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QUESTION BANK

DEPARTMENT: CIVIL   **SEMESTER: V**

SUBJECT CODE / Name: CE 2302 / STRUCTURAL ANALYSIS-I

UNIT 3 – ARCHES

PART – A (2 marks)

1. **Write the types of arches based on the number of hinges.**

    (AUC Apr/May 2012, Nov/Dec 2013, May/June 2014)

    - ➢ Two hinged arch
    - ➢ Three hinged arch
    - ➢ Fixed arch
    - ➢ Four hinged
    - ➢ One hinged
    - ➢ Support reactions

2. **What is meant by degree of static indeterminacy of a structure?   (AUC Apr/May 2012)**

    If the conditions of statics i.e., $\sum H=0$, $\sum V=0$ and $\sum M=0$ alone are not sufficient to find either external reactions or internal forces in a structure, the structure is called a statically indeterminate structure.

3. **Write the difference between circular arch and parabolic arch.   (AUC Nov/Dec 2012)**

S.No	Circular arch	Parabolic arch
1	The calculation part is difficult in this circular type of arches.	The calculation part is easier in this parabolic type of arches.
2	The equation to find the height 'y' under the section is $$y = \frac{4y_C}{L^2}\ (L-2x)$$	The equation to find the height 'y' under the section is $R^2 = x^2 + (R - y_C + y)^2$ Here R can be determined by $$(2R_C - y_C)\ y_C = \left(\frac{L}{2}\right)^2$$

4. **Give the equation for temperature effect in arches.   (AUC Nov/Dec 2012)**

    Horizontal thrust, H = $\dfrac{l\alpha TEI}{\int_{o}^{l} y^2 dx}$

5. **Derive the expression for the horizontal thrust in a three hinged parabolic arch carrying UDL over entire span.** **(AUC Apr/May 2011)**
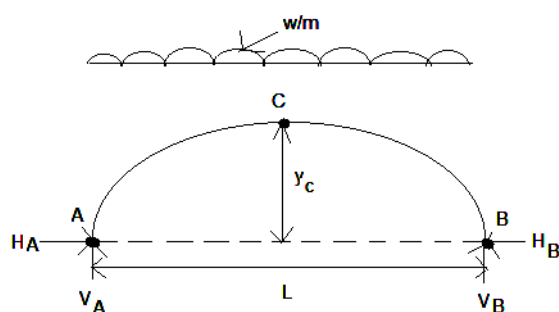
Here $V_A = \dfrac{wl}{2}$; $V_B = \dfrac{wl}{2}$

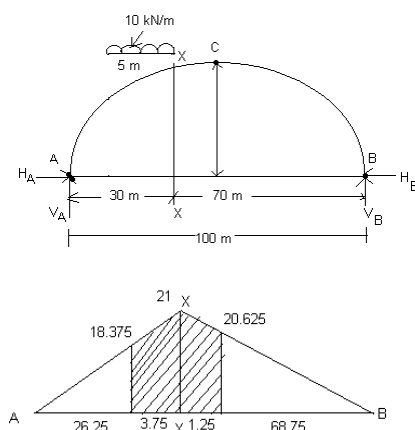Taking moment about C,

$$V_A \times \frac{l}{2} - \left(\frac{wl}{2} X \frac{l}{2} \right) - H_A \times \frac{l}{2} = 0$$

$$\frac{wl}{2} \times \frac{l}{2} - \left(\frac{wl}{2} X \frac{l}{2} \right) = H_A \times \frac{l}{2}$$

Horizontal thrust, $H_A = 0$

6. **Find the maximum bending moment at a section 30 m from the left end of the three hinged stiffening girder of span 100 m when a UDL of 10 kN/m, 5 m length crosses the girder.**
(AUC Apr/May 2011)

Maximum BM = $10 \times \left[(18.375 \times 3.75) + \left(\frac{1}{2} \times 3.75 \times 2.625 \right) + (20.625 \times 1.25) + \right.$

$$\left. \left(\frac{1}{2} \times 1.25 \times 0.375 \right) \right]$$

$$= 10 \times 99.84$$

$$= 998.4 \text{ kNm}$$

7. **Which theorem is utilized in solving the two hinged arch? State the theorem.**

(AUC Nov/Dec 2010)

Eddy's theorem is utilized in solving the two hinged arch.

Eddy's theorem states that "The bending moment at any section of an arch is proportional to the vertical intercept between the linear arch (or theoretical arch) and the centre line of the actual arch".

8. **What is the degree of static indeterminacy of the fixed arch?** **(AUC Nov/Dec 2010)**

The degree of static indeterminacy of the fixed arch is three.

9. **Name the different types of arch as per structure configuration.**
   - ➢ Curved arch
   - ➢ Parabolic arch
   - ➢ Elliptical arch
   - ➢ Polygonal arch

10. **Give an expression for the determination of horizontal thrust of a two hinged arch considering bending deformation only.**

$$\text{Horizontal thrust, } H = \frac{\int_0^l \mu y\, dx}{\int_0^l y^2\, dx}$$

11. **What is a three hinged arch and two hinged arch?**

| Sl.No.	Two hinged arches	Three hinged arches
1	Statically indeterminate to first degree	Statically determinate
2	Might develop temperature stresses	Increase in temperature causes increase in central rise. No stresses.
3	Structurally more efficient	Easy to analyse. But in construction, the central hinge may involve additional expenditure.
4	Will develop stresses due to sinking of supports	Since this is determinate, no stresses due to support sinking.

12. **Explain the transfer of load to the arches.**
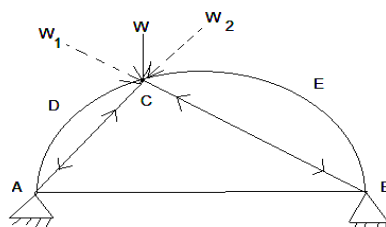
We have a load W acting at a point C on an arch. If there were straight members CA and CB, the load W would transfer directly to the supports A and B in the form of thrusts along the notational members CA and CB.

13. **Give the applications of two hinged arches.** **(AUC May/June 2014)**
   - Two hinged arches are more practical.
   - They are statically indeterminate structures.
   - We have to invoke Castigliano's II theorem to sort out the support reactions.
   - This is applicable to all shapes of arches.

14. Differentiate between the cable and arch.

Sl.No	Arch	Cable
1	An arch is essentially a compression member which can also take bending moments and shears.	A cable can take only tension.
2	Bending moments and shears will be absent if the arch is parabolic and the loading uniformly distributed.	The girder will take the bending moment and shears in the bridge and the cable, only tension.

15. Write down the expression for the horizontal thrust when the two hinged arch is subjected to uniformly distributed load throughout the span.

Horizontal thrust, $H = \dfrac{\int_0^l \mu y \, dx}{\int_0^l y^2 \, dx}$
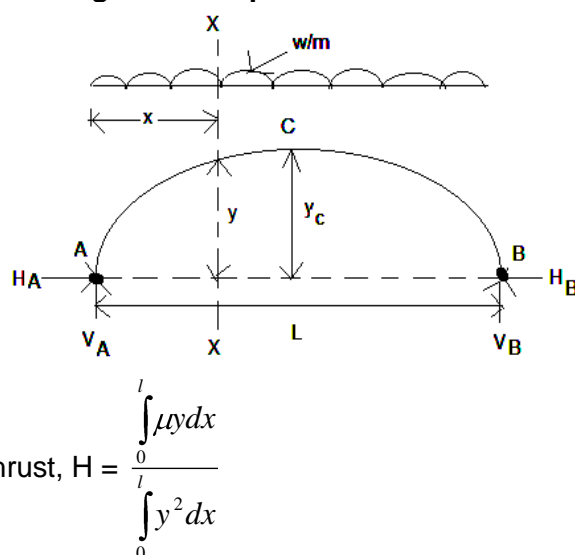
Where μ = bending moment at the section for loading portion and unloading portion.

        y = height in the arch where the section is acting.

16. Explain the term Horizontal thrust.   **(AUC Nov/Dec 2013)**

        The horizontal force is calculated by equating the bending moment at the central hinge to zero is called as Horizontal thrust. i.e., $\sum M = 0$.
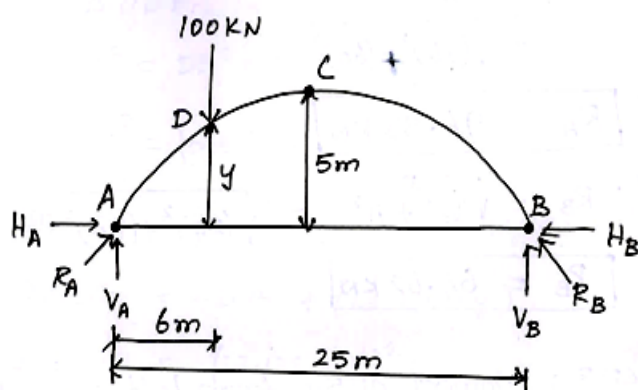
$$M_X = \mu_X - H_y$$

Horizontal thrust, $H = \dfrac{\int_0^l \mu y \, dx}{\int_0^l y^2 \, dx}$

PART – B (16 marks)

1. **A circular (three hinged) arch of span 25 m with a central rise of 5 m is hinged at the crown and the end supports. It carries a point load of 100 kN at 6 m from the left support. Calculate**

    i.   **The reaction at the supports and**

    ii. **Moment at 5 m from the left support.**   **(AUC Apr/May 2012)**

    Solution:

step 1 : Vertical reactions and Horizontal thrust (H) :-

i) Vertical reactions V_A and V_B :-

Taking moment about A,

$$(100 \times 6) - V_B (25) = 0$$

$$\boxed{V_B = 24 \ KN}$$

$\Sigma V = 0,$

$$V_A + V_B = 100$$

$$\therefore V_A + 24 = 100$$

$$\boxed{V_A = 76 \ KN}$$

ii) Horizontal thrust (H) :-

Taking moment about C,

$$- V_B \times 12 \cdot 5 + (H \times 5) = 0$$

$$H = \frac{24 \times 12 \cdot 5}{5}$$

$$\boxed{H = 60 \ KN}$$

Step 2 : Reactions R_A and R_B :-
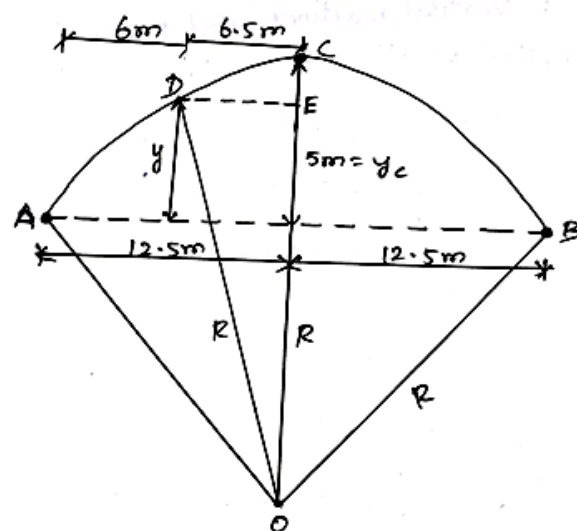
$$R_A = \sqrt{V_A{}^2 + H^2}$$

$$= \sqrt{(76)^2 + (60)^2}$$

$$\boxed{R_A = 96.83 \text{ KN}}$$

$$R_B = \sqrt{V_B{}^2 + H^2} = \sqrt{(24)^2 + (60)^2}$$

$$\boxed{R_B = 64.62 \text{ KN}}$$

Step 3 : Moment at 5m from left support :-

To find R :-

$$(2R - y_c)y_c = \left(\tfrac{l}{2}\right)^2$$

$$(2R - 5)5 = (12.5)^2$$

$$10R - 25 = 156.25$$

$$\boxed{R = 18.125 \text{ m}}$$

To find y :-

In $\triangle ODE$,

$$OD^2 = DE^2 + OE^2$$

$$(R)^2 = (6.5)^2 + (R - y_c + y)^2$$

$$\left(18.125\right)^2 = \left(6.5\right)^2 + \left(18.125 - 5 + y\right)^2$$

$$328.52 = 42.25 + \left(13.125 + y\right)^2$$

$$286.27 = \left(13.125 + y\right)^2$$

Taking square root on both sides,

$$16.92 = 13.125 + y$$

$$\boxed{y = 3.795 \text{ m}}$$

$$\left(BM\right)_D = V_A \times 6 - H \times 3.795$$

$$= \left(76 \times 6\right) - \left(60 \times 3.795\right)$$

$$\boxed{\left(BM\right)_D = 228.3 \text{ KN m.}}$$

2. **A parabolic two hinged arch has a span of 40 m and a rise of 5 m. a concentrated load 10 kN acts at 15 m from the left support. The second moment of area varies as the secant of the inclination of the arch axis. Calculate the horizontal thrust and reactions at the hinge. Also calculate maximum bending moment at the section.** **(AUC Apr/May 2012)**

Solution:

step 1: Vertical Reactions :-

$$\Sigma V = 0$$

$$V_A + V_B = 10$$

Taking moment about A,

$$-V_B \times 40 + \left(10 \times 15\right) = 0$$

$$\boxed{V_B = 3.75 \ kN}$$

$$\therefore \ V_A = 10 - 3.75$$

$$\boxed{V_A = 6.25 \ kN}$$

Step 2 : **Horizontal Thrust (H) :-**

$$H = \frac{\int_0^l \mu y \, dx}{\int_0^l y^2 \, dx}$$

$$H = \frac{\int_0^{15} \mu_1 \, y \, dx + \int_{15}^{40} \mu_2 \, y \, dx}{\int_0^{40} y^2 \, dx} = \frac{Nr(1) + N_r(2)}{Dr}$$

where, μ_1 = Beam bending moment in Ax.

μ_2 = Beam bending moment in xB.

i) **Denominator :-**

$$y = \frac{4 \, y_c}{l^2} \times (l - x) = \frac{4 \times 5}{(40)^2} (40x - x^2)$$

$$y = 0.5 x - 0.0125 x^2 .$$

$$\therefore \ Dr = \int_0^{40} y^2 \, dx$$

$$= \int_0^{40} (0.5x - 0.0125 \, x^2)^2 dx$$

$$= \int_0^{40} (0.25x^2 + 1.56 \times 10^{-4} x^4 - 0.0125 \, x^3) dx$$

$$= \left[\frac{0.25 x^3}{3} + \frac{1.56 \times 10^{-4} \, x^5}{5} - \frac{0.0125 \, x^4}{4} \right]_0^{40}$$

$$= \left[(5333.33 + 3194.88 - 8000) - 0 \right]$$

$$\boxed{Dr = 528.21}$$

ii) Numerator (1) :-

$$NY(1) = \int_0^{15} \mu_1 \, y \, dx$$

here, $\mu_1 = V_A x_1 = 6.25 x_1 = 6.25 x$

$$NY(1) = \int_0^{15} 6.25x \, (0.5x - 0.0125x^2) \, dx$$

$$= \int_0^{15} (3.125 x^2 - 0.078 x^3) \, dx$$

$$= \left[\frac{3.125 x^3}{3} - \frac{0.078 x^4}{4} \right]_0^{15}$$

$$= \left[(3515.63 - 987.18) - 0 \right]$$

$$\boxed{NY(1) = 2528.45}$$

iii) Numerator (2) :-

$$NY(2) = \int_{15}^{40} \mu_2 \, y \, dx$$

here, $\mu_2 = V_A \times x_2 - 10(x_2 - 15)$

$$= 6.25x - 10x + 150$$

$$\mu_2 = 150 - 3.75x$$

$$NY(2) = \int_{15}^{40} (150 - 3.75x)(0.5x - 0.0125x^2) \, dx$$

$$= \int_{15}^{40} (75x - 1.875x^2 - 1.875x^2 + 0.047x^3) \, dx$$

$$= \int_{15}^{40} (75x - 3.75x^2 + 0.047x^3) \, dx$$

$$= \left[\frac{75x^2}{2} - \frac{3.75x^3}{3} + \frac{0.047x^4}{4} \right]_{15}^{40}$$

$$= \left[(60000 - 80000 + 30080) - (8437.5 - 4218.75 + 594.84) \right]$$

$$\boxed{N_Y(2) = 5266.41}$$

$$\therefore \ H = \frac{N_Y(1) + N_Y(2)}{D_r} = \left(\frac{2528.45 + 5266.41}{528.21}\right)$$

$$\boxed{H = 14.76 \, kN}$$

step 3: Reactions at A and B :-

$$R_A = \sqrt{V_A^2 + H^2} = \sqrt{(6.25)^2 + (14.76)^2}$$

$$\boxed{R_A = 16.03 \, kN}$$

$$R_B = \sqrt{V_B^2 + H^2} = \sqrt{(3.75)^2 + (14.76)^2}$$

$$\boxed{R_B = 15.22 \, kN}$$

step 4: Maximum Bending Moment :-

$$M_x = V_A \times 15 - H \times y$$

here,

$$y = \frac{4 y_c}{l^2} x(l-x) = \frac{4 \times 5}{(40)^2} \times 15 (40-15)$$

$$\boxed{y = 4.68 \, m}$$

$$\therefore \ M_x = (6.25 \times 15) - (14.76 \times 4.68)$$

$$\boxed{M_{max} = 24.67 \, kNm}$$

3. A symmetrical three hinged circular arch has a span of 13 m and a rise to the central hinge of 3 m. it carries a vertical load of 15 kN at 3 m from the left hand end. Find
    i.   The reactions at the supports,
    ii.   Magnitude of the thrust at the springing,
    iii.   Bending moment at 5 m from the left hand hinge and
    iv.   The maximum positive and negative bending moment.   (AUC Nov/Dec 2012)
    Solution:
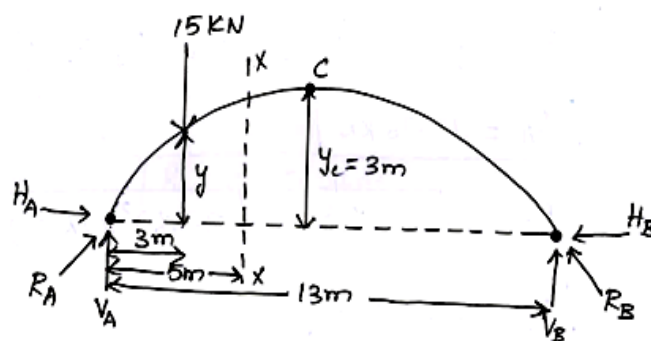
step 1: Vertical Reactions :-

$$\Sigma v = 0.$$

$$V_A + V_B = 15$$

Taking moment about A,

$$-V_B \times 13 + (15 \times 3) = 0$$

$$\boxed{V_B = 3.46 \, KN}$$

$$V_A + 3.46 = 15$$

$$\boxed{V_A = 11.54 \, KN}$$

Step 2: Horizontal thrust :-

Taking moment about c,

$$V_A \times 6.5 - H_A \times Y_c - 15 \times 3.5 = 0$$

$$11.54 \times 6.5 - 3 H_A - 15 \times 3.5 = 0$$

$$\boxed{H_A = 7.5 \, KN} (\rightarrow)$$

$$\Sigma H = 0$$

$$H_A = -H_B$$

$$\boxed{H_B = 7.5 \, KN} (\leftarrow)$$

step 3: Resultant Reactions :-

$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(11.54)^2 + (7.5)^2}$$

$$\boxed{R_A = 13.76 \, KN}$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(3.46)^2 + (7.5)^2}$$

$$\boxed{R_B = 8.25 \, KN}$$

Step 4: Bending Moment at 5m from left support.

In the Bending moment at $x = 5m$ from the left support, we find the radius and y value by using the formula.
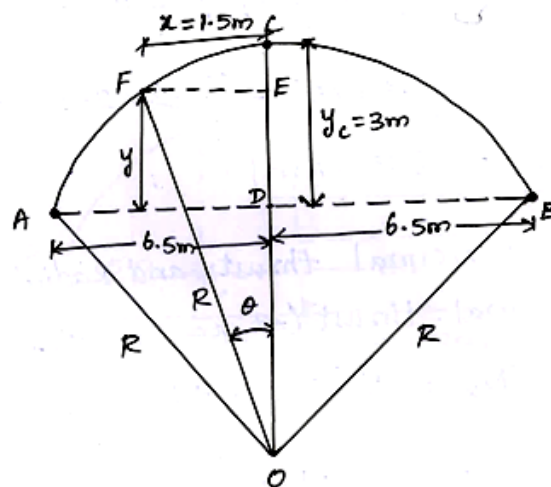
To find Radius(R):-

$$(2R - y_c)y_c = \left(\frac{\ell}{2}\right)^2$$

$$(2R - 3) \times 3 = \left(\frac{13}{2}\right)^2$$

$$\boxed{R = 8.54m}$$

To find y :-

In $\triangle$ OFE,

$$R^2 = x^2 + (R - y_c + y)^2$$

$$R^2 = (1.5)^2 + (R - 3 + y)^2$$

$$(8.54)^2 = (1.5)^2 + (8.54 - 3 + y)^2$$

$$70.68 = (5.54 + y)^2$$

$$8.41 = 5.54 + y$$

$$\boxed{y = 2.86 \, m} \quad \text{at } x = 1.5m \text{ from centre.}$$

$$B.M = V_A \times 5 - H_A(y) - 15 \times 2$$

$$= (11.54 \times 5) - (7.5 \times 2.86) - (15 \times 2)$$

$$\boxed{B.M = 6.25 \, KNm}$$

step 5 : Normal thrust and Radial shear :-

i) Normal thrust (at $x = 5m$ from A).

$$N_x = V \sin\theta + H \cos\theta$$

here,

$$\theta = \tan^{-1}\left(\frac{FE}{OE}\right)$$

$$= \tan^{-1}\left(\frac{1.5}{8.4}\right)$$

$$\boxed{\theta = 10° \ 7'}$$

V = Net vertical shear force at $x = 5m$ from A.

$$= V_A - 15$$

$$= 11.54 - 15$$

$$\boxed{V = -3.46 \ KN}$$

$$N_x = (-3.46 \times \sin(10°7')) + (7.5 \times \cos(10°7'))$$
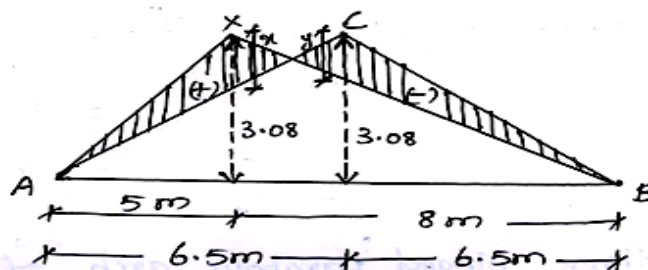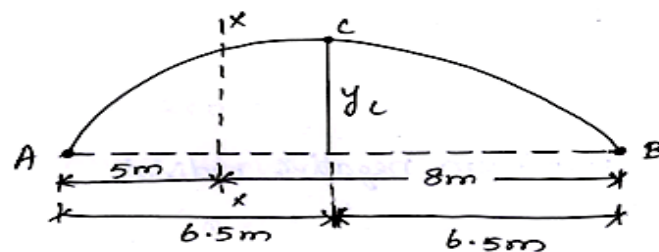
$$\boxed{N_x = 5.77 \ KN}$$

ii) Radial shear :-

$$R_x = V\cos\theta - H\sin\theta$$

$$= (-3.46 \times \cos(10°7')) - (7.5 \times \sin(10°7'))$$

$$\boxed{R_x = -4.72 \ KN}$$

step 6 : Maximum Bending Moment :-

Maximum positive (sagging) ordinate, $= \dfrac{x(l-x)}{l}$

$$= \dfrac{5(13-5)}{13}$$

$$= 3.08$$

Net Maximum positive ordinate, $x = 3.08 - \dfrac{3.08}{6.5} \times 5$

$$x = 0.71$$

Maximum positive moment $=$ load $\times$ ordinate

$$= 15 \times 0.71$$

$$= 10.65 \ kNm$$

Maximum Negative (hogging) ordinate $= \dfrac{x(l-x)}{l}$

$$= \dfrac{5(13-5)}{13}$$

$$= 3.08$$

Net maximum negative ordinate $= 3.08 - \dfrac{3.08}{8} \times 6.5$

$$= 0.58$$

Maximum Negative moment $=$ Load $\times$ ordinate

$$= 15 \times 0.58$$

$$= 8.7 \ kNm.$$

4. **A two hinged parabolic arch of span 25 m and rise 5 m carries an udl of 38 kN/m covering a distance of 10 m from left end. Find the horizontal thrust, the reactions at the hinges and the maximum negative moment.   (AUC Nov/Dec 2012, 2013, May/June 2014)**

Solution:
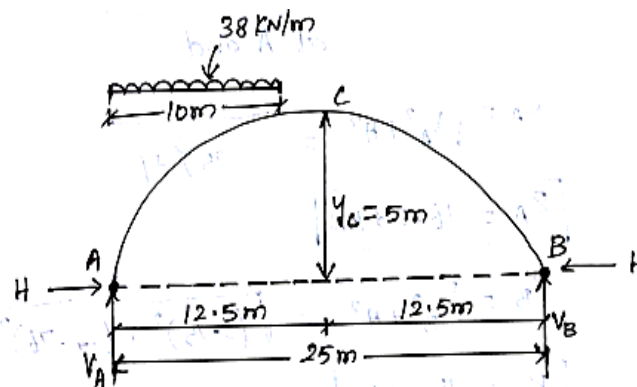
step 1: Vertical Reactions:-

$$\Sigma V = 0$$

$$V_A + V_B = 38 \times 10 = 380$$

Taking moment about A,

$$-V_B \times 25 + 38 \times \frac{(10)^2}{2} = 0$$

$$\boxed{V_B = 76 \, kN}$$

$$\therefore \quad V_A = 380 - 76$$

$$\boxed{V_A = 304 \, kN}$$

step 2: Horizontal thrust (H):-

$$H = \frac{\int_0^l (\mu_1 y + \mu_2 y) dx}{\int_0^l y^2 dx}$$

$$y = \frac{4 y_c}{l^2}(lx - x^2) = \frac{4 \times 5}{(25)^2}(25x - x^2)$$

$$y = 0.8x - 0.032x^2$$

i) **Denominator :-**

$$Dr = \int_0^{25} y^2 dx = \int_0^{25}(0.8x - 0.032x^2) dx$$

$$= \int_0^{25}(0.64x^2 + 1.02 \times 10^{-3} x^4 - 0.051 x^3) dx$$

$$= \left[\frac{0.64x^3}{3} + \frac{1.02 \times 10^{-3} x^5}{5} - \frac{0.051 x^4}{4} \right]_0^{25}$$

$$= \left[(3333.33 + 1992.18 - 4980.46) - 0 \right]$$

$$\boxed{Dr = 345.05}$$

ii) <u>Numerator (1)</u> :- (loaded portion)

$$\mu_1 = V_A \times x - \frac{38 \times x^2}{2} = 304x - 19x^2$$

$$N_r(1) = \int_0^{10} \mu_1 \, y \, dx = \int_0^{10} (304x - 19x^2) \times (0.8x - 0.032x^2) \, dx$$

$$= \int_0^{10} (243.2x^2 - 9.728x^3 - 15.2x^3 + 0.608x^4) \, dx$$

$$= \int_0^{10} (243.2x^2 - 24.93x^3 + 0.608x^4) \, dx$$

$$= \left[\frac{243.2x^3}{3} - \frac{24.93x^4}{4} + \frac{0.608x^5}{5} \right]_0^{10}$$

$$= \left[(81066.67 - 62325 + 12160) - 0 \right]$$

$$\boxed{N_r(1) = 30901.67}$$

iii) <u>Numerator (2)</u> :-

$$\mu_2 = V_B \times (1-x) = 76(25-x) = 1900 - 76x$$

$$N_r(2) = \int_{10}^{25} \mu_2 \, y \, dx$$

$$= \int_{10}^{25} (1900 - 76x)(0.8x - 0.032x) \, dx$$

$$= \int_{10}^{25} (1520x - 60.8x^2 - 60.8x^2 + 2.43x^3) \, dx$$

$$= \left[\frac{1520x^2}{2} - \frac{121.6x^3}{3} + \frac{2.43x^4}{4} \right]_{10}^{25}$$

$$= \left[(475000 - 633333.33 + 237304.68) \right.$$

$$\left. - (76000 - 40533.33 + 6075) \right]$$

$$= 78971.35 - 41541.67$$

$$\boxed{N_r(2) = 37429.68}$$

$$H = \frac{N_r(1) + N_r(2)}{D_r}$$

$$= \left(\frac{30901 \cdot 67 + 37429 \cdot 68}{345 \cdot 05} \right)$$

$$\boxed{H = 198 \cdot 03 \ KN}$$

Step 3: Resultant Reactions:-

$$R_A = \sqrt{V_A^2 + H^2} = \sqrt{(304)^2 + (198.03)^2}$$

$$\boxed{R_A = 362 \cdot 81 KN}$$

$$R_B = \sqrt{V_B^2 + H^2} = \sqrt{(76)^2 + (198.03)^2}$$

$$\boxed{R_B = 212 \cdot 11 \ KN}$$

Step 4 : Maximum Bending Moment:-

The maximum negative bending moment occurs at right of the span.

$$x = \frac{3}{4} l = \frac{3}{4} \times 25 = 18 \cdot 75 \ m \quad \text{from } A.$$

$$l - x = 25 - 18 \cdot 75 = 6 \cdot 25 \ m \quad \text{from } B.$$

$$B \cdot M = V_B \times 6 \cdot 25 - H \times y_x$$

here,

$$y = \frac{4 y_c}{l^2} \ x(l-x)$$

$$= \frac{4 \times 5}{(25)^2} \times 18 \cdot 75 \times 6 \cdot 25$$

$$y = 3 \cdot 75 \ m.$$

$$\therefore M_x = (76 \times 6 \cdot 25) - (198 \cdot 03 \times 3 \cdot 75)$$

$$\boxed{M_x = -267 \cdot 61 \ KNm.}$$

5. **Derive the expression for horizontal thrust in a two hinged parabolic arch carrying a point load P at distance one fourth spans from left support. Assume I = I_o sec θ.**

(AUC Apr/May 2011)

Solution:
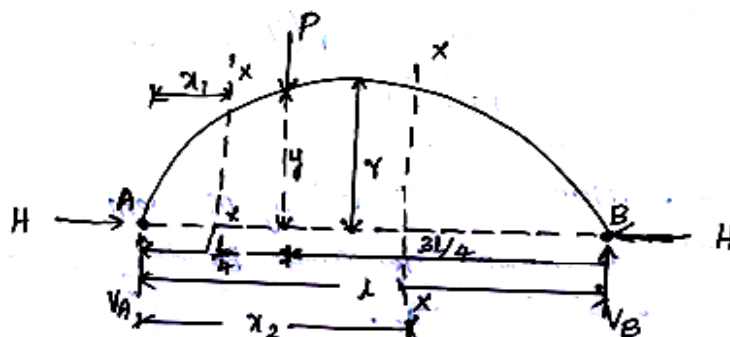
Step 1: Vertical Reactions :-

$$\Sigma V = 0$$

$$V_A + V_B = P$$

Taking moment about A,

$$-V_B \times l + P\tfrac{l}{4} = 0$$

$$\boxed{V_B = P/4}$$

$$V_A = P - P/4$$

$$\boxed{V_A = \tfrac{3P}{4}}$$

Step 2: Horizontal Thrust (H):-

$$H = \frac{\int \mu y \, dx}{\int y^2 \, dx} = \frac{\int_0^{l/4} \mu_1 y \, dx + \int_{l/4}^{l} \mu_2 y \, dx}{\int_0^{l} y^2 \, dx}$$

$$y = \frac{4 y_c}{l^2} x (l - x) = \frac{4r}{l^2}(lx - x^2).$$

i.) Denominator :-

$$Dr = \int_0^{l} y^2 \, dx = \int_0^{l} \left[\frac{4r}{l^2}(lx - x^2) \right]^2 dx$$

$$= \int_0^l \frac{16r^2}{l^4} \left(l^2 x^2 + x^4 - 2lx^3 \right) dx$$

$$= \frac{16r^2}{l^4} \int_0^l \left(l^2 x^2 + x^4 - 2lx^3 \right) dx$$

$$= \frac{16r^2}{l^4} \left[\frac{l^2 x^3}{3} + \frac{x^5}{5} - \frac{2lx^4}{4} \right]_0^l$$

$$= \frac{16r^2}{l^4} \left[\left(\frac{l^5}{3} + \frac{l^5}{5} - \frac{l^5}{4} \right) - 0 \right]$$

$$= \frac{16r^2 l^5}{l^4} \left[\frac{1}{3} + \frac{1}{5} - \frac{1}{4} \right]$$

$$= 16r^2 l \left(0.283 \right)$$

$$\boxed{Dr = 4.528 \, r^2 l}$$

ii) Numerator (1) :-

$$Nr(1) = \int_0^{l/4} \mu_1 \, y \, dx .$$

here, $\mu_1 = V_A \times x, = \frac{3Px}{4}$

$$Nr(1) = \int_0^{l/4} \frac{3Px}{4} \left[\frac{4r}{l^2} \left(lx - x^2 \right) \right] dx$$

$$= \frac{3Pr}{l^2} \int_0^{l/4} \left(lx^2 - x^3 \right) dx$$

$$= \frac{3Pr}{l^2} \left[\frac{lx^3}{3} - \frac{x^4}{4} \right]_0^{l/4}$$

$$= \frac{3Pr}{l^2} \left[\frac{l}{3} \left(\frac{l}{4} \right)^3 - \frac{1}{4} \left(\frac{l}{4} \right)^4 \right]$$

$$= \frac{3Pr}{l^2} \left[\frac{l^4}{192} - \frac{l^4}{1024} \right]$$

$$= \frac{3Pr l^4}{l^2} \left(\frac{1}{192} - \frac{1}{1024} \right)$$

$$\boxed{Nr(1) = 0.0126 \, Pr l^2}$$

iii) Numerator (2):-

$$Nr(2) = \int_{l/4}^{l} \mu_2 \, y \, dx$$

here, $\mu_2 = V_A \times x_2 - P\left(x_2 - l/4\right)$

$$= \frac{3Px}{4} - Px + \frac{Pl}{4}$$

$$\mu_2 = \frac{Pl}{4} - \frac{Px}{4}$$

$$Nr(2) = \int_{l/4}^{l} \left(\frac{Pl}{4} - \frac{Px}{4}\right)\left(\frac{4r}{l^2}(lx - x^2)\right) dx$$

$$= \frac{4r}{l^2} \int_{l/4}^{l} \left(\frac{Pl^2 x}{4} - \frac{Plx^2}{4} - \frac{Plx^2}{4} + \frac{Px^3}{4}\right) dx$$

$$= \frac{4r}{l^2} \int_{l/4}^{l} \left(\frac{Pl^2 x}{4} - \frac{Plx^2}{2} + \frac{Px^3}{4}\right) dx$$

$$= \frac{4r}{l^2}\left[\frac{Pl^2 x^2}{8} - \frac{Plx^3}{6} + \frac{Px^4}{16}\right]_{l/4}^{l}$$

$$= \frac{4r}{l^2}\left[\left(\frac{Pl^4}{8} - \frac{Pl^4}{6} + \frac{Pl^4}{16}\right) - \left(\frac{Pl^4}{128} - \frac{Pl^4}{384} + \frac{Pl^4}{4096}\right)\right]$$

$$= \frac{4r}{l^2}\left[\frac{Pl^4}{48} - \frac{67Pl^4}{12288}\right]$$

$$= \frac{4rPl^4}{l^2}(0.015)$$

$$\boxed{Nr(2) = 0.06\,Prl^2}$$

$$\therefore \quad H = \frac{Nr(1) + Nr(2)}{Dr} = \frac{0.0126\,Prl^2 + 0.06\,Prl^2}{4.528\,r^2 l}$$

$$= \frac{0.0726\,Prl^2}{4.528\,r^2 l}$$

$$\boxed{H = \frac{0.016\,Pl}{r}}$$

6. **A three hinged parabolic arch has supports at different levels having span 20 m and carries a UDL of 30 kN/m over the left half of the span. The left support is 5 m below the crown and the right support is 4 m below the crown. Draw the BMD. Also find the normal thrust and radial shear at a section 4 m from the left support.   (AUC Apr/May 2011)**

Solution:
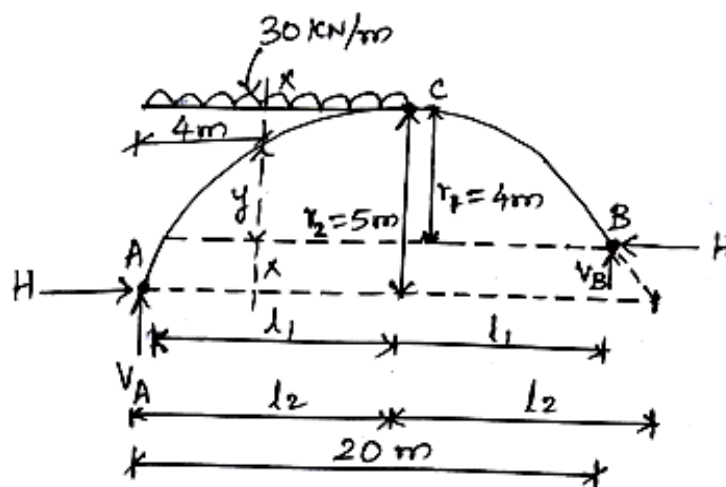
step 1: To find l_1 and l_2 :-

$$\frac{l_1}{l_2} = \sqrt{\frac{r_1}{r_2}}$$

$$\frac{l_1}{20-l_1} = \sqrt{\frac{4}{5}} = 0.89$$

$$l_1 = 0.89\,(20-l_1)$$

$$l_1 = 17.8 - 0.89\,l_1$$

$$1.89\,l_1 = 17.8$$

$$\boxed{l_1 = 9.42\ m}$$

$$l_2 = 20 - l_1 = 20 - 9.42$$

$$\boxed{l_2 = 10.58\ m}$$

step 2: Vertical and Horizontal Reactions :-

Considering the portion to left of c and taking moment,

$$V_A \times 10.58 - H \times 5 - 30 \times \frac{(10.58)^2}{2} = 0$$

$$10.58\,V_A - 5H = 1679 \rightarrow ①$$

Taking moment about right of c,

$$-V_B \times 9.42 + H \times 4 = 0$$

$$-9.42 V_B + 4H = 0 \quad \rightarrow ②$$

By $\Sigma V = 0$,

$$V_A + V_B = 30 \times 10 = 300$$

$$V_B = 300 - V_A \quad \rightarrow ③$$

Sub ③ in ②,

$$-9.42 (300 - V_A) + 4H = 0$$

$$-2826 + 9.42 V_A + 4H = 0$$

$$+9.42 V_A + 4H = 2826 \quad \rightarrow ④$$

By solving eqn. ① and ④,

$$\boxed{V_A = 233.12 \ KN}$$

$$\boxed{H = 157.49 \ KN}$$

③ $\Rightarrow$

$$\boxed{V_B = 66.88 \ KN}$$

<u>Step 3</u> : <u>Bending Moment</u> at $\underline{x = 4m \ from \ A}$:-

$$BM_x = V_A \times 4 - H \times y - 30 \times \frac{(4)^2}{2}$$

here, $$y = \frac{4 y_c}{L^2} \times (L - x)$$

$$= \frac{4 \times 5}{(2 \times 10.58)^2} \times 4 ((2 \times 10.58) - 4)$$

$$\boxed{y = 3.05 \ m}$$

$$M_x = (233.12 \times 4) - (157.49 \times 3.05) - 240$$

$$\boxed{M_x = 212.14 \ kNm}$$

step 4 : <u>Normal thrust and Radial shear</u> :-
  i) <u>Normal thrust</u> :-

$$N = V \sin\theta + H \cos\theta .$$

here,

V = Net vertical shear force at $x = 4m$ from A,

$= V_A - 30 \times 4$

$= 233.12 - 120$

$$\boxed{V = 113.12 \ kN}$$

$H = 157.49 \ kN$

$\theta = \tan^{-1}\left(\dfrac{4y_c}{(21.16)^2} \times (21.16 - 2 \times 4)\right)$

$= \tan^{-1}\left(\dfrac{4 \times 5}{(21.16)^2} \times (21.16 - 8)\right)$

$= \tan^{-1}(0.588)$

$$\boxed{\theta = 30° \ 27'}$$

$N = 113.12 \times \sin(30° \ 27') + 157.49 \times \cos(30°27')$

$= 57.33 + 135.77$

$$\boxed{N = 193.1 \ kN}$$
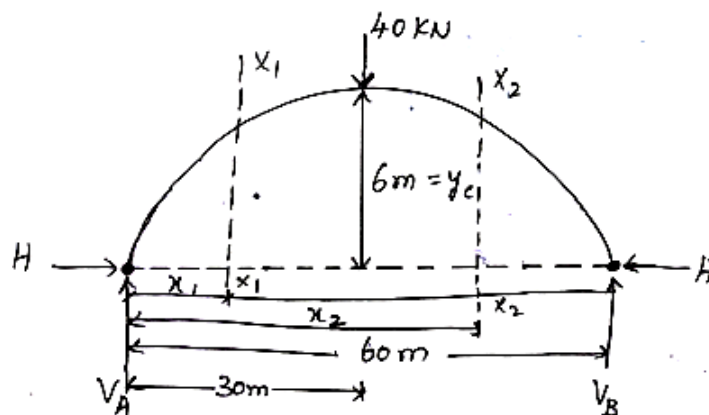
ii) Radial shear :-

$R = V \cos\theta - H \sin\theta$

$= 113.12 \times \cos(30° 27') - 157.49 \times \sin(30°27')$

$$\boxed{R = 17.7 \ kN}$$

7. **A parabolic two hinged arch of span 60 m and central rise of 6 m is subjected to a crown load of 40 kN. Allowing rib shortening and temperature rise of 20° C, determine horizontal thrust, H. $I_C = 6 \times 10^5 \ cm^4$, $A_C = 1000 \ cm^2$, $E = 1 \times 10^4 \ MPa$, $\alpha = 11 \times 10^{-6} \ /°C$, $I = I_C \sec\theta$.**

(AUC Nov/Dec 2010)

Solution:

Horizontal thrust, $H = H_1 + H_2$

here, H_1 = horizontal thrust under the load.

H_2 = horizontal thrust due to temperature rise

$$H_2 = \frac{l \alpha T EI}{\int_0^l y^2 ds}$$

$$H_1 = \frac{\int \mu y \, dx}{\int y^2 dx}$$

Step 1 : Vertical Reactions :-

$$\underline{\underline{\Sigma V = 0}}$$

$$V_A + V_B = 40$$

Taking moment about A,

$$-V_B \times 60 + (40 \times 30) = 0$$

$$\boxed{V_B = 20 \, kN}$$

$$V_A = 40 - 20$$

$$\boxed{V_A = 20 \, kN}$$

Step 2 : Horizontal thrust (H_1) :-

$$H_1 = \frac{\int \mu y \, dx}{\int y^2 dx} = \frac{\int_0^{30} \mu_1 y \, dx + \int_{30}^{60} \mu_2 y \, dx}{\int_0^{60} y^2 dx}$$

$$H_1 = \frac{N_r(1) + N_r(2)}{Dr}$$

i.) **Denominator :-**

$$Dr = \int_0^{60} y^2 dx$$

here, $y = \dfrac{4 y_c}{l^2} \times (l - x) = \dfrac{4 \times 6}{(60)^2} (lx - x^2)$

$$= \frac{24}{(60)^2} (60x - x^2)$$

$$\boxed{y = 0.4x - 0.0067x^2}$$

$$Dr = \int_0^{60} (0.4x - 0.0067x^2)^2 \, dx$$

$$= \int_0^{60} (0.16x^2 + 4.48 \times 10^{-5} x^4 - 0.0054 x^3) \, dx$$

$$= \left[\frac{0.16 x^3}{3} + \frac{4.48 \times 10^{-5} x^5}{5} - \frac{0.0054 x^4}{4} \right]_0^6$$

$$= \left[(11520 + 6967.29 - 17496) - 0 \right]$$

$$\boxed{Dr = 991.29}$$

ii) Numerator (1) :-

$$N_Y(1) = \int_0^{30} \mu_1 y \, dx = \int_0^{30} (20x)(0.4x - 0.0067x^2) \, dx$$

$$= \int_0^{30} (8x^2 - 0.134x^3) \, dx$$

$$= \left(\frac{8x^3}{3} - \frac{0.134x^4}{4} \right)_0^{30}$$

$$= \left[(72000 - 27135) - 0 \right]$$

$$\boxed{N_Y(1) = 44865}$$

iii) Numerator (2) :-

$$N_Y(2) = \int_{30}^{60} \mu_2 y \, dx$$

here, $\mu_2 = 20x - 40x + 1200$

$$\mu_2 = 1200 - 20x$$

$$N_Y(2) = \int_{30}^{60} (1200 - 20x)(0.4x - 0.0067x^2) \, dx$$

$$= \int_{30}^{60} (480x - 8.04x^2 - 8x^2 + 0.134x^3) \, dx$$

$$= \int_{30}^{60} (480x - 16.04x^2 + 0.134x^3) \, dx$$

$$= \left[\frac{480x^2}{2} - \frac{16.04x^3}{3} + \frac{0.134x^4}{4}\right]_{30}^{60}$$

$$= \Big[(864000 - 1154880 + 434160)$$
$$\qquad - (216000 - 144360 + 27135)\Big]$$

$$\boxed{N_Y(2) = 44505}$$

$$\therefore H_1 = \frac{N_Y(1) + N_Y(2)}{D_Y}$$

$$= \left(\frac{44865 + 44505}{991.28}\right)$$

$$\therefore \boxed{H_1 = 90.16 \, kN}$$

Step 3 : Increased Horizontal thrust :-

$$H_2 = \frac{l\alpha T E I}{\int_0^l y^2 ds}$$

here,

$$l = 60 \, m \; ;$$

$$\alpha = 11 \times 10^{-6}/°c .$$

$$T = 20°c .$$

$$E = 1 \times 10^4 MPa = 1 \times 10^{10} N/m^2 .$$

$$I = I_c \sec\theta$$

$$I_c = 6 \times 10^5 cm^4 = 6 \times 10^5 \times 10^{-8} = 0.006 \, m^4 .$$

$$y = \frac{4Y}{l^2}(lx - x^2)$$

$$\theta = \frac{dy}{dx} = \frac{4Y}{l^2}(l - 2x)$$

$$= \frac{4 \times 6}{(60)^2}(60 - 2(30))$$

$$\theta = 0 .$$

$$\therefore I = I_c \sec\theta$$

$$= \frac{0.006}{\cos(0)}$$

$$I = 0.006 \, m^4$$

$$y = \frac{4y_c}{l^2}(lx - x^2)$$

$$= \frac{4 \times 6}{(60)^2}(60x - x^2)$$

$$y = 0.4x - 0.0067x^2$$

$$H_2 = \frac{l\,\alpha T E I}{\int_0^{60} y^2\,dx}$$

here,

$$Dr = \int_0^{60} y^2\,dx = \int_0^{60}(0.4x - 0.0067x^2)^2\,dx$$

$$= \int_0^{60}(0.16x^2 + 4.489 \times 10^{-5}x^4 - 5.36 \times 10^{-3}x^3)\,dx$$

$$= \left[\frac{0.16x^3}{3} + \frac{4.489 \times 10^{-5}x^5}{5} - \frac{5.36 \times 10^{-3}x^4}{4}\right]_0^{60}$$

$$= \left[(11520 + 6981.29 - 17366.4) - 0\right]$$

$$\boxed{Dr = 1134.89}$$

$$\therefore H_2 = \left(\frac{60 \times 11 \times 10^{-6} \times 20 \times 1 \times 10^{10} \times 0.006}{1134.89}\right)$$

$$H_2 = 697.86\ N$$
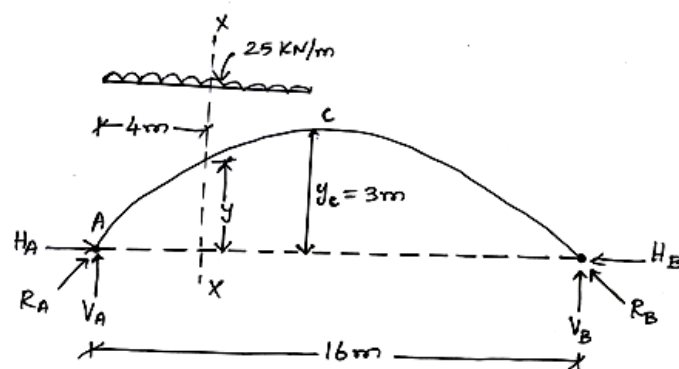
$$\boxed{H_2 = 0.697\,kN}$$

$$H = H_1 + H_2 = 90.16 + 0.697$$

$$\boxed{H = 90.857\,kN}$$

8. **A parabolic 3 hinged arch carries a UDL of 25 kN/m on the left half of the span. It has a span of 16 m and a central rise of 3 m. Determine the resultant reaction at supports. Find also the bending moment, normal thrust and radial shear at a section 4 m from left support.**

(AUC Nov/Dec 2010, 2013, May/June 2014)

Solution:

step 1 : Vertical Reactions V_A and V_B :-

$$\Sigma V = 0$$

$$V_A + V_B = (25 \times 8) = 200 \rightarrow ①$$

Taking moment about A,

$$-V_B \times 16 + 25 \times \left(\frac{8^2}{2}\right) = 0$$

$$\boxed{V_B = 50 \; KN}$$

$$① \Rightarrow \quad V_A + V_B = 200$$

$$V_A + 50 = 200$$

$$V_A = 200 - 50$$

$$\boxed{V_A = 150 \; KN}$$

step 2 : Horizontal Reactions :-

Taking moment about c,

$$V_A \times 8 - 25 \times \frac{8^2}{2} - H_A \times 3 = 0$$

$$(150 \times 8) - \left(25 \times \frac{8^2}{2}\right) - 3H_A = 0$$

$$\boxed{H_A = 133 \cdot 33 \; KN}$$

$$\therefore \quad \boxed{H_B = 133 \cdot 33 \; KN}$$

Step 3 : Resultant Reactions at A and B :-

$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(150)^2 + (133.33)^2}$$

$$\boxed{R_A = 200.69 \, kN}$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(50)^2 + (133.33)^2}$$

$$\boxed{R_B = 142.39 \, kN}$$

step 4 : Bending Moment at x = 4m from A ;-

$$BM = V_A \times 4 - 25 \times \frac{4^2}{2} - H_A \times y$$

To find y :-

For parabolic arches,

$$y = \frac{4 y_c}{L^2} x(L-x)$$

$$= \frac{4 \times 3}{(16)^2} \times 4 \, (16-4)$$

$$\boxed{y = 2.25 \, m}$$

$$B.M = (150 \times 4) - \left(25 \times \frac{4^2}{2}\right) - (133.33 \times 2.25)$$

$$\boxed{B.M = 100 \, kNm}$$

step 5 : Radial shear force at x = 4m from A :-

$$S.F \; ; \quad R_x = V_x \cos\theta - H \sin\theta$$

here, V = Net vertical shear force at x = 4m from A.

$$= V_A - w \times 4 = 150 - (25 \times 4)$$

$$\boxed{V = 50 \, kN}$$

H = Horizontal shear force = 133.33 kN

$$\theta = \tan^{-1}\left[\frac{4r}{L^2}(L-2x)\right]$$

$$= \tan^{-1}\left[\frac{4 \times 3}{(16)^2}(16-2(4))\right]$$

$$= \tan^{-1}(0.375)$$

$$\boxed{\theta = 20° \, 33'}$$

$$R = 50 \times \cos(20°33') - 133.33 \times \sin(20°33')$$
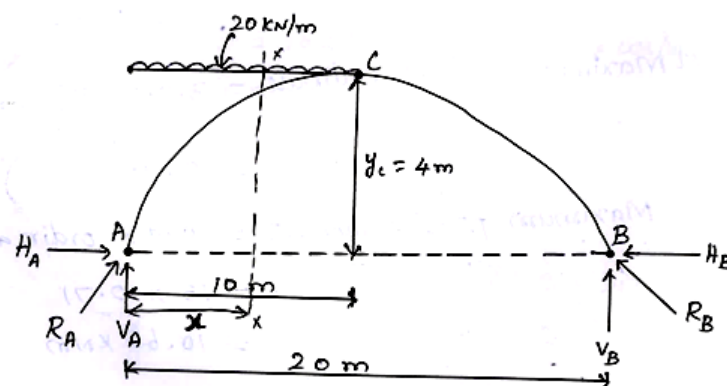
$$\boxed{R = 0.016 \quad KN}$$

Step 6: Normal thrust at $x = 4m$ from A :-

Normal thrust, $N_x = V_x \sin\theta + H \cos\theta$

$$= 50 \times \sin(20°33') + 133.33 \times \cos(20°33')$$

$$\boxed{P_N = 142.39 \quad KN}$$

9. **A three hinged parabolic arch of span 20 m and rise 4m carries a UDL of 20 kN/m over the left half of the span. Draw the BMD.**

   Solution:

Step 1: Vertical Reactions :-

$$\Sigma V = 0$$

$$V_A + V_B = 200$$

Taking moment about A,

$$-V_B \times 20 + \left(20 \times \frac{(10)^2}{2}\right) = 0$$

$$\boxed{V_B = 50 \ KN}$$

$$V_A + 50 = 200$$

$$\boxed{V_A = 150 \ KN}$$

Step 2: Horizontal Reactions :-

$$\Sigma H = 0$$

$$H_A + H_B = 0$$

$$H_A = -H_B$$

Taking moment about c,

$$V_A \times 10 - 20 \times \frac{(10)^2}{2} - H_A \times y_c = 0$$

$$(150 \times 10) - 1000 - 4 H_A = 0$$

$$\boxed{H_A = 125 \, KN}$$

$$\boxed{H_B = -125 \, KN} \quad \boxed{= 125 \, KN}$$

step 3 : Resultant Reactions :-

$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(150)^2 + (125)^2}$$

$$\boxed{R_A = 195.25 \, KN}$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(50)^2 + (125)^2}$$

$$\boxed{R_B = 134.63 \, KN}$$

step 4 : Maximum Bending Moment :-

$$BM_{xx} = V_A \times x - \frac{20 \times x^2}{2} - H_A \times y$$

here,

$$y = \frac{4 y_c}{l^2} \times (l - x)$$

$$y = \frac{4 \times 4}{(20)^2} \times (20x - x^2)$$

$$y = 0.8x - 0.04x^2$$

$$BM = 150x - 10x^2 - 125 (0.8x - 0.04x^2)$$

$$= 150x - 10x^2 - 100x + 5x^2$$

$$BM = 50x - 5x^2$$

Differentiate with respect to x.
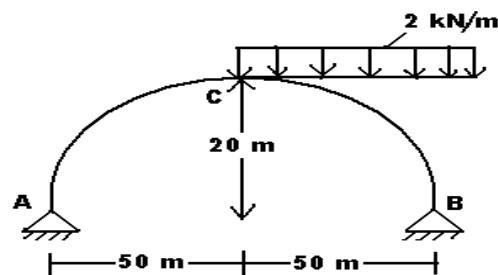
$$\frac{dM}{dx} = 50 - 10x$$
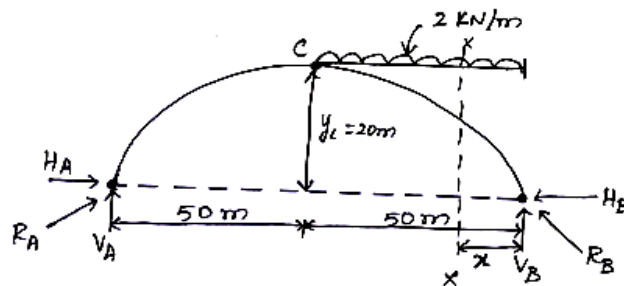
$$0 = 50 - 10x$$

$$\boxed{x = 5m}$$

$$\therefore \quad BM = 50x - 5x^2 = (50 \times 5) - 5 \times (5)^2$$

$$\boxed{BM = 125 \, KNm}$$

10. A three hinged parabolic arch of span 100m and rise 20m carries a uniformly distributed load of 2KN/m length on the right half as shown in the figure. Determine the maximum bending moment in the arch.

Solution:

Step 1: Vertical Reactions :-

$$\Sigma V = 0$$

$$V_A + V_B = 100$$

Taking moment about B,

$$V_A \times 100 - \frac{2 \times (50)^2}{2} = 0$$

$$\boxed{V_A = 25 \, KN}$$

$$V_B = 100 - 25$$

$$\boxed{V_B = 75 \, KN}$$

Step 2 : **Horizontal Reactions :-**

$$\Sigma H = 0$$

$$H_A = H_B$$

Taking moment about c,

$$V_B \times 50 - 2 \times \frac{(50)^2}{2} - H_B \times y_c = 0$$

$$(75 \times 50) - (50)^2 - 20 H_B = 0$$

$$\boxed{H_B = 62 \cdot 5 \ kN}$$

$$\boxed{H_A = 62 \cdot 5 \ kN}$$

Step 3 : **Resultant Reactions :-**

$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(25)^2 + (62 \cdot 5)^2}$$

$$\boxed{R_A = 67 \cdot 31 \ kN}$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(75)^2 + (62 \cdot 5)^2}$$

$$\boxed{R_B = 97 \cdot 63 \ kN}$$

Step 4 : **Maximum Bending Moment :-**

The maximum bending moment will occur at a section xx at x distance from B.

$$BM = -V_B \times x + \frac{2 \times x^2}{2} + H_B \times y$$

here,
$$y = \frac{4 y_c}{l^2} \, x \, (l - x)$$

$$= \frac{4 \times 20}{(100)^2} \times x \, (100 - x)$$

$$y = 0 \cdot 8 x - 0 \cdot 008 x^2$$

$$M_x = -75x + x^2 + 62 \cdot 5 \, (0 \cdot 8x - 0 \cdot 008x^2)$$

$$= -75x + x^2 + 50x - 0 \cdot 5x^2$$

$$M_x = -25x + 0 \cdot 5x^2$$

For maximum bending moment, $\dfrac{dM_x}{dx} = 0$.

Differentiate with respect to x,

$$\frac{dM_x}{dx} = -25 + x$$

$$0 = -25 + x$$

$$\boxed{x = 25m}$$

$$\therefore M_x = -25x + 0.5x^2$$

$$= (-25 \times 25) + (0.5 \times (25)^2)$$

$$\boxed{M_x = -312.5 \, kNm}$$