

QUESTION BANK**DEPARTMENT: CIVIL****SEMESTER: V****SUBJECT CODE / Name: CE 2302 / STRUCTURAL ANALYSIS-I****UNIT 2 – MOVING LOADS AND INFLUENCE LINES****(DETERMINATE & INDETERMINATE STRUCTURES WITH REDUNDANCY****RESTRICTED TO ONE)****PART - A (2 marks)****1. What is the use of influence line diagram (ILD)? (AUC Apr/May 2012)**

- Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads.
- Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

2. State Muller Breslau's principle. (AUC Nov/Dec 2012 & 2013, Apr/May 2012, May/June 2014)

Muller-Breslau principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,

- We remove from the structure the resistant to that force quantity and
- We apply on the remaining structure a unit displacement corresponding to that force quantity.

3. What are influence lines? (AUC Nov/Dec 2012, May/June 2014)

An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

4. Explain the use of Beggs deformer. (AUC Apr/May 2011)

It permits extremely accurate work in indirect model analysis.

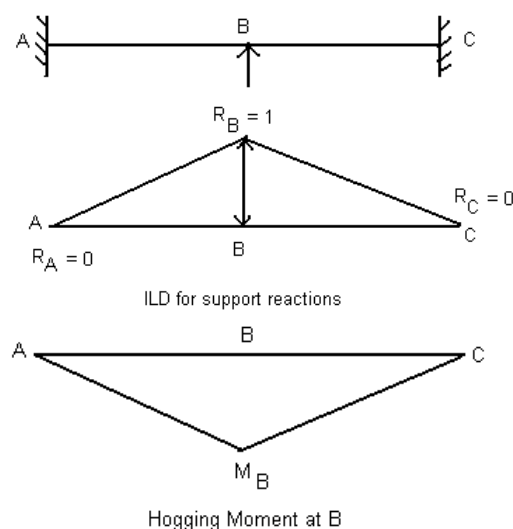
For best results the deformer should be used in a room with controlled temperature and humidity so as to avoid disturbance of model deflections due to differential heating.

Extended periods of use of this deformer may cause considerable eye strain.

5. State the three equilibrium equations. (AUC Nov/Dec 2010)

$$\sum H = 0; \sum V = 0; \sum M = 0;$$

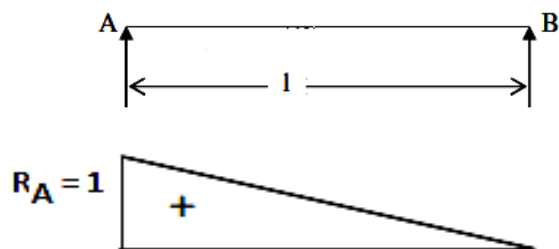
6. Sketch the shapes of the influence lines for the support reaction and the hogging moment at the continuous support B of a two span continuous beam ABC. Assume the extreme ends A and C to be fully fixed. (AUC Apr/May 2011)



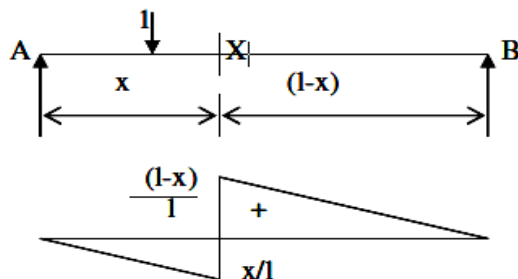
7. Give the condition at which maximum absolute bending moment occurs in a simply supported beam when a number of point loads are moving on it. (AUC Nov/Dec 2010)

When a series of point loads crosses a simply supported beam, the absolute maximum bending moment will occur near midspan under the load W_{cr} , where W_{cr} is the maximum load in a series of point loads.

8. Draw the ILD for reaction at the left support of a simply supported beam.



9. Sketch the influence line diagram for shear force at any section of a simply supported beam.



10. How will you obtain degree of static determinacy?

If the conditions of statics i.e., $\sum H=0$, $\sum V=0$ and $\sum M=0$ are alone sufficient to find either external reactions or internal forces in a structure, the structure is called a statically determinate structure.

11. What is degree of kinematic indeterminacy?

Members of structure deform due to external loads. The minimum number of parameters required to uniquely describe the deformed shape of structure is called "Degree of kinematic indeterminacy".

12. What are the types of connections possible in the model of begg's deformeter?

(i) Hinged connection (ii) Fixed connection (iii) Floating connection

13. What is meant by absolute maximum bending moment in a beam?

When a given load system moves from one end to the other end of a girder, depending upon the position of the load, there will be a maximum bending moment for every section. The maximum of these bending moments will usually occur near or at the midspan. The maximum of maximum bending moments is called the absolute maximum bending moment.

14. Where do you get rolling loads in practice?

Shifting of load positions is common enough in buildings. But they are more pronounced in bridges and in gantry girders over which vehicles keep rolling.

15. Name the type of rolling loads for which the absolute maximum bending moment occurs at the mid span of a beam?

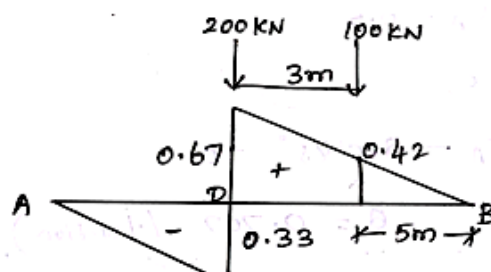
- (i) Single concentrated load
- (ii) udl longer than the span
- (iii) udl shorter than the span
- (iv) Also when the resultant of several concentrated loads crossing a span, coincides with a concentrated load then also the maximum bending moment occurs at the centre of the span.

PART - B (16 marks)

1. Two point loads of 100 kN and 200 kN spaced 3 m apart cross a girder of span 12 m from left to right with the 100 kN leading. Draw the ILD for shear force and bending moment and find the values of maximum shear force and bending moment at a section 4 m from the left hand support. Also evaluate the absolute maximum bending moment due to the given loading system. (AUC Apr/May 2012, Nov/Dec 2013)

Solution:

a.) Maximum positive shear force:



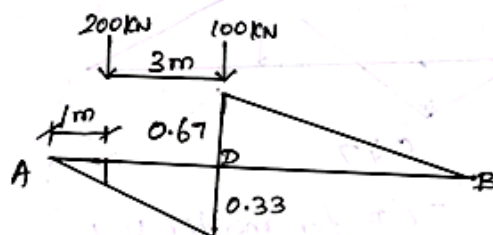
$$\text{positive ordinate under 200 kN} = \frac{l-x}{l} = \frac{12-4}{12} = 0.67$$

$$\text{ordinate under 100 kN load} = \frac{0.67}{8} \times 5 = 0.42$$

$$\text{Maximum positive shear force} = (200 \times 0.67) + (100 \times 0.42)$$

$$+ve \text{ SF} = 176 \text{ kN.}$$

b.) Negative shear force:-



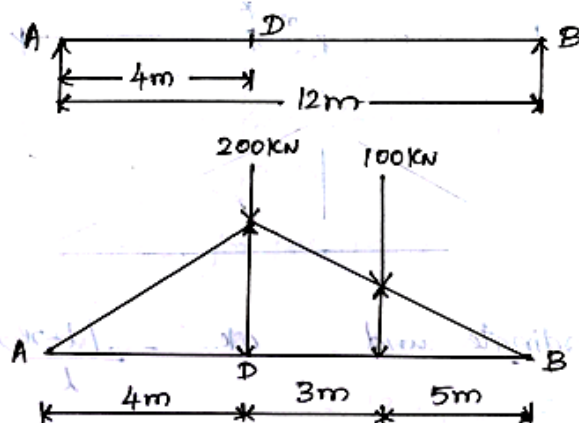
$$\text{negative ordinate under 100 kN} = \frac{x}{l} = \frac{4}{12} = 0.33$$

$$\text{ordinate under 200 kN load} = \frac{0.33}{4} \times 1 = 0.083$$

$$\text{Maximum Negative shear force} = (100 \times 0.33) + (200 \times 0.083)$$

$$-ve \text{ SF} = 49.6 \text{ kN}$$

c.) Maximum Bending Moment :-



$$\begin{aligned} \text{maximum ordinate of ILD (200kN)} &= \frac{x(1-x)}{1} \\ &= \frac{4 \times (12-4)}{12} \\ &= 2.67 \end{aligned}$$

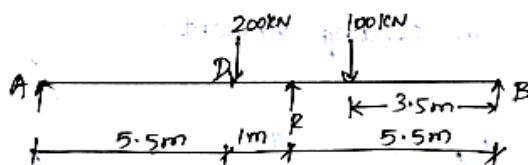
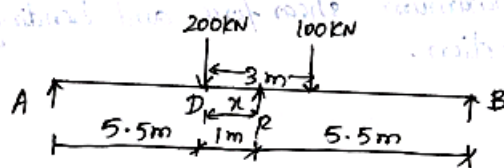
$$\text{maximum ordinate at 100kN} = \frac{2.67}{8} \times 5 = 1.67$$

$$\text{Maximum BM} = \text{Load} \times \text{Ordinate}$$

$$= [(200 \times 2.67) + (100 \times 1.67)]$$

$$\text{Max. BM} = 701 \text{ kNm}$$

d.) Absolute Maximum bending moment :-



$$\text{ordinate under } 200 \text{ kN} = \frac{x(1-x)}{1} = \frac{5.5(12-5.5)}{12}$$

$$= 2.98$$

$$\text{ordinate under } 100 \text{ kN} = \frac{2.98}{6.5} \times 3.5 = 1.6$$

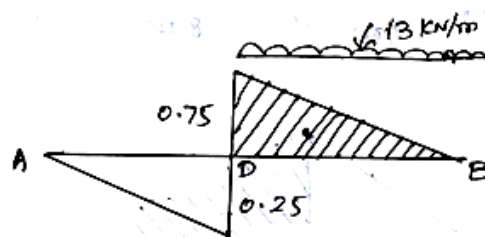
$$\begin{aligned} \text{Absolute Max. BM} &= (200 \times 2.98) + (100 \times 1.6) \\ &= 756 \text{ kNm} \end{aligned}$$

2. A simply supported beam has a span of 16 m is subjected to a UDL (dead load) of 5 kN/m and a UDL (live load) of 8 kN/m (longer than the span) traveling from left to right. Draw the ILD for shear force and bending moment at a section 4 m from the left end. Use these diagrams to determine the maximum shear force and bending moment at this section.

(AUC Apr/May 2012)

Solution:

Step 1: Maximum +ve shear force :-



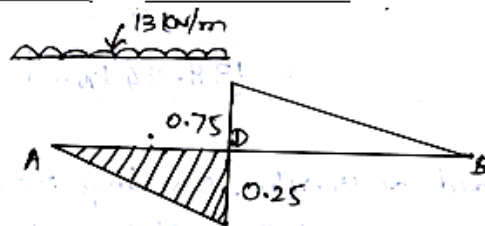
$$\text{ILD ordinate for positive} = \frac{1-x}{1} = \frac{16-4}{16} = 0.75$$

$$\text{Maximum positive SF} = \text{load} \times \text{Area}$$

$$= 13 \times \left[\frac{1}{2} \times 12 \times 0.75 \right]$$

$$+ve \text{ SF} = 58.5 \text{ kN}$$

Step 2: Maximum -ve shear force :-



$$\text{ILD for negative ordinate} = \frac{x}{1} = \frac{4}{16} = 0.25$$

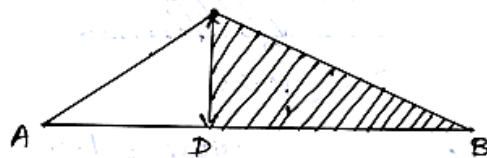
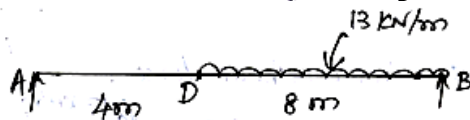
$$\text{Maximum Negative SF} = 13 \times \left[\frac{1}{2} \times 4 \times 0.25 \right]$$

$$-ve \text{ SF} = 6.5 \text{ kN}$$

step 3: Maximum Bending moment :-

$$l/4 = \frac{16}{4} = 4 \text{ m}$$

Since the udl is acting longer than the span.
Maximum bending moment occur right of D.



$$\begin{aligned} \text{Maximum ordinate at D} &= \frac{x(1-x)}{1} = \frac{4(12-4)}{12} \\ &= 2.67 \end{aligned}$$

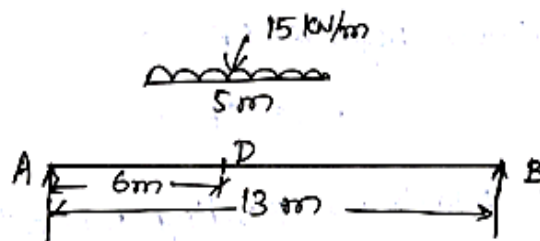
$$\begin{aligned} \text{Maximum B.M} &= \text{load} \times \text{Area} \\ &= 13 \times \left[\frac{1}{2} \times (8 \times 2.67) \right] \end{aligned}$$

$$\boxed{\text{Max. B.M.} = 138.84 \text{ kNm}}$$

3. A live load of 15 kN/m, 5 m long moves on a girder simply supported on a span of 13 m. Find the maximum bending moment that can occur at a section 6 m from the left end.

(AUC Nov/Dec 2012, May/June 2014)

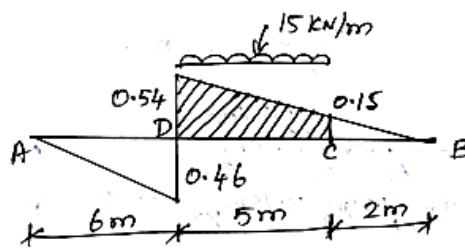
Solution:



step 1: maximum positive shear force :-

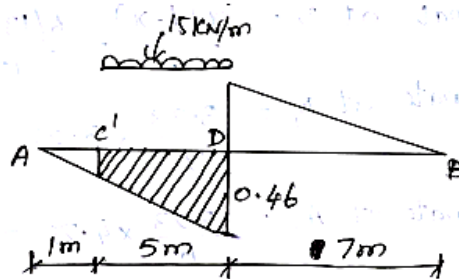
$$\text{ILD for positive ordinate at D} = \frac{1-x}{1} = \frac{13-6}{13} = 0.54$$

$$\text{ordinate under at C} = \frac{0.54}{7} \times 2 = 0.15$$



$$\begin{aligned}\text{Maximum positive shear force} &= \text{Load} \times \text{Area} \\ &= 15 \times \left[\left(\frac{0.54 + 0.15}{2} \right) \times 5 \right] \\ &= 25.87 \text{ kN}.\end{aligned}$$

Step 2: Maximum Negative shear force :-



$$\text{ILD for negative ordinate} = \frac{x_1}{l} = \frac{6}{13} = 0.46$$

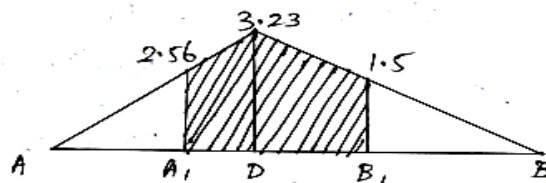
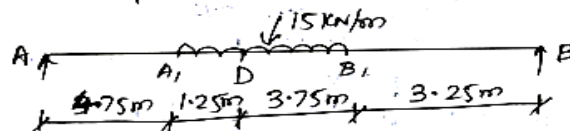
$$\text{ordinate at } C' = \frac{0.46}{6} \times 1 = 0.077$$

$$\begin{aligned}\text{Maximum Negative SF} &= 15 \times \left[\left(\frac{0.46 + 0.077}{2} \right) \times 5 \right] \\ &= 20.14 \text{ kN}\end{aligned}$$

Step 3: Maximum Bending Moment :-

Maximum bending moment will occur at D and udl will be placed at $l/4$ distance.

$$l/4 = 5/4 = 1.25 \text{ m}.$$



$$\text{ordinate at D} = \frac{x(1-x)}{1} = \frac{6(13-6)}{13} = 3.23$$

$$\text{ordinate at B}_1 = \frac{3.23}{7} \times 3.25 = 1.5$$

$$\text{ordinate at A}_1 = \frac{3.23}{6} \times 4.75 = 2.56$$

$$\text{Maximum BM} = \text{load} \times \text{Area}$$

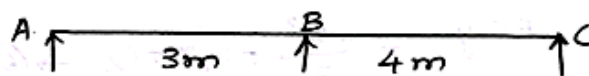
$$= 15 \times \left[(2.56 \times 1.25) + \left(\frac{1}{2} \times 0.67 \times 1.25 \right) + (1.5 \times 3.75) + \left(\frac{1}{2} \times 1.73 \times 3.75 \right) \right]$$

$$= 15 \times [12.48]$$

$$\boxed{\text{Max. BM} = 187.2 \text{ kNm}}$$

4. Draw the influence line for M_B of the continuous beam ABC simply supported at A & C using Muller Breslau's principle. AB = 3 m, BC = 4 m. EI is constant. (AUC Apr/May 2011)

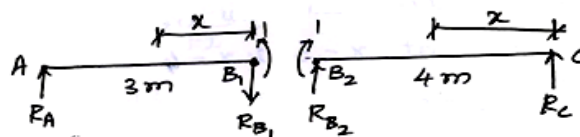
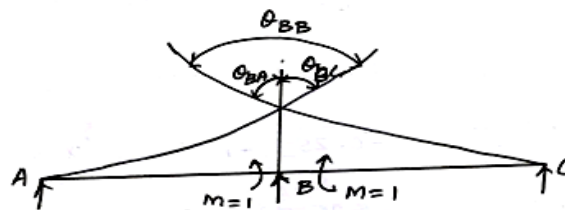
Solution:



To get IL for M_B :-

- Apply a unit BM at D.
- Determine the deflection y_{XB} any x and slope θ_B at B.

$$\text{IL ordinate at any } x = \frac{y_{XB}}{\theta_B}$$



Due to $M=1$ at B,

$$R_A \times 3 = 1$$

$$R_A = \frac{1}{3} = 0.333$$

$$R_{B1} = 0.333 \downarrow$$

$$R_{B2} = 0.333 \uparrow$$

Taking moments about B,

$$R_c \times 4 = 1$$

$$R_c = \frac{1}{4} = 0.25$$

The two regions AB and BC will be considered separately.

BM at any x is,

$$M_x = -EI \frac{d^2y}{dx^2} = 0.25x - 0.333(x-4)$$

$$EI \frac{d^2y}{dx^2} = -0.25x + 0.333(x-4)$$

$$EI \frac{dy}{dx} = -0.25 \frac{x^2}{2} + 0.333 \frac{(x-4)^2}{2} \rightarrow (1)$$

$$EI y = -0.25 \frac{x^3}{6} + 0.333 \frac{(x-4)^3}{6} + C_1x + C_2$$

The boundary conditions are

$$y = 0 \text{ at } x = 0 \text{ \& } x = 4$$

$$\therefore (2) \Rightarrow C_2 = 0$$

$$0 = -0.25 \frac{(4)^3}{6} + 4C_1 + 0$$

$$C_1 = 0.67$$

$$\therefore EI y_{AB} = -0.25 \frac{x^3}{6} + 0.67x + 0.333 \frac{(x-4)^3}{6} \rightarrow (3)$$

$$(1) \Rightarrow EI \frac{dy}{dx} = -0.25 \frac{x^2}{2} + C_1 + 0.333 \frac{(x-4)^2}{2}$$

At $x = 4$,

$$\theta_{BC} = \left(\frac{dy}{dx} \right)_{BC} = \frac{1}{EI} \left[-0.25 \frac{(4)^2}{2} + 0.67 + 0.333 \frac{(0)^2}{2} \right]$$

$$\theta_{BC} = -\frac{1.334}{EI}$$

$$(3) \Rightarrow y_D = y_4 = \frac{1}{EI} \left[-0.25 \frac{(4)^3}{6} + (0.67 \times 4) + \frac{0.333(0)}{6} \right]$$

$$y_D = \frac{0.013}{EI}$$

For the zone AB,

$$M_x = 1 - 0.333x$$

$$EI \frac{d^2y}{dx^2} = 0.333x - 1$$

$$EI \frac{dy}{dx} = 0.333 \frac{x^2}{2} - x + C_3 \rightarrow (4)$$

$$EI y = 0.333 \frac{x^3}{6} - \frac{x^2}{2} + C_3 x + C_4 \rightarrow (5)$$

$$\text{At } x=0; y = \frac{0.013}{EI},$$

$$(5) \Rightarrow 0.013 = C_4$$

$$\text{At } x=3; y=0,$$

$$(5) \Rightarrow 0 = 0.333 \frac{(3)^3}{6} - \frac{(3)^2}{2} + 3C_3 + 0.013$$

$$C_3 = 0.99 = 1.$$

$$(4) \Rightarrow EI \frac{dy}{dx} = 0.333 \frac{x^2}{2} - x + 1$$

$$\text{At } x=0;$$

$$\theta_{BA} = \left(\frac{dy}{dx} \right) = \frac{1}{EI}$$

$$\therefore EI y = 0.333 \frac{x^3}{6} - \frac{x^2}{2} + x + 0.013 \rightarrow (6)$$

$$y_{XB} = \frac{1}{EI} \left[0.333 \frac{x^3}{6} - \frac{x^2}{2} + x + 0.013 \right] \rightarrow (7)$$

$$\theta_{BB} = \theta_{BA} - \theta_{BC}$$

$$= \frac{1}{EI} - \left(-\frac{1.33}{EI} \right) = \frac{1}{EI} + \frac{1.33}{EI}$$

$$\boxed{\theta_{BB} = \frac{2.33}{EI}}$$

For the region CB,

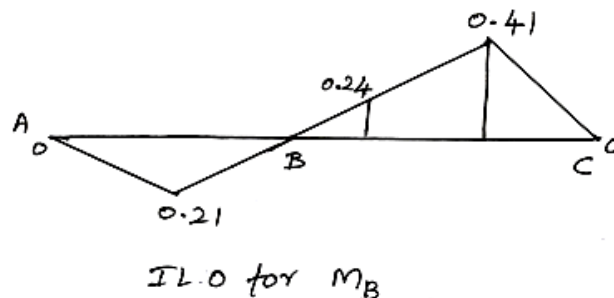
$$IL \text{ ordinate for } M_B = \frac{y_{XB}}{\theta_{BB}} = \left[\frac{-\frac{0.25x^3}{6} + 0.67x}{2.33} + \frac{0.333(x-4)^3}{6} \right] \rightarrow (8)$$

For the region BA,

$$IL \text{ ordinate for } M_B = \frac{y_{XB}}{\theta_{BB}} = \left[\frac{\frac{0.333x^3}{6} - \frac{x^2}{2} + x + 0.013}{2.33} \right] \rightarrow (9)$$

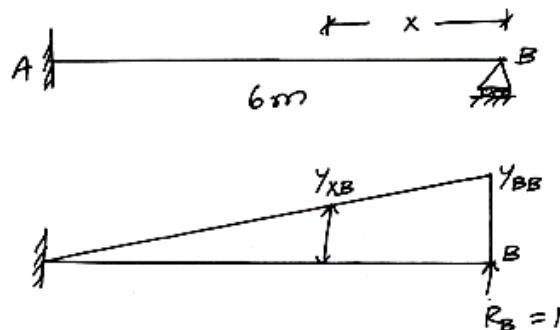
The influence line ordinates are tabulated,

x from C	0	1.75	3.5	4	5.25	7
I.L.O	0	0.41	0.24	0	-0.21	0



5. Draw the influence line diagram for the propped reaction of a propped cantilever beam having span 6 m. Take $EI = \text{constant}$. (AUC Apr/May 2011)

Solution:



a) To generate the ILD for R_B :-

- Remove the restraint due to R_B (remove support B)
- Apply a unit displacement (upward) at B.

When $R_B = 1$, then y_{xB} is the displacement at section x due to unit load applied at B,

$$M_x = -EI \frac{d^2 y}{dx^2} = R_B \cdot x = 1x$$

$$EI \frac{d^2 y}{dx^2} = -x$$

$$EI \frac{dy}{dx} = -\frac{x^2}{2} + C_1 \rightarrow (1)$$

$$EI y = -\frac{x^3}{6} + C_1 x + C_2 \rightarrow (2)$$

At $x = 6$,

$$y = 0; \frac{dy}{dx} = 0$$

(1) & (2) we get,

$$(1) \Rightarrow 0 = -\frac{x^2}{2} + C_1$$

$$C_1 = \frac{x^2}{2} = \frac{(6)^2}{2} = 18$$

$$\boxed{C_1 = 18}$$

$$(2) \Rightarrow 0 = -\frac{x^3}{6} + C_1 x + C_2$$

$$C_2 = \frac{x^3}{6} - C_1 x = \frac{(6)^3}{6} - (18 \times 6)$$

$$\boxed{C_2 = -72}$$

Hence,

$$(2) \Rightarrow y_{xB} = \frac{1}{EI} \left[-\frac{x^3}{6} + 18x - 72 \right] \rightarrow (3)$$

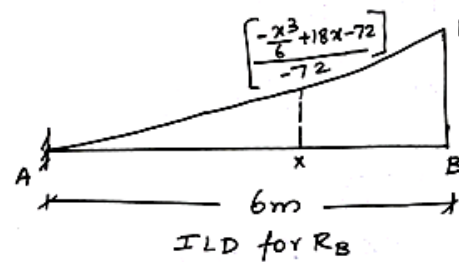
$$(2) \Rightarrow y_{BB} \text{ (at } x=0) = \left[0 + C_1(0) + C_2 \right] \frac{1}{EI}$$

$$y_{BB} = -\frac{72}{EI} \rightarrow (4)$$

$$\text{IL ordinate for } R_B \text{ at } x = \frac{y_{xB}}{y_{BB}}$$

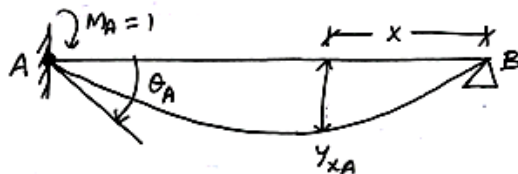
$$= \frac{\frac{1}{EI} \left[-\frac{x^3}{6} + 18x - 72 \right]}{-\frac{72}{EI}}$$

$$R_B = \frac{-\frac{x^3}{6} + 18x - 72}{-72}$$



b) ILD for M_A :-

- i) Introduce a hinge at A
- ii) Apply a unit rotation at A.



Due to $M_A = 1$, $\Rightarrow R_B \times 12 = 1 \Rightarrow R_B = \frac{1}{6}$

$$M_x = -EI \frac{d^2 y}{dx^2} = \frac{x}{6}$$

$$EI \frac{d^2 y}{dx^2} = -\frac{x}{6}$$

$$EI \frac{dy}{dx} = -\frac{x^2}{12} + C_1 \rightarrow (5)$$

$$EI y = -\frac{x^3}{36} + C_1 x + C_2 \rightarrow (6)$$

At $x=0; y=0 \Rightarrow \boxed{C_2 = 0}$

At $x=6; y=0 \Rightarrow 0 = -6 + 6C_1$

$$\boxed{C_1 = 1}$$

$$(6) \Rightarrow y_{xA} = \frac{1}{EI} \left(-\frac{x^3}{36} + x \right)$$

$$\frac{dy}{dx} = \theta_{xA} = \frac{1}{EI} \left(-\frac{x^2}{12} + 1 \right)$$

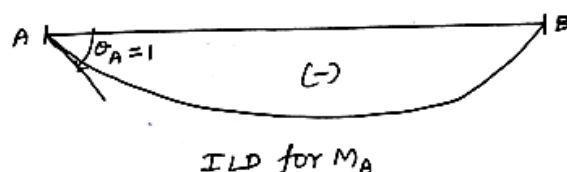
At $x=6$,

$$\theta_A = -\frac{2}{EI}$$

When we divide y_{XA} by θ_A ,

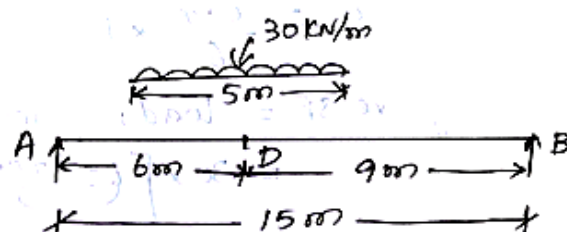
$$\text{IL ordinate at } x \text{ for } M_A = \frac{\frac{1}{EI} \left(-\frac{x^3}{36} + x \right)}{-\frac{2}{EI}}$$

$$M_A = \left(\frac{x^3}{72} - \frac{x}{2} \right)$$

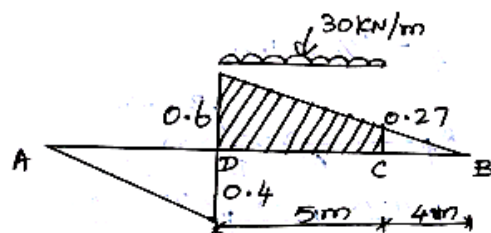


6. A simply supported beam has a span of 15 m and subjected to an UDL of 30 kN/m, 5 m long travelling from left to right. Draw the ILD for shear force and bending moment at a section 6 m from the left end. Use these diagrams for calculating the maximum BM and SF at this section. (AUC Nov/Dec 2010)

Solution:



step 1: Maximum positive SF :-



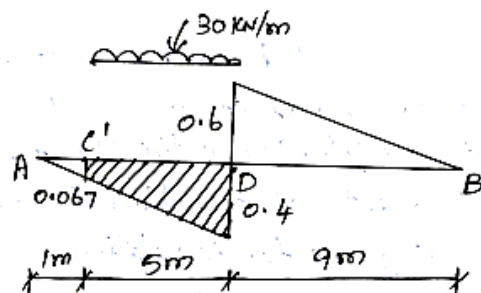
$$\text{ordinate at D} = \frac{l-x}{l} = \frac{15-6}{15} = 0.6$$

$$\text{ordinate at C} = \frac{0.6}{9} \times 4 = 0.27$$

$$\begin{aligned} \text{Maximum positive SF} &= \text{load} \times \text{Area} \\ &= 30 \times \left[\left(\frac{0.6+0.27}{2} \right) \times 5 \right] \\ &= 65.25 \text{ kN} \end{aligned}$$

step 2: Maximum Negative SF :-

$$\text{ordinate at D} = \frac{x}{l} = \frac{6}{15} = 0.4$$

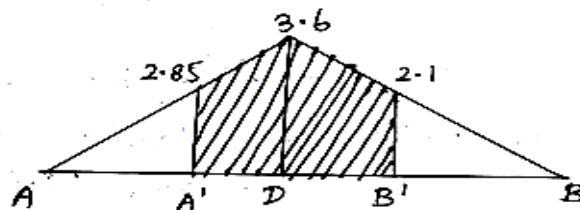
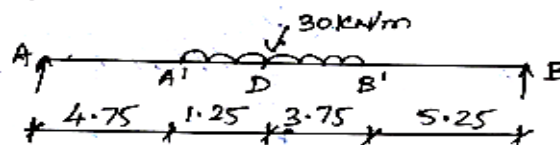


$$\text{ordinate at } C' = \frac{0.4}{6} \times 1 = 0.067$$

$$\begin{aligned} \text{Maximum Negative SF} &= \text{load} \times \text{Area} \\ &= 30 \times \left[\left(\frac{0.4 + 0.067}{2} \right) \times 5 \right] \\ &= 35 \text{ kN} \end{aligned}$$

step 3: maximum Bending Moment :-

$$l/4 = 5/4 = 1.25 \text{ m from left of D.}$$



$$\text{ordinate at D} = x \left(\frac{l-x}{l} \right) = 6 \left(\frac{15-6}{15} \right) = 3.6$$

$$\text{ordinate at B'} = \frac{3.6}{9} \times 5.25 = 2.1$$

$$\text{ordinate at A'} = \frac{3.6}{6} \times 4.75 = 2.85$$

$$\begin{aligned} \text{Maximum BM} &= \text{load} \times \text{Area} \\ &= 30 \left[(2.85 \times 1.25) + \left(\frac{1}{2} \times 0.75 \times 1.25 \right) \right. \\ &\quad \left. + (2.1 \times 3.75) + \left(\frac{1}{2} \times 1.5 \times 3.75 \right) \right] \\ &= 30 \times 14.72 \end{aligned}$$

$$\boxed{\text{Max. BM} = 441.6 \text{ kNm}}$$

7. Draw the ILD for the forces in members U_2L_2 and U_2L_3 of the truss shown in figure.

(AUC Nov/Dec 2010)

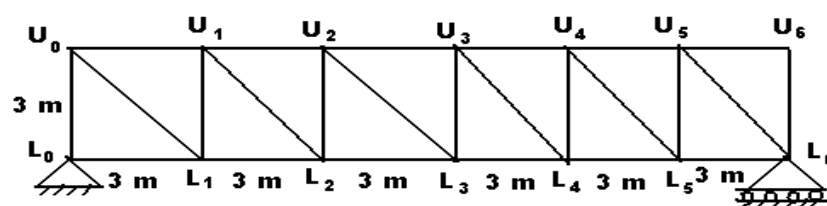
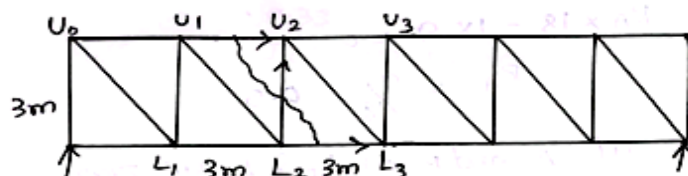


Fig.

Solution:

a) ILD for the member U_2L_2 :-



The nature of shear force in the panel L_2L_3 change as the load moves from L_2 to L_3 .

When unit load is at L_2 , shear force is negative and force in U_2L_2 is tension. When it is at L_3 , the force in U_2L_2 is compression.

To find ILD at L_2 and at L_3 and join them to get the ILD for shear in the panel L_2L_3 .

Unit load is at L_2 , shear in $L_2L_3 = -R_B$

$$R_A + R_B = 1$$

Taking moment about B

$$R_A \times 18 - 1 \times 12 = 0$$

$$R_A = \frac{12}{18} = 0.67$$

$$R_B = 0.33$$

Shear in panel, at $L_2L_3 = -R_B = -0.33$

When the load is at L_3 ,

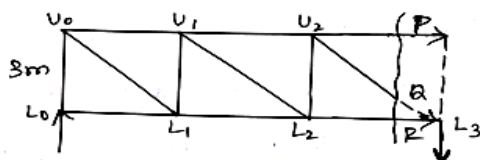
$$R_A + R_B = 1$$

$$R_A \times 18 - 1 \times 9 = 0$$

$$R_A = \frac{9}{18} = 0.5$$

∴ ILD at A and B (or) L_0 & L_6 are zero.

b.) ILD for the member $U_2 L_3$:-



i.) Unit load at L_2 :-

taking moments about B,

$$R_A \times 18 - 1 \times 12 = 0$$

$$R_A = 0.67$$

Since $\sum V = 0$;

$$R_A - 1 - Q \cos 45^\circ = 0$$

$$Q = \frac{0.67 - 1}{\cos 45^\circ} = 0.47 \text{ (comp)}$$

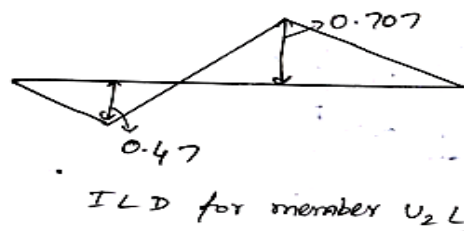
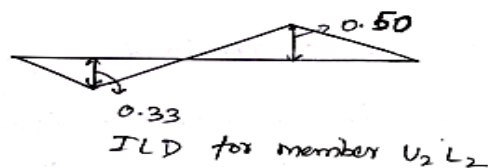
ii.) Unit load at L_3 :-

$$R_A = 0.5$$

Since $\sum V = 0$;

$$R_A - Q \cos 45^\circ = 0$$

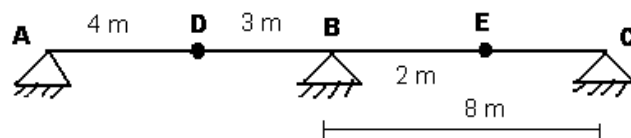
$$Q = 0.707 \text{ (tension)}$$



8. A beam ABC is supported at A, B and C as shown in Fig. It has the hinge at D. Draw the influence lines for

- Reactions at A, B and C
- Shear to the right of B
- Bending moment at E

(AUC Nov/Dec 2012, 2013)

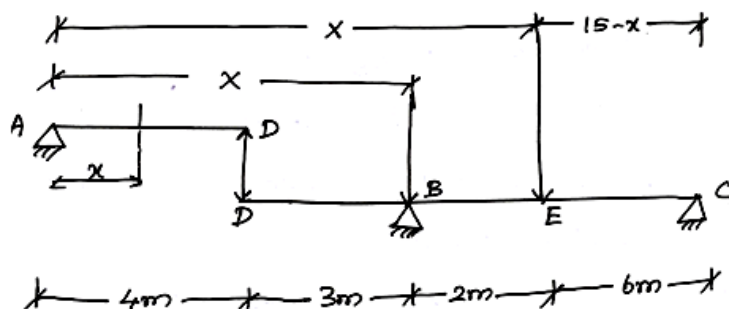


Solution:

a.) Influence lines for R_A , R_B and R_C :-

i) IL for R_A :-

Due to hinge at D, AD will behave as a s/s beam. The reaction at D, when it acts upward at D on AD, downward action at D of DBC. Any load on DBC will have no effect on AD.



When a unit load is on AD,

$$R_A = \frac{4-x}{4}$$

$$\text{At } x=0; R_A = 1.$$

$$\text{At } x=4; R_A = 0$$

$$\text{When } x > 4; R_A = 0.$$

ii) IL for R_B :-

When a unit load is on AD,

$$R_A = 1 - \frac{x}{4}$$

$$R_D = 1 - R_A = \frac{x}{4}.$$

Taking moment about C,

$$R_B \times 8 - R_D \times 11 = 0$$

$$8R_B - \frac{x \times 11}{4} = 0$$

$$R_B = \frac{11x}{4 \times 8} = \frac{11x}{32}$$

When $x = 0$; $R_B = 0$.

At $x = 4$; $R_B = 1.375$

When the load is on DBC, the reaction at D is zero.

Taking moment about C,

$$R_B \times 8 - 1(15-x) = 0$$

$$R_B = \frac{15-x}{8}$$

When $x = 4$; $R_B = 1.375$

$x = 7$; $R_B = 1$

$x = 15$; $R_B = 0$

(iii) IL for R_c :-

When the unit load is on AD, 'load' at D $= \frac{x}{4} \downarrow$.

Taking moments about B,

$$-\frac{x}{4} \times 3 - R_c \times 8 = 0$$

$$R_c = -\frac{3x}{32}$$

When $x = 0$; $R_c = 0$.

$x = 4$; $R_c = -0.375$.

When the load is over DBC, $R_D = 0$.

Taking moments about B,

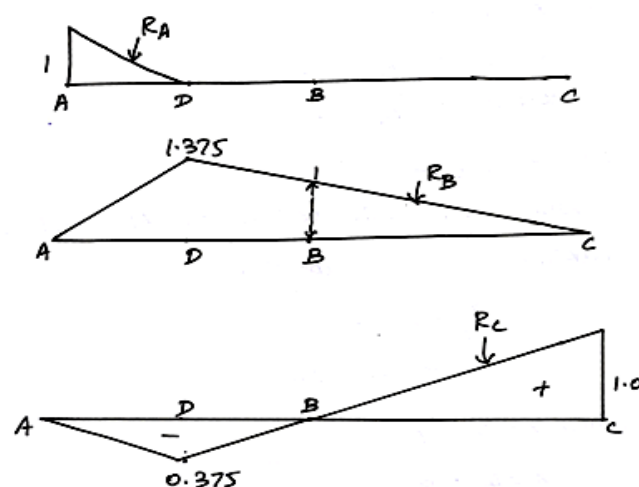
$$-1(7-x) - R_c \times 8 = 0$$

$$R_c = \frac{x-7}{8}$$

$x = 4$; $R_c = -0.375$

$x = 7$; $R_c = 0$

$x = 15$; $R_c = 1$



b.) ILD for shear to the right of B (F_B):-

when the load is on AD, $F_B = -R_C$

$$R_C = -\frac{3x}{32} ; F_B = \frac{3x}{32}$$

$$x=0 ; F_B = 0$$

$$x=4 ; F_B = 0.375$$

When the load is over DB, $F_B = -R_C$

$$R_C = \frac{x-7}{8} ; F_B = \frac{7-x}{8}$$

$$x=4 ; F_B = 0.375$$

$$x=7 ; F_B = 0$$

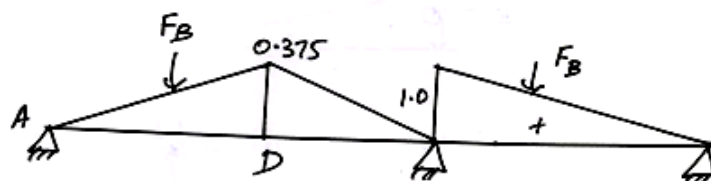
When the load is over BC, $F_B = R_B$

$$R_B \times 8 - 1(15-x) = 0$$

$$F_B = R_B = \frac{15-x}{8}$$

$$x=7 ; F_B = 1$$

$$x=15 ; F_B = 0$$



c.) ILD for BM at E (M_E):-

When unit load is on AD,

$$M_E = R_c \times 6 = -\frac{3x}{32} \times 6$$

$$M_E = -\frac{18x}{32}$$

$$x=0; \quad M_E = 0$$

$$x=4; \quad M_E = -2.25$$

When the load is b/n D & E,

$$M_E = R_c \times 6 = \frac{x-7}{8} \times 6 = \frac{3}{4}(x-7)$$

$$x=4; \quad M_E = -2.25$$

$$x=7; \quad M_E = 0$$

$$x=9; \quad M_E = 1.5$$

When the load is b/n E & C:-

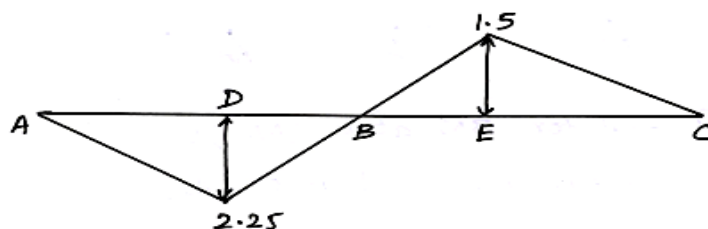
$$M_E = R_c \times 6 - 1(x-9)$$

$$R_c = \frac{x-7}{8}$$

$$M_E = \frac{(x-7)6}{8} - (x-9)$$

$$x=9 \text{ m}; \quad M_E = 1.5$$

$$x=15; \quad M_E = 0$$



9. Explain the procedure and applications of Beggs deformer.

(AUC May/June 2014)

Introduced by professor G.E. Beggs of Princeton University in 1922, Beggs' Deformer addresses all the minute experimental considerations in applying M-B Principle for model analysis.

Fig. 3.25 (b) shows an experimental setup using Beggs' Deformer. Fig. 3.25 (c) shows a single Beggs' Deformer gauge.

The gauge is made up of 2 metal bars held together by a pair of spring loaded screws. The bars can be separated by a precise distance with the aid of several pairs of plugs (Fig 3.26). Of the 2 bars, one is called the fixed bar. This has to be fixed to the drawing board with a pair of wood screws. To the other bar, the model, suitably designed and shaped to simulate any given structure, is attached. Three types of connections with the model are possible. (Fig. 3.27).

1. Hinged connection, in which the model is pivoted to the moving bar. A hole in the model engages into a pin on the moving bar.

2. Fixed connection in which the model is clamped to the moving bar using a fixing plate and 4 screws.

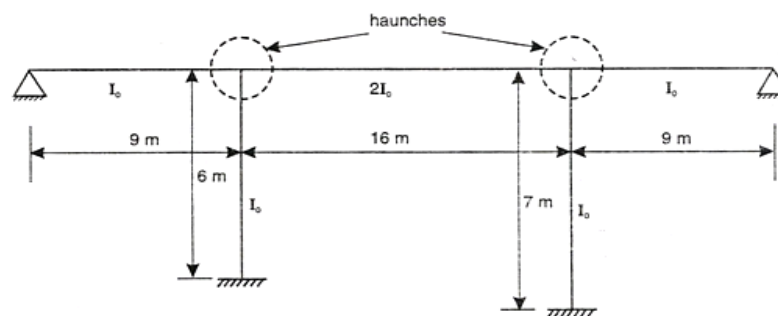
3. Floating connection in which neither of the beams is fixed to the drawing board but is kept afloat on a plate of glass, supported on steel balls which rest on another plate of glass. Here the model is attached to both the beams of the gauge using serrated metal strips about 3 mm wide and 25 mm long. To complete the floating gauge connection, the model has to be cut in the region, between the 2 bars of the gauge while normal plugs are in position.

(a) Calibration of plugs

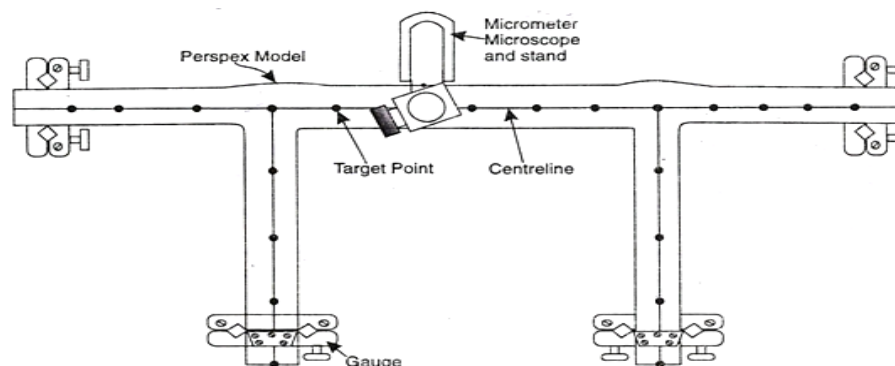
For determining how much displacement is effected by each pair of plugs, normally a cantilever arrangement is employed. The arrangement is shown in Fig. 3.28 (a) to (e). The cantilever can be 100 mm long from the face of fixity to the target point.

The normal position is when both the slots in the gauge are fitted with normal plugs. A micrometer microscope is positioned over the target. This instrument is capable of measuring movements in the x and y co-ordinate directions correct to 0.004 mm. The initial readings are noted with normal plugs in position.

For x displacements we use the 2 pairs of thrust plugs. First remove the normal plugs, and introduce two large thrust plugs. The target points would move in x negative direction. The FM microscope would measure the x movement in the microscope units. (It is not even necessary to convert this into microns or millimeters since the units would cancel off.) Next we introduce two small thrust plugs. This would move the target in x -positive direction. The FMM readings would indicate the actual movement due to small thrust plugs. The net movement between the x positive and x negative extremes is the displacement effected by thrust plugs.



(a) Line Diagram of Structure



(b) Experimental Setup

To calibrate the moment plugs (o, O),

- (i) Observe Y reading of target with normal plugs in place
- (ii) Observe Y reading with moment plugs as in Fig 3.28 (e) (small plug above and large plug below). This would cause target to move downwards (Y-negative).
- (iii) Observe Y reading with moment plugs inter charged causing upward (Y-positive) movement of target.

The difference between readings in (iii) and (ii) above divided by the length of the cantilever would give the calibration value of moment plugs.

(b) Filar Micrometer Microscope

This comes with a heavy metallic stand and can be set up above one target point on the model at a time.

Because of the large magnification what we think is a circular target looks like a figure with jagged edges. So special care must be taken in making targets in the form of black filled in circles.

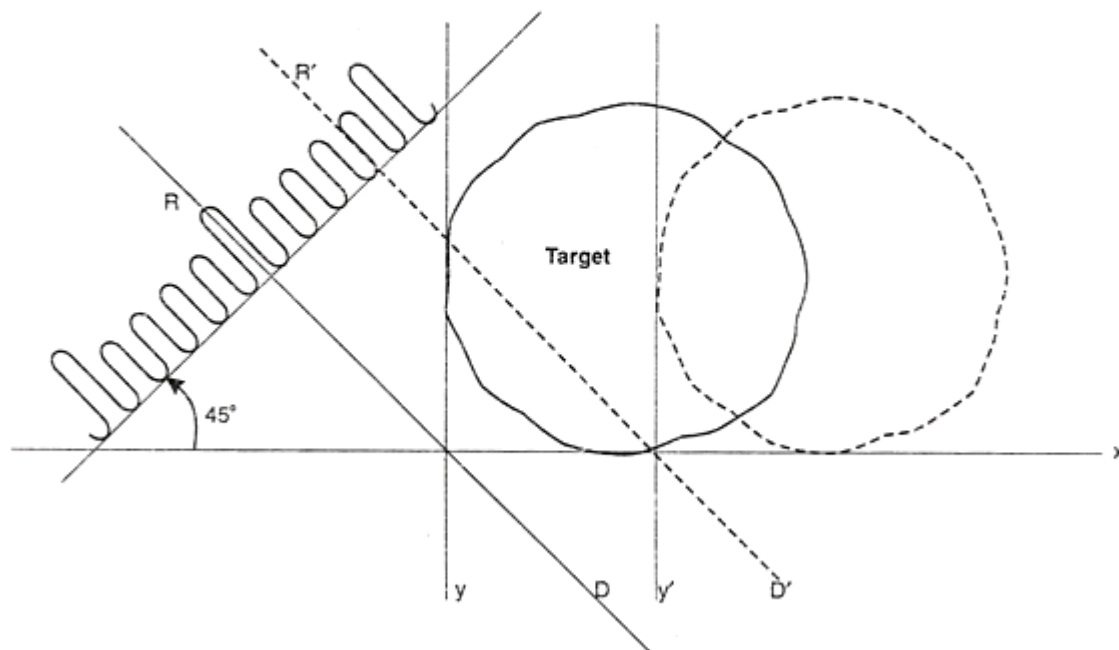


Fig. 3.29

In the field of a *f. m.* microscope a single diagonal scale serves for both *x* and *y* displacements. The diagonal scale is the main scale which is served by an outside drum scale.

The 3 intersecting lines *x*, *y* and *D* can be bodily moved in the field of view of the microscope.

If we want the *x* movement of the target we first make *y* line tangential to the target and take the main scale reading on *D*-line. After the target has shifted, we again bring *y* line tangential to the new target position shown dotted. This line is marked as *y'*. Now the *D* line has moved along the diagonal scale to *D'*. *R'* is the new main scale reading. We can use the drum to bring the *D'* line to the nearest whole main scale reading to get the fraction of the distance from the whole main scale reading.

Recent trends in *f.m.* microscope is to adopt digital indicators in which the displacement of the target can be read off a monitor attached to the *f.m.m.*