$
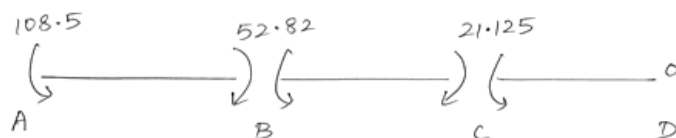
Step 5 : Bending Moment diagram :-

6. **Analyze the frame shown in figure by slope deflection method. Take EI = constant.**
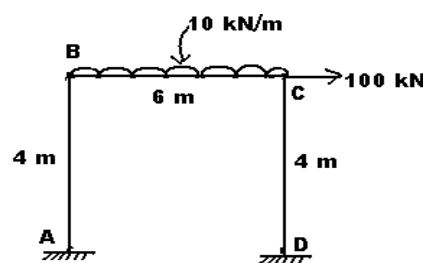
(AUC Apr/May 2011)

Fig.

Solution:

Step 1 : Fixed End Moment :-

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{10 \times (6)^2}{12} = -30 \text{ KNm}.$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{10 \times (6)^2}{12} = 30 \text{ KNm}$$

Step 2 : Slope deflection Equation :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B\right)$$

$$= 0 + \frac{2EI}{4}\left(\theta_B\right)$$

$$= 0.5 EI \, \theta_B.$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(\theta_A + 2\theta_B)$$

$$= 0 + \frac{2EI(2\theta_B)}{4}$$

$$= EI\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_c)$$

$$= -30 + \frac{2EI(2\theta_B)}{6} + \frac{2EI\theta_c}{6}$$

$$= -30 + 0.67EI\theta_B + 0.33EI\theta_c$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(\theta_B + 2\theta_c)$$

$$= 30 + \frac{2EI}{6}(\theta_B) + \frac{2EI(2\theta_c)}{6}$$

$$= 30 + 0.33EI\theta_B + 0.67EI\theta_c$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}(2\theta_c + \theta_D)$$

$$= 0 + \frac{2EI(2\theta_c)}{4}$$

$$= EI\theta_c$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}(\theta_c + 2\theta_D)$$

$$= 0 + \frac{2EI\theta_c}{4}$$

$$= 0.5EI\theta_c$$

Step 3 : Equilibrium Equations :-

$$M_{BA} + M_{BC} = 0 \longrightarrow ①$$

$$M_{CB} + M_{CD} = 0 \longrightarrow ②$$

$$① \Rightarrow EI\theta_B - 30 + 0.67EI\theta_B + 0.33EI\theta_c = 0$$

$$1.67EI\theta_B + 0.33EI\theta_c = 30 \longrightarrow ③$$

$$② \Rightarrow 30 + 0.33EI\theta_B + 0.67EI\theta_c + EI\theta_c = 0$$

$$0.33EI\theta_B + 1.67EI\theta_c = -30 \longrightarrow ④$$

By solving ③ & ④ we get,

$$\theta_B = \frac{22.47}{EI}$$

$$\theta_C = -\frac{22.78}{EI}$$
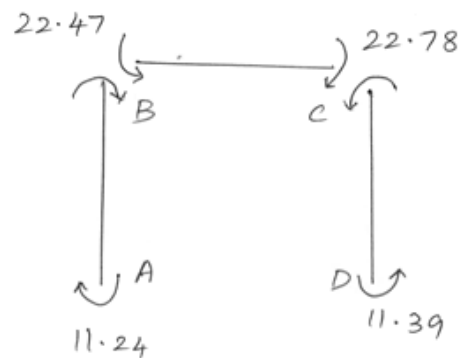
step 4 : Final Moments :-

$M_{AB} = 11.24 \ kNm$

$M_{BA} = 22.47 \ kNm$

$M_{BC} = -22.47 \ kNm$

$M_{CB} = 22.15 \ kNm$
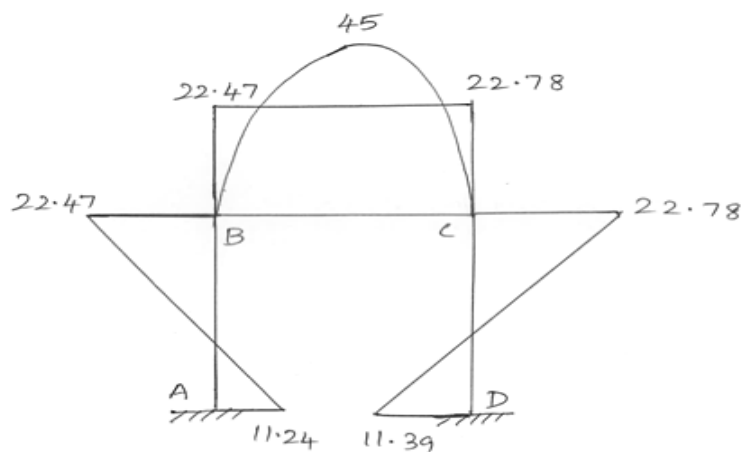
$M_{CD} = -22.78 \ kNm$

$M_{DC} = -11.39 \ kNm$

step 5 : Bending Moment Diagram :-

7. Analyze the continuous beam ABC shown in figure by slope deflection method. Draw also the bending moment diagram. Take EI = constant. **(AUC Nov/Dec 2010)**
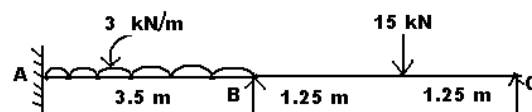

Fig.

Solution:

Step:1 : Fixed End Moments :-

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{3 \times (3.5)^2}{12} = -3.06 \, kNm$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{3 \times (3.5)^2}{12} = 3.06 \, kNm$$

$$M_{FBC} = -\frac{wl}{8} = -\frac{15 \times 2.5}{8} = -4.68 \, kNm$$

$$M_{FCB} = \frac{wl}{8} = \frac{15 \times 2.5}{8} = 4.68 \, kNm$$

Step 2 : slope Deflection Equation :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B + \frac{3\delta}{l}\right)$$

$$= -3.06 + \frac{2EI \, \theta_B}{3.5}$$

$$= -3.06 + 0.57 \, EI \, \theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B + \frac{3\delta}{l}\right)$$

$$= 3.06 + \frac{4EI \, \theta_B}{3.5}$$

$$= 3.06 + 1.14 \, EI \, \theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_c + \frac{3\delta}{l}\right)$$

$$= -4.68 + \frac{4EI \, \theta_B}{2.5} + \frac{2EI \, \theta_c}{2.5}$$

$$= -4.68 + 1.6 \, EI \, \theta_B + 0.8 \, EI \, \theta_c$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(\theta_B + 2\theta_c + \frac{3\delta}{l}\right)$$

$$= 4.68 + \frac{2EI \, \theta_B}{2.5} + \frac{4EI \, \theta_c}{2.5}$$

$$= 4.68 + 0.8 \, EI \, \theta_B + 1.6 \, EI \, \theta_c$$

Step 3: Equilibrium Equations :-

$$M_{BA} + M_{BC} = 0 \rightarrow ①$$

$$M_{CB} = 0 \rightarrow ②$$

① $\Rightarrow M_{BA} + M_{BC} = 0$

$3.06 + 1.14\, EI\, \theta_B - 4.68 + 1.6\, EI\, \theta_B + 0.8\, EI\, \theta_C = 0$

$2.74\, EI\, \theta_B + 0.8\, EI\, \theta_C = 1.62 \rightarrow ③$

② $\Rightarrow M_{CB} = 0$

$4.68 + 0.8\, EI\, \theta_B + 1.6\, EI\, \theta_C = 0$

$0.8\, EI\, \theta_B + 1.6\, EI\, \theta_C = -4.68 \rightarrow ④$

By solving ③ & ④ we get,

$$\theta_B = \frac{1.69}{EI}$$

$$\theta_C = -\frac{3.77}{EI}$$

Step 4: Final Moments :-

$M_{AB} = -3.06 + 0.57\, EI\, \theta_B$

$\quad = -3.06 + 0.57\, EI\left(\frac{1.69}{EI}\right)$

$M_{AB} = -2.09\ KNm$

$M_{BA} = 3.06 + 1.14\, EI\, \theta_B$

$\quad = 3.06 + 1.14\, EI\left(\frac{1.69}{EI}\right)$

$M_{BA} = 4.98\ KNm$

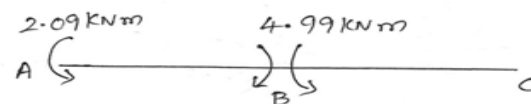$M_{BC} = -4.68 + 1.6\, EI\, \theta_B + 0.8\, EI\, \theta_C$

$\quad = -4.68 + 1.6\, EI\left(\frac{1.69}{EI}\right) + 0.8\, EI\left(\frac{-3.77}{EI}\right)$
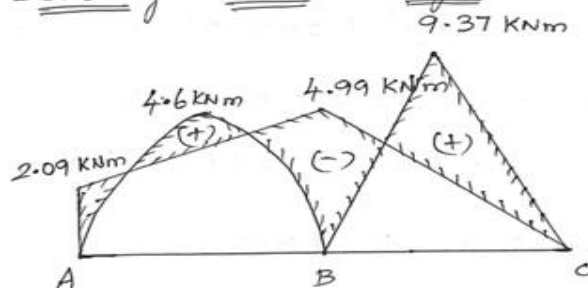
$M_{BC} = -4.99\ KNm$

$M_{CB} = 4.68 + 0.8\, EI\, \theta_B + 1.6\, EI\, \theta_C$

$\quad = 4.68 + 0.8\, EI\left(\frac{1.69}{EI}\right) + 1.6\, EI\left(\frac{-3.77}{EI}\right)$

$M_{CB} = 0$

step 5: Bending Moment diagram:-

8. **Analyze the portal frame ABCD shown in figure by slope deflection method. Take EI = constant.** **(AUC Nov/Dec 2010, May/June 2014)**
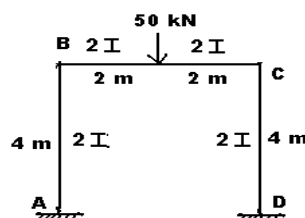

Fig.

Solution:

Step 1: Fixed End Moment :-

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = -\frac{wl}{8} = -\frac{50 \times 4}{8} = -25 \, KNm$$

$$M_{FCB} = \frac{wl}{8} = \frac{50 \times 4}{8} = 25 \, KNm$$

Step 2 : Slope Deflection Equation :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B\right)$$

$$= 0 + \frac{2E(2I)}{4}\left(\theta_B\right)$$

$$= EI\,\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B\right)$$

$$= 0 + \frac{2E(2I)}{4}\left(2\theta_B\right)$$

$$= 2EI\,\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -25 + \frac{2E(4I)}{4}(2\theta_B) + \frac{2E(4I)\theta_C}{4}$$

$$= -25 + 4EI\theta_B + 2EI\theta_C$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(\theta_B + 2\theta_C)$$

$$= 25 + \frac{2E(4I)\theta_B}{4} + \frac{2E(4I)(2\theta_C)}{4}$$

$$= 25 + 2EI\theta_B + 4EI\theta_C$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}(2\theta_C + \theta_D)$$

$$= 0 + \frac{2E(2I)}{4}(2\theta_C) + \frac{2E(2I)\theta_D}{4}$$

$$= 2EI\theta_C$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}(\theta_C + 2\theta_D)$$

$$= 0 + \frac{2E(2I)\theta_C}{4}$$

$$= EI\theta_C$$

Step 3 : Equilibrium Equations :-

$$M_{BA} + M_{BC} = 0 \longrightarrow \text{①}$$

$$M_{CB} + M_{CD} = 0 \longrightarrow \text{②}$$

$$\text{①} \Rightarrow 2EI\theta_B - 25 + 4EI\theta_B + 2EI\theta_C = 0$$

$$6EI\theta_B + 2EI\theta_C = 25 \longrightarrow \text{③}$$

$$\text{②} \Rightarrow 25 + 2EI\theta_B + 4EI\theta_C + 2EI\theta_C = 0$$

$$2EI\theta_B + 6EI\theta_C = -25 \longrightarrow \text{④}$$

By solving ③ & ④ we get,

$$\theta_B = \frac{6.25}{EI} \quad ; \quad \theta_C = -\frac{6.25}{EI}$$

step 4 : Final Moments :-

$M_{AB} = 6.25\ KNm$

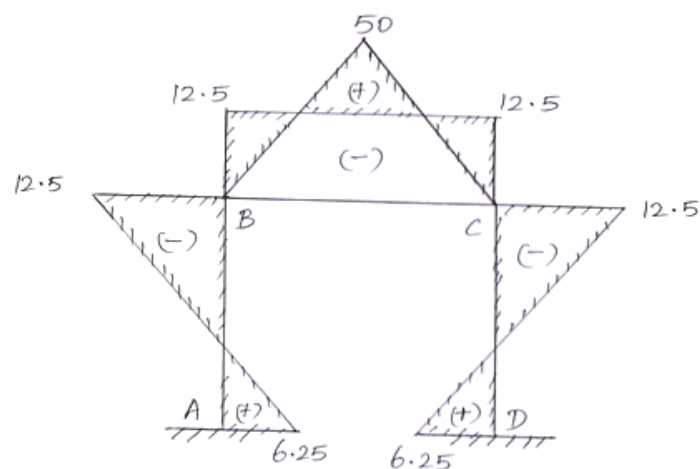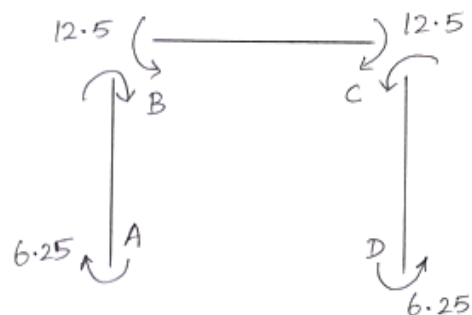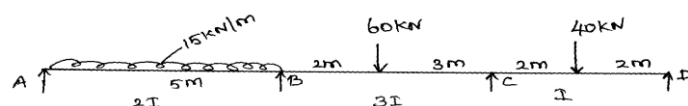$M_{BA} = 12.5\ KNm$

$M_{BC} = -12.5\ KNm$

$M_{CB} = 12.5\ KNm$

$M_{CD} = -12.5\ KNm$

$M_{DC} = -6.25\ KNm$

9. **Analyse the continuous beam given in figure by slope deflection method and draw the B.M.D.** **(AUC May/June 2014)**

Solution:

Step 1 : Fixed End Moment :

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{15 \times (5)^2}{12} = -31.25 \, KNm$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{15 \times (5)^2}{12} = 31.25 \, KNm$$

$$M_{FBC} = -\frac{wab^2}{l^2} = -\frac{60 \times 2 \times (3)^2}{(5)^2} = -43.2 \, KNm$$

$$M_{FCB} = \frac{wab^2}{l^2} = \frac{60 \times 2 \times (3)^2}{(5)^2} = 43.2 \, KNm$$

$$M_{FCD} = -\frac{wl}{8} = -\frac{40 \times 4}{8} = -20 \, KNm$$

$$M_{FDC} = \frac{wl}{8} = \frac{40 \times 4}{8} = 20 \, KNm$$

Step 2 : Slope Deflection Equation :-

Unknowns $= \theta_A, \theta_B, \theta_c, \theta_D$

$\delta = 0$ (No deflection)

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -31.25 + \frac{2E(2I)}{5}(2\theta_A) + \frac{2E(2I)\theta_B}{5}$$

$$= -31.25 + 1.6EI\theta_A + 0.8EI\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(\theta_A + 2\theta_B)$$

$$= 31.25 + \frac{2E(2I)\theta_A}{5} + \frac{2E(2I)}{5}(2\theta_B)$$

$$= 31.25 + 0.8EI\theta_A + 1.6EI\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_c)$$

$$= -43.2 + \frac{2E(3I)}{5}(2\theta_B) + \frac{2E(3I)\theta_c}{.5}$$

$$= -43.2 + 2.4EI\theta_B + 1.2EI\theta_c$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(\theta_B + 2\theta_C)$$

$$= 43.2 + \frac{2E(3I)\theta_B}{5} + \frac{2E(3I)(2\theta_C)}{5}$$

$$= 43.2 + 1.2EI\theta_B + 2.4EI\theta_C$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}(2\theta_C + \theta_D)$$

$$= -20 + \frac{2EI(2\theta_C)}{4} + \frac{2EI\theta_D}{4}$$

$$= -20 + EI\theta_C + 0.5EI\theta_D$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}(\theta_C + 2\theta_D)$$

$$= 20 + \frac{2EI\theta_C}{4} + \frac{2EI(2\theta_D)}{4}$$

$$= 20 + 0.5EI\theta_C + EI\theta_D$$

step 3 : Equilibrium Equations :-

$$M_{AB} = 0 \longrightarrow \text{①}$$

$$M_{BA} + M_{BC} = 0 \longrightarrow \text{②}$$

$$M_{CB} + M_{CD} = 0 \longrightarrow \text{③}$$

$$M_{DC} = 0 \longrightarrow \text{④}$$

① $\Rightarrow$ $-31.25 + 1.6EI\theta_A + 0.8EI\theta_B = 0 \longrightarrow \text{⑤}$

② $\Rightarrow$ $31.25 + 0.8EI\theta_A + 1.6EI\theta_B - 43.2$
$$+ 2.4EI\theta_B + 1.2EI\theta_C = 0$$

$$0.8EI\theta_A + 4EI\theta_B + 1.2EI\theta_C = 11.95 \longrightarrow \text{⑥}$$

③ $\Rightarrow$ $43.2 + 1.2EI\theta_B + 2.4EI\theta_C - 20$
$$+ EI\theta_C + 0.5EI\theta_D = 0$$

$$1.2EI\theta_B + 3.4EI\theta_C + 0.5EI\theta_D = -23.2 \longrightarrow \text{⑦}$$

④ ⟹ $20 + 0.5 EI \theta_C + EI \theta_D = 0 \rightarrow$ ⑧

Adding ⑤ & ⑧,

$0.8 EI\theta_A + 4 EI \theta_B + 1.7 EI \theta_C + EI\theta_D = -8.05 \rightarrow$ ⑨

Adding ⑤ & ⑦,

$1.6 EI \theta_A + 2 EI\theta_B + 3.4 EI\theta_C + 0.5 EI\theta_D = -23.2 \rightarrow 10$

Adding ⑥ & ⑦,

$0.8 EI\theta_A + 5.2 EI \theta_B + 4.6 EI\theta_C + 0.5 EI\theta_D = -11.25 \rightarrow$ ⑪

By solving ⑨, ⑩ & ⑪,

$$\theta_A = \theta_B = \theta_c = \theta_D = 0$$
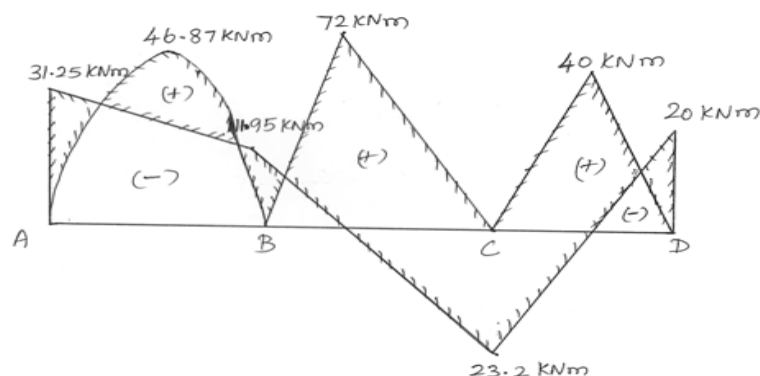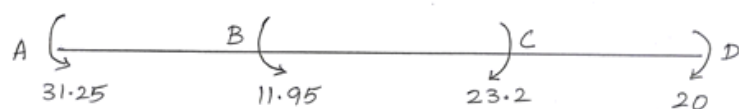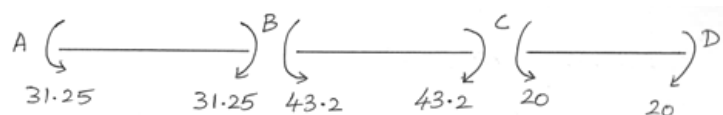
step 4 :   Final Moments :-

$M_{AB} = -31.25 \ KNm$

$M_{BA} = 31.25 \ KNm$

$M_{BC} = -43.2 \ KNm$
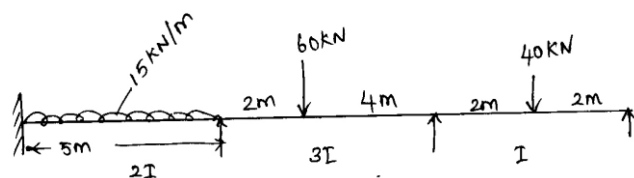
$M_{CB} = 43.2 \ KNm$

$M_{CD} = -20 \ KNm$

$M_{DC} = 20 \ KNm$

10. Analyse the continuous beam given in figure by slope deflection method and draw the B.M.D.

Solution:

Step1 : Fixed End Moment :-

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{15 \times (5)^2}{12} = -31.25 \, KNm.$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{15 \times (5)^2}{12} = 31.25 \, KNm.$$

$$M_{FBC} = -\frac{w \, ab^2}{l^2} = -\frac{60 \times 2 \times (4)^2}{(6)^2} = -53.33 \, KNm.$$

$$M_{FCB} = \frac{w \, ab^2}{l^2} = \frac{60 \times 2 \times (4)^2}{(6)^2} = 53.33 \, KNm.$$

$$M_{FCD} = -\frac{wl}{8} = -\frac{40 \times 4}{8} = -20 \, KNm.$$

$$M_{FDC} = \frac{wl}{8} = \frac{40 \times 4}{8} = 20 \, KNm.$$

Step 2 : SLOPE DEFLECTION EQUATION :-

Unknowns, $= \theta_B, \theta_c, \theta_D$

$\theta_A = 0$; $\delta = 0$ (NO deflection)

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -31.25 + \frac{2E(2I)}{l}(\theta_B)$$

$$= -31.25 + \frac{4EI \, \theta_B}{5}$$

$$= -31.25 + 0.8 \, EI \, \theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(\theta_A + 2\theta_B)$$

$$= +31.25 + \frac{4EI}{5}(2\theta_B)$$

$$= +31.25 + 1.6 \, EI \, \theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -53.33 + \frac{2E(3I)}{6}(2\theta_B) + \frac{2E(3I)\theta_C}{6}$$

$$= -53.33 + 2EI\theta_B + EI\theta_C$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(\theta_B + 2\theta_C)$$

$$= 53.33 + \frac{2E(3I)\theta_B}{6} + \frac{2E(3I)}{6}(2\theta_C)$$

$$= 53.33 + EI\theta_B + 2EI\theta_C$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}(2\theta_C + \theta_D)$$

$$= -20 + \frac{2EI}{4}(2\theta_C) + \frac{2EI\theta_D}{4}$$

$$= -20 + EI\theta_C + 0.5EI\theta_D$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}(\theta_C + 2\theta_D)$$

$$= 20 + \frac{2EI\theta_C}{4} + \frac{2EI(2\theta_D)}{4}$$

$$M_{DC} = 20 + 0.5EI\theta_C + EI\theta_D$$

Step 3 : Equilibrium Equations :-

$$M_{BA} + M_{BC} = 0 \rightarrow ①$$

$$M_{CB} + M_{CD} = 0 \rightarrow ②$$

$$M_{DC} = 0 \rightarrow ③$$

① $\Rightarrow 31.25 + 1.6EI\theta_B - 53.33 + 2EI\theta_B + EI\theta_C = 0$

$$3.6EI\theta_B + EI\theta_C = 22.08 \rightarrow ④$$

② $\Rightarrow 53.33 + EI\theta_B + 2EI\theta_C - 20 + EI\theta_C + 0.5EI\theta_D = 0$

$$EI\theta_B + 3EI\theta_C + 0.5EI\theta_D = -33.33 \rightarrow ⑤$$

③ $\Rightarrow 20 + 0.5EI\theta_C + EI\theta_D = 0$

$$0.5EI\theta_C + EI\theta_D = -20 \rightarrow ⑥$$

$$④ + ⑥ \Rightarrow 3.6EI\theta_B + 1.5EI\theta_C + EI\theta_D = 2.08 \rightarrow ⑦$$

$$⑤ - \frac{⑦}{2} \Rightarrow 0.8\theta_B - 2.25\theta_C = 34.37 \rightarrow ⑧$$

$$⑧ + ④ \times 2.25 \Rightarrow \theta_B = \frac{9.44}{EI} ; \theta_C = -\frac{11.9}{EI} ; \theta_D = -\frac{14.05}{EI}$$

Step 4: Final Moments :-

$$M_{AB} = -31.25 + 0.8EI\left(\frac{9.44}{EI}\right)$$
$$= -23.69 \text{ kNm}$$

$$M_{BA} = 31.25 + 1.6EI\left(\frac{9.44}{EI}\right)$$
$$= 46.35 \text{ kNm}$$

$$M_{BC} = -53.33 + 2EI\left(\frac{9.44}{EI}\right) + EI\left(-\frac{11.9}{EI}\right)$$
$$= -46.35 \text{ kNm}$$

$$M_{CB} = 53.33 + EI\left(\frac{9.44}{EI}\right) + 2EI\left(-\frac{11.9}{EI}\right)$$
$$= 38.93 \text{ kNm}$$

$$M_{CD} = -20 + EI\left(-\frac{11.9}{EI}\right) + 0.5EI\left(-\frac{14.05}{EI}\right)$$
$$= -38.93 \text{ kNm}$$

$$M_{DC} = 20 + 0.5EI\left(-\frac{11.9}{EI}\right) + EI\left(-\frac{14.05}{EI}\right)$$
$$= 0$$

step 5 : Bending Moment diagram :-