QUESTION BANK

DEPARTMENT: CIVIL **SEMESTER: V**

SUBJECT CODE / Name: CE 2302 / STRUCTURAL ANALYSIS-I

UNIT 1- DEFLECTION OF DETERMINATE STRUCTURES

PART - A (2 marks)

1. **State and explain the principle of virtual work.**

(AUC Apr/May 2012 & 2011, Nov/Dec 2012 & 2013, May/June 2014)

The principle of virtual work is based on the conservation of energy for a structure which implies that work done on a structure by external loads is equal to workdone on a structure by internal loads.

2. **Write down the Castigliano's first theorem.** **(AUC Nov/Dec 2010)**

The partial derivative of the total strain energy with respect to an applied force or moment gives the displacement or rotation at the point of application of the force and in the direction of application of the force.

3. **What is the significance of unit load method?** **(AUC Apr/May 2012)**

The external load is removed and the unit load is applied at the point, where the deflection or rotation is to found.

4. **State the basic unit load formula.** **(AUC Nov/Dec 2010)**

i) Find the forces P1, P2, ……. in all the members due to external loads.
ii) Remove the external loads and apply the unit vertical point load at the joint if the vertical deflection is required and find the stress.
iii) Apply the equation for vertical and horizontal deflection.

5. **Explain Mohr's correction.** **(AUC Apr/May 2011)**

The Williot diagram does not give the true deflection of the joints but the same can be modified and correlated to the true deflection by applying certain correction is known as Mohr's correction.

6. **Differentiate perfect and imperfect frame.** **(AUC Nov/Dec 2012)**

A structural frame that is stable under loads imposed upon it from any direction is known as perfect frame.

A structural frame is unstable if one of its members were removed or one of its fixed ends became hinged is known as imperfect frame.

7. State Maxwell's Reciprocal theorem?

This theorem states that 'work done by the forces of the first state on the corresponding displacements of the second state is equal to the work done by the forces of the second state on the corresponding displacements of the first state'.

8. Determine the free end slope of a cantilever beam having length 'L' due to an applied moment 'M' at free end using the principle of virtual work?

$$\Delta = \int_0^l \frac{mM}{EI} dx$$

Here m = - x and M = $-\frac{wx^2}{2}$

$$\Delta = \frac{wl^2}{8}$$

9. Distinguish between pin jointed and rigidly jointed structures.

Pin jointed structures:

i) The joints permit change of angle between connected members.
ii) The joints are incapable of transferring any moment to the connected members and Vice versa.
iii) The pins transmit forces between connected members by developing shear.

Rigidly jointed structures:

i) The members connected at a rigid joint will maintain the angle between them even under deformation due to loads.
ii) Members can transmit both forces and moments between themselves through the joint.
iii) Provision of rigid joints normally increases the redundancy of the structures.

10. What are the assumptions made in the analysis of pin jointed trusses?

i) All the members are pin jointed.
ii) External loads are transmitted to the structure only at the joint.
iii) Pins do not transfer any moment to any of the connected members.
iv) Pins allow the connected members to change the angles between them.

11. Explain Williot's diagram.

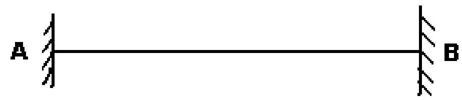
A graphical method used to determining the deflection of a framed structure under the load is known as Williot's diagram.

12. Give the equation that is used for the determination of deflection at a given point in truss and frames?

For truss, $\Delta = \frac{\Sigma kFl}{AE}$

For frames, $\Delta = \int_0^l \frac{mM}{EI} dx$

13. Find the static indeterminacy of below figure.



Static indeterminacy = No. of unknowns – No. of conditions

= 6 – 3

= 3

14. Define internally and externally indeterminate structures.

Internally indeterminate structures:

In a pin jointed frames redundancy caused by too many members is called internally indeterminate structures or internal redundancy.

Externally indeterminate structures:

In a pin jointed frames redundancy caused by too many supports is called externally indeterminate structures or external redundancy.

15. Define degree of freedom.

In a structure the number of independent joint displacement that the structures can undergo are known as degree of freedom. It is also known as kinematic indeterminacy.

16. Write any two important assumptions made in the analysis of trusses?

i) All the members are pin jointed.

ii) External loads are transmitted to the structure only at the joint.

17. State the difference between strain energy method and unit load method in the determination of deflection of structures?

In the unit load method, one has to analyze the frame twice to find the load and deflection.

While in the strain energy method, only one analysis is needed to find the load and deflection.

18. Name any four methods used for computation of deflection in structures?

Double integration method, Macaulay's method, Conjugate beam method, Moment area method, Method of elastic weights, Virtual work method- Dummy unit load method, Strain energy method and Williot Mohr diagram method.

19. Define static indeterminacy of structures. (AUC Nov/Dec 2013)

If the conditions of statics i.e. $\sum H = 0$, $\sum V = 0$ and $\sum M = 0$ alone are not sufficient to find either external reactions or internal forces in a structure. The structure is called static indeterminacy of structures.

20. Define static determinate structures. (AUC Nov/Dec 2013)

If the conditions of statics i.e. $\sum H = 0$, $\sum V = 0$ and $\sum M = 0$ alone are sufficient to find either external reactions or internal forces in a structure. The structure is called static determinacy of structures.

PART - B (16 marks)

1. **Determine the horizontal displacement at the roller support of the rigid jointed frame shown in figure. Take $E = 2\times10^5$ MPa and $I_1 = 30\times10^8$ mm^4. (AUC Apr/May 2012)**

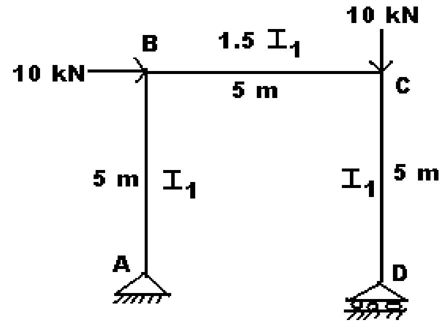
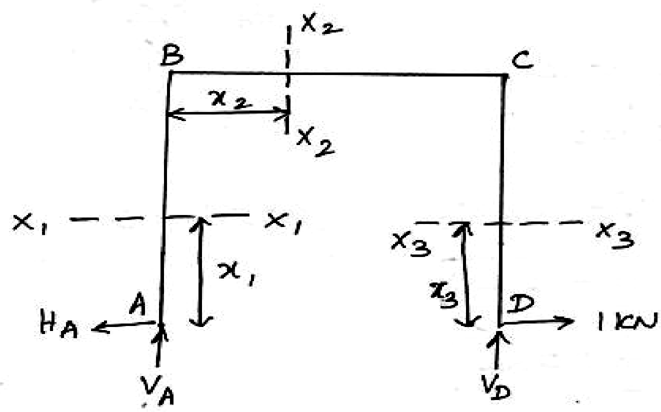


Fig.

Solution:

step 1 : Virtual Moments (m) :-



$\Sigma H = 0$

$$\boxed{H_A = 1\,KN}$$

$\Sigma V = 0$

$V_A + V_D = 0$

$\Sigma M @ D = 0$

$V_A \times 5 = 0$

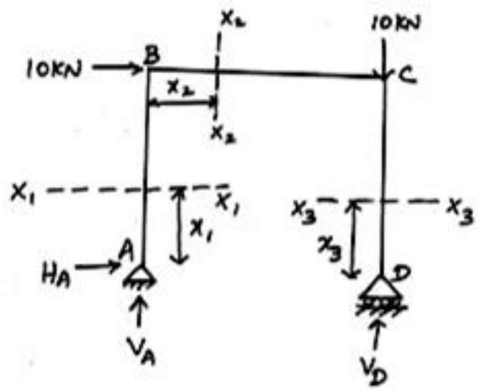
$$\boxed{V_A = V_D = 0}$$

$m_1 = +x_1$ (limits 0 to 5m)

$m_2 = 1 \times 5 = 5$ (limits 0 to 5m)

$m_3 = -x_3$ (limits 0 to 5m)

Step 2: Real Moments (M):-



$\Sigma H = 0$

$H_A + 10 = 0$

$H_A = -10$ KN

$\boxed{H_A = 10 \text{ KN}}$ $(\leftarrow)$

$\Sigma V = 0$

$V_A + V_D = 10$

$\Sigma M @ D = 0$

$(V_A \times 5) + (H_A \times 0) + (10 \times 5) = 0$

$\boxed{V_A = -10 \text{ KN}}$

$\boxed{V_D = 20 \text{ KN}}$

$M_1 = 10x_1$ (limits 0 to 5m)

$M_2 = (10 \times 5) - 10x_2$ (limits 0 to 5m)

$M_3 = 20 \times 0 = 0$ (limits 0 to 5m)

Step 3: Virtual Work Equation:-

$$(\Delta_D)_h = \int_0^l \frac{mM}{EI}\, dx$$

$$= \frac{1}{EI}\int_0^5 x(10x)\, dx_1 + \frac{1}{E(1.5I)}\int_0^5 5(50-10x_2)\, dx_2 + \frac{1}{EI}\int_0^5 0(-x_3)\, dx_3$$

$$= \frac{1}{EI}\int_0^5 10x^2\, dx + \frac{1}{1.5EI}\int_0^5 (250-50x)\, dx + 0$$

$$= \frac{1}{EI}\left[\frac{10x^3}{3}\right]_0^5 + \frac{1}{1.5EI}\left[250x - \frac{50x^2}{2}\right]_0^5$$

$$= \frac{416.67}{EI} + \frac{625}{1.5\,EI}$$

$$= \frac{833.34}{EI}$$

$$= \frac{833.34}{2\times10^{8}\times30\times10^{-4}}$$

$$(\Delta_D)_h = 0.00139\ m$$

$$\boxed{(\Delta_D)_h = 1.39\ mm}$$

$\therefore$ The Horizontal deflection at D is 1.39 mm.

2. **Determine the vertical deflection of point C in the truss shown in figure. The cross sectional area of members AD and DE are 1500 mm^2 while those of other members are 1000 mm^2. Take E = 200 GPa. (AUC Apr/May 2012)**

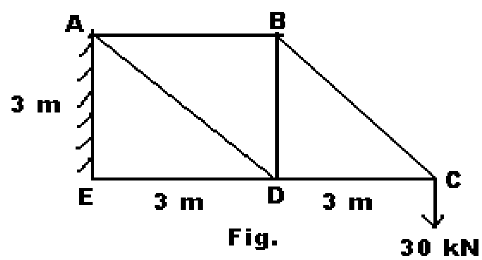
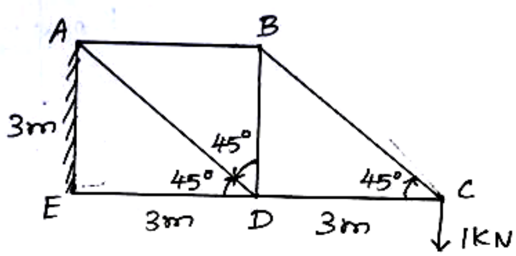


Fig.

Solution:

Step 1: Virtual forces (K):-



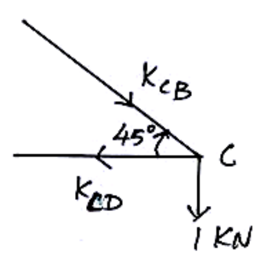
At Joint :-

$\Sigma V = 0$

$$1 + K_{CB}\sin 45^\circ = 0$$

$$K_{CB} = -1.414\ KN\ (Comp)$$

$$\boxed{K_{CB} = 1.414\ KN\ (Tensile)}$$



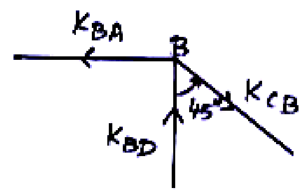
$\Sigma H = 0$

$K_{CD} = K_{CB} \cos 45° = -1.414 \cos 45°$

$= -1 \text{ kN (T)}$

$\boxed{K_{CD} = 1 \text{ kN (Comp)}}$

At Joint B :-



$\Sigma V = 0$

$K_{BD} = K_{CB} \sin 45°$

$= 1.414 \times \sin 45°$

$\boxed{K_{BD} = 1 \text{ kN (Comp)}}$

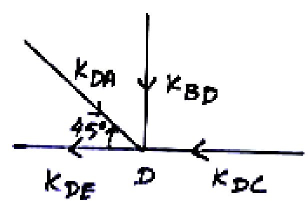
$\Sigma H = 0$

$K_{BA} = K_{CB} \cos 45°$

$= 1.414 \times \cos 45°$

$\boxed{K_{BA} = 1 \text{ kN (Tensile)}}$

At Joint D:-



$\Sigma V = 0$

$K_{BD} + K_{DA} \sin 45° = 0$

$1 + K_{DA} \sin 45° = 0$

$K_{DA} = -1.414 \text{ kN (Comp)}$

$\boxed{K_{DA} = 1.414 \text{ kN (Tensile)}}$

$\Sigma H = 0$

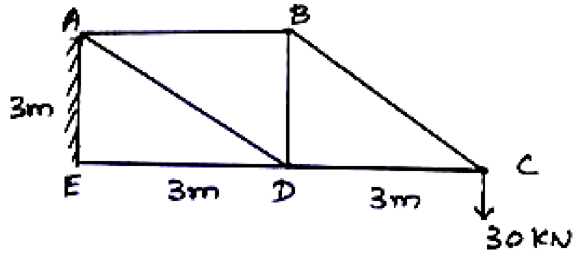
$K_{DE} + K_{DC} = K_{DA} \cos 45°$

$K_{DE} + 1 = -1.414 \times \cos 45°$

$K_{DE} = -1 - 1 = -2 \text{ kN (Tensile)}$

$\boxed{K_{DE} = 2 \text{ kN (Comp)}}$

Step 2 : Real Forces (F):-



Since there is no other external loads except the point c. So, the external load is 30KN is applied at c. So, we multiply the virtual members forces into 30 times we get real forces.

$F_{CB} = 1.414 \times 30 = 42.42\ kN\ (T)$

$F_{CD} = 1 \times 30 = 30\ kN\ (C)$

$F_{BD} = 1 \times 30 = 30\ kN\ (C)$

$F_{BA} = 1 \times 30 = 30\ kN\ (T)$

$F_{DA} = 1.414 \times 30 = 42.42\ kN\ (T)$

$F_{DE} = 2 \times 30 = 60\ kN\ (C)$

S.No.	Member	K	F (kN)	L (m)	ΣKFL (kN·m)
1.	BC	1.414	42.42	4.24	254.32
2.	CD	-1	-30	3	90
3.	BD	-1	-30	3	90
4.	BA	1	30	3	90
5.	DA	1.414	42.42	4.24	254.32
6.	DE	-2	-60	3	360
				ΣKFL =	1138.64

Step 3 : Virtual Work Equation :-

$$(\Delta_c)_v = \frac{\Sigma KFL}{AE}$$

For AD and DE members $= 1500 mm^2$

For other members $= 1000 mm^2$

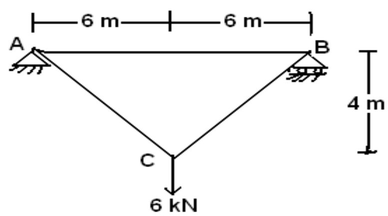
$$(\Delta_c)_v = \frac{\Sigma KFL}{AE}$$

$$= \left[\frac{(254.32 + 360) \times 10^6}{1500 \times 2 \times 10^5}\right] + \left[\frac{(254.32 + 90 + 90 + 90) \times 10^6}{1000 \times 2 \times 10^5}\right]$$

$$(\Delta_c)_v = 4.67 mm$$

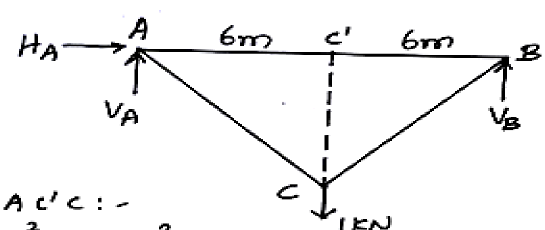
$\therefore$ The vertical displacement at $c = 4.67 mm$

3. **Determine the vertical and horizontal displacements of the point C of the pin jointed frame shown in figure. The cross sectional area of AB is 125 square mm and of AC and BC are 175 square mm each. E = 2 x 10^5 N per square mm. (AUC Nov/Dec 2012 & 2013)**



Solution:

Step 1: Vertical Virtual force (K_v):-



In Δ $Ac'c$:-

$$Ac^2 = Ac'^2 + cc'^2$$

$$Ac = \sqrt{(6)^2 + (4)^2}$$

$$\boxed{Ac = 7.21 m}$$

$$\boxed{BC = 7.21 m}$$

$\Sigma V = 0$

$$V_A + V_B = 1$$

Taking moment about A,

$$-V_B \times 12 + (1 \times 6) = 0$$

$\boxed{V_B = 0.5\ kN}$

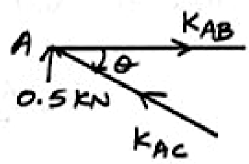
$\boxed{V_A = 0.5\ kN}$

At Joint A :-

In Δ AC'C,

$\tan\theta = \frac{4}{6}$

$\boxed{\theta = 33°49'}$



$\Sigma V = 0$

$0.5 + K_{AC} \sin(33°49') = 0$

$K_{AC} = -0.89\ kN$ (Comp)

$\boxed{K_{AC} = 0.89\ kN}$ (Tensile)

$\Sigma H = 0$

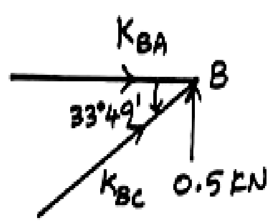
$K_{AB} = K_{AC} \cos(33°49')$

$= -0.89 \times \cos(33°49')$

$= -0.74\ kN$ (Tensile)

$\boxed{K_{AB} = 0.74\ kN}$ (comp)

At Joint B :-



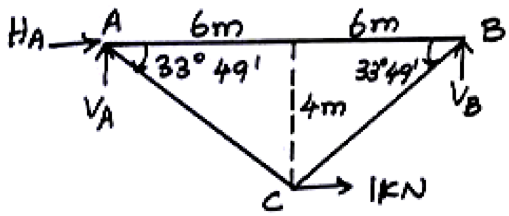
$\Sigma V = 0$

$0.5 + K_{BC} \sin(33°49') = 0$

$K_{BC} = -0.89\ kN$ (Comp)

$\boxed{K_{BC} = 0.89\ kN}$ (Tensile)

Step 2: Horizontal Virtual Force (K_h) :-



$\Sigma H = 0$

$H_A + 1 = 0$

$H_A = -1 kN \ (\rightarrow)$

$\boxed{H_A = 1 kN} \ (\leftarrow)$

$\Sigma V = 0$

$V_A + V_B = 0$

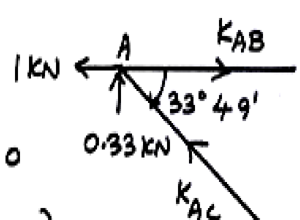
Taking moment about A,

$-V_B \times 12 - 1 \times 4 = 0$

$\boxed{V_B = -0.33 kN}$

$\boxed{V_A = 0.33 kN}$

At Joint A :-



$\Sigma V = 0$

$0.33 + K_{AC} \sin(33°49') = 0$

$K_{AC} = -0.59 kN \ (comp)$

$\boxed{K_{AC} = 0.59 kN}$ (Tensile)

$\Sigma H = 0$

$K_{AB} = 1 + K_{AC} \sin(33°49')$

$= 1 - (0.59 \times \sin(33°49'))$

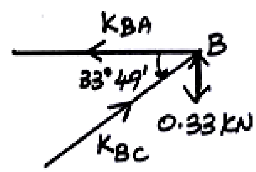
$\boxed{K_{AB} = 0.67 \text{ KN}}$ (Tensile)

At Joint B :-

$\Sigma V = 0$

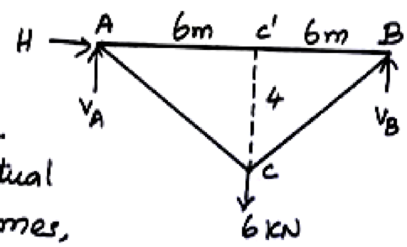
$K_{BC} \sin(33°49') = 0.33$

$\boxed{K_{BC} = 0.59 \text{ KN}}$ (comp)



Step 3 : Real Forces (F) :-

Since there is no other external loads acting except the point c. So, we multiply the virtual member forces into 6 times, we get real forces.



$F_{AC} = 0.89 \times 6 = 5.34 \text{ KN}$ (Tensile)

$F_{AB} = 0.74 \times 6 = 4.44 \text{ KN}$ (comp)

$F_{BC} = 0.89 \times 6 = 5.34 \text{ KN}$ (Tensile)

S.NO	Member	K_V	K_h	F (KN)	L (m)	$\Sigma K_V F L$	$\Sigma K_H F L$
1.	AB	-0.74	0.67	-4.44	12	39.42	-35.69
2.	AC	0.89	0.59	5.34	7.21	34.26	22.71
3.	BC	0.89	-0.59	5.34	7.21	34.26	-22.71

Step 4 : Virtual Work Equation :-

$$(\Delta_c)_V = \frac{\Sigma K_V F L}{AE}$$

$$= \left[\frac{39.42 \times 10^6}{125 \times 2 \times 10^5} + \frac{(34.26 + 34.26) \times 10^6}{175 \times 2 \times 10^5}\right]$$

$$\boxed{(\Delta_c)_v = 3.53\ mm}$$

$$(\Delta_c)_h = \frac{\Sigma K_h F L}{AE}$$

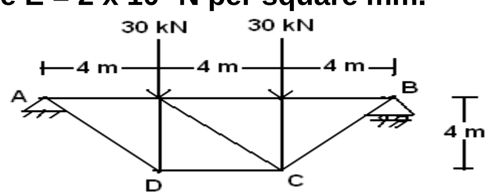
$$= \left[\frac{-35.69 \times 10^6}{125 \times 2 \times 10^5} + \frac{(22.71 - 22.71) \times 10^6}{175 \times 2 \times 10^5}\right]$$

$$\boxed{(\Delta_c)_h = -1.43\ mm}$$

$\therefore$ vertical displacement at c $= 3.53\ mm\ (\uparrow)$

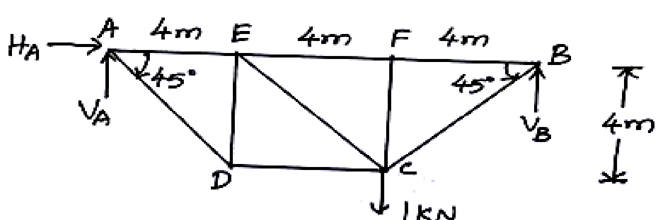
Horizontal displacement at c $= 1.43\ mm\ (\downarrow)$

4. **The steel truss shown in figure is anchored at A and supported on rollers at B. if the truss is so designed that, under the given loading, all tension members are stressed to 110 N per square mm and all compression members to 85 N per square mm. Find the vertical deflection of the point C. Take E = 2 x 10^5 N per square mm. (AUC Nov/Dec 2012)**



Solution:

Step:1 : Virtual Forces (K_v) :-



$\Sigma H = 0$

$\boxed{H_A = 0}$

$\Sigma V = 0$

$V_A + V_B = 1$

$\Sigma M @ A = 0,$

$-V_B \times 12 + 1 \times 8 = 0$

$\boxed{V_B = 0.67\ KN}$ $\boxed{V_A = 0.33\ KN}$

ΔAED,

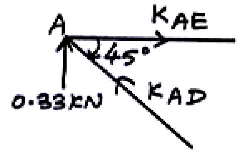
$\tan\theta = \frac{4}{4} = 1$

$\boxed{\theta = 45^\circ}$

At Joint A :-

$AD = EC = BC = \sqrt{4^2+4^2} = 5.65m$

$\Sigma V = 0$



$0.33 + K_{AD} \sin 45^\circ = 0$

$K_{AD} = -0.46 \text{ KN} \ (\text{Comp})$

$\boxed{K_{AD} = 0.46 \text{ KN}}$ (Tensile)

$\Sigma H = 0$

$K_{AE} = K_{AD} \cos 45^\circ$

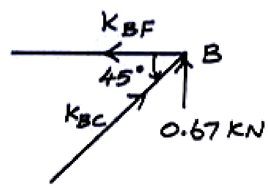
$= -0.46 \times \cos 45^\circ$

$= -0.32 \text{ KN}$ (Tensile)

$\boxed{K_{AE} = 0.32 \text{ KN}}$ (Comp)

At Joint B :-

$\Sigma V = 0$



$0.67 + K_{BC} \sin 45^\circ = 0$

$K_{BC} = -0.94 \text{ KN} \ (\text{comp})$

$\boxed{K_{BC} = 0.94 \text{ KN}}$ (Tensile)

$\Sigma H = 0$

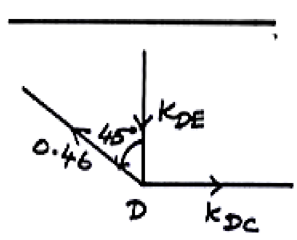
$K_{BF} = K_{BC} \cos 45^\circ = -0.94 \times \cos 45^\circ$

$= -0.66 \text{ KN}$ (Tensile)

$\boxed{K_{BF} = 0.66 \text{ KN}}$ (Comp)

At Joint D :-

$\Sigma V = 0$



$K_{DE} = 0.46 \times \sin 45^\circ$

$\boxed{K_{DE} = 0.32 \text{ KN}}$ (Comp)

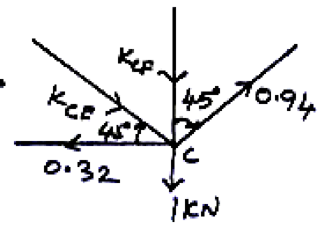
$\Sigma H = 0$

$K_{DC} = 0.46 \times \cos 45^\circ$

$\boxed{K_{DC} = 0.32 \text{ KN}}$ (Tensile)

At Joint C :-

$\Sigma H = 0$



$0.32 = K_{CE} \cos 45^\circ + 0.94 \times \cos 45^\circ$

$K_{CE} = -0.48\ kN$ (comp)

$\boxed{K_{CE} = 0.48\ kN}$ (Tensile)

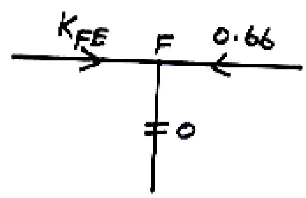
$\Sigma V = 0$

$K_{CF} + 1 + K_{CE} \sin 45^\circ = 0.94 \times \sin 45^\circ$

$K_{CF} + 1 - 0.48 \times \sin 45^\circ = 0.94 \times \sin 45^\circ$

$\boxed{K_{CF} = 0}$

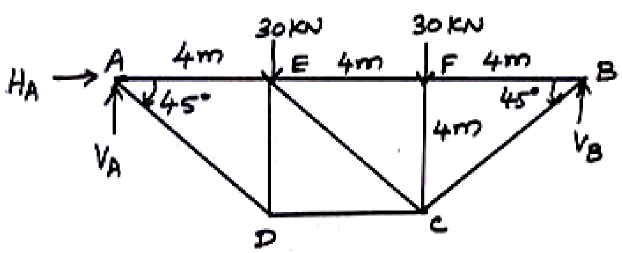
At Joint F:-



$\Sigma H = 0$

$\boxed{K_{FE} = 0.66\ kN}$ (comp)

Step 2: Real Forces (F):-



$\Sigma V = 0$

$V_A + V_B = 60$

$\Sigma M @ A = 0,$

$-V_B \times 12 + (30 \times 8) + (30 \times 4) = 0$

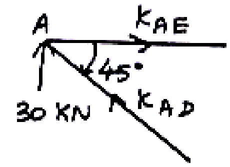
$\boxed{V_B = 30\ kN}$

$\boxed{V_A = 30\ kN}$

$\Sigma H = 0.$

$\boxed{H_A = 0}$

At Joint A :-



$\Sigma V = 0$

$30 + K_{AD} \sin 45° = 0$

$K_{AD} = -42.42$ KN (comp)

$\boxed{K_{AD} = 42.42 \text{ KN}}$ (Tensile)

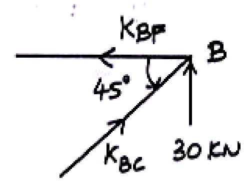
$\Sigma H = 0$

$K_{AE} = K_{AD} \cos 45° = -42.42 \times \cos 45°$

$= -30$ KN (Tensile)

$\boxed{K_{AE} = 30 \text{KN}}$ (Comp)

At Joint B :-



$\Sigma V = 0$

$30 + K_{BC} \sin 45° = 0$

$K_{BC} = -42.42$ KN (comp)

$\boxed{K_{BC} = 42.42 \text{ KN}}$ (T)

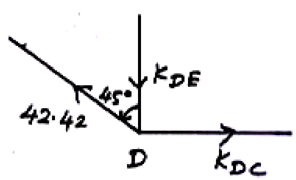
$\Sigma H = 0$

$K_{BF} = K_{BC} \cos 45° = -42.42 \times \cos 45°$

$= -30$ KN (T)

$\boxed{K_{BF} = 30 \text{ KN}}$ (comp)

At Joint D :-



$\Sigma V = 0$

$K_{DE} = 42.42 \times \sin 45°$

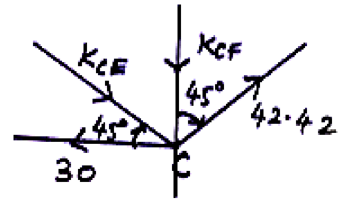
$\boxed{K_{DE} = 30 \text{ KN}}$ (Comp)

$\Sigma H = 0$

$K_{DC} = 42.42 \times \cos 45°$

$\boxed{K_{DC} = 30 \text{ KN}}$ (Tensile)

At Joint C :-



$\Sigma H = 0$

$30 = K_{CE} \cos 45° + 42.42 \times \cos 45°$

$\boxed{K_{CE} = 0}$

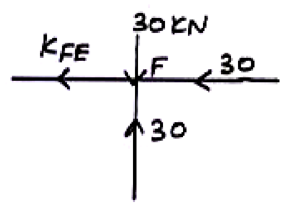
$\Sigma V = 0$

$K_{CF} + K_{CE} \times \sin 45° = 42.42 \times \sin 45°$

$K_{CF} + 0 = 30$

$\boxed{K_{CF} = 30 \text{ kN}}$ (Comp)

At Joint F :.



$\Sigma H = 0$

$K_{FE} + 30 = 0$

$K_{FE} = -30 \text{ kN}$ (T)

$\boxed{K_{FE} = 30 \text{ kN}}$ (Comp)

Step 3: Virtual Work Equation :-

Members	Length (m)	K_v	F (kN)	ΣKFL (kNm)
AD	5.65	0.46	42.42	110.25
AE	4	-0.32	-30	38.40
BC	5.65	0.94	42.42	225.29
BF	4	-0.66	-30	79.20
DC	4	0.32	30	38.40
DE	4	-0.32	-30	38.40
CE	5.65	0.48	0	0
CF	4	0	-30	0
EF	4	-0.66	-30	79.20

$$(\Delta_c)_v = \frac{\Sigma k_v F L}{AE}$$

here, Area is

For all tension members, $A = 110\ mm^2$.

For all compression members, $A = 85\ mm^2$.

For Tension members,

$$(\Delta_c)_v = \left[\frac{(110.25 + 225.29 + 38.40) \times 10^6}{1100 \times 2 \times 10^5}\right]$$

$$(\Delta_c)_v = 16.99\ mm.$$

For compression members,

$$(\Delta_c)_v = \left[\frac{(38.40 + 79.20 + 38.40 + 79.20) \times 10^6}{85 \times 2 \times 10^5}\right]$$

$$(\Delta_c)_v = 13.83\ mm$$

$$\therefore (\Delta_c)_v = 16.99 + 13.83$$

$$\boxed{(\Delta_c)_v = 30.82\ mm}$$

$\therefore$ The vertical deflection (or) displacement at C = 30.82 mm (↑)

5. Determine the horizontal deflection of joint C as shown in figure. Take E = 200 kN/mm² and A = 600 mm² for all the members. (AUC Apr/May 2011)

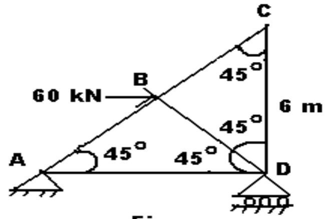
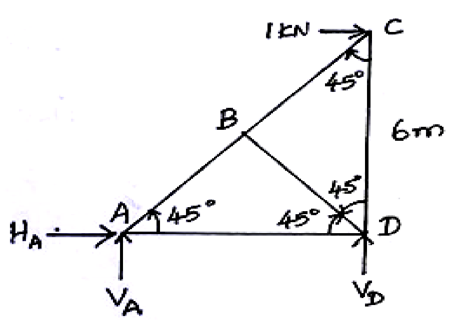


Fig.

Solution:

Step 1: Virtual Forces (k_h):-



In Δ ADC :-

$$\tan 45^\circ = \frac{6}{AD}$$

$$\boxed{AD = 6m}$$

$$\sin 45^\circ = \frac{6}{AC}$$

$$\boxed{AC = 8.48m}$$

IN Δ ABD :.

$$\cos 45^\circ = \frac{AB}{6}$$

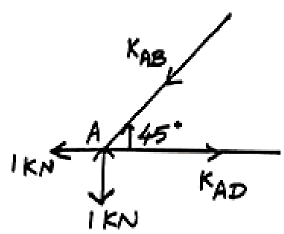
$$\boxed{AB = 4.24m}$$

$$\sin 45^\circ = \frac{BD}{6}$$

$$\boxed{BD = 4.24m}$$

$$\boxed{BC = 4.24m}$$

At Joint A :-



$\Sigma V = 0$

$$V_A + V_D = 0$$

$\Sigma M @ D = 0$

$$V_A \times 6 + 1 \times 6 = 0$$

$$\boxed{V_A = -1\,KN}$$

$$\boxed{V_B = 1\,KN}$$

$\Sigma H = 0$

$$H_A + 1 = 0$$

$$\boxed{H_A = -1KN}$$

$\Sigma V = 0$

$$1 + K_{AB} \sin 45^\circ = 0$$

$$K_{AB} = -1.41\,KN \text{ (comp)}$$

$$\boxed{K_{AB} = 1.41\,KN} \text{ (T)}$$

$\Sigma H = 0$

$1 + K_{AB} \cos 45° = K_{AD}$

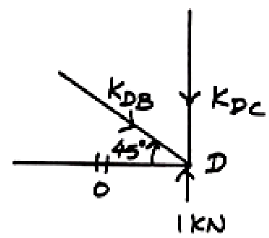
$1 - 1.41 \times \cos 45° = K_{AD}$

$\boxed{K_{AD} = 0}$

At Joint D:-

$\Sigma V = 0$

$K_{DC} + K_{DB} \sin 45° = 1 \rightarrow (1)$



$\Sigma H = 0$

$K_{DA} + K_{DB} \cos 45° = 0$

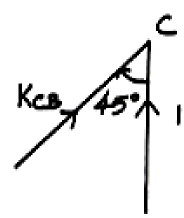
$0 + K_{DB} \cos 45° = 0$

$\boxed{K_{DB} = 0}$

$(1) \Rightarrow \boxed{K_{DC} = 1\ KN}$ (comp)

At Joint C:-

$\Sigma V = 0$



$K_{CB} \sin 45° + 1 = 0$

$K_{CB} = -1.41\ KN$ (comp)

$\boxed{K_{CB} = 1.41\ KN}$ (T)

$\Sigma H = 0$

$1 + K_{AB} \cos 45° = K_{AD}$

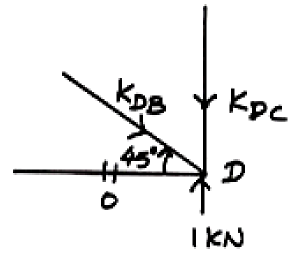
$1 - 1.41 \times \cos 45° = K_{AD}$

$\boxed{K_{AD} = 0}$

At Joint D:-

$\Sigma V = 0$

$K_{DC} + K_{DB} \sin 45° = 1 \rightarrow (1)$



$\Sigma H = 0$

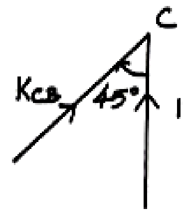
$K_{DA} + K_{DB} \cos 45° = 0$

$0 + K_{DB} \cos 45° = 0$

$\boxed{K_{DB} = 0}$

① ⇒ $\boxed{K_{DC} = 1 \text{ KN}}$ (Comp)

At Joint C :-



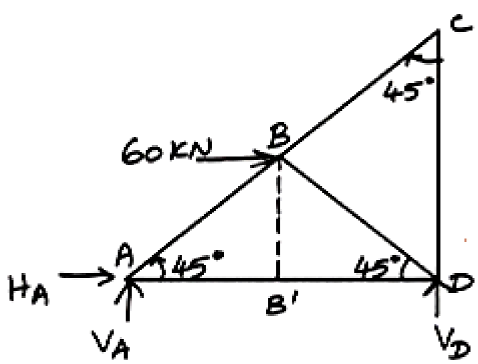
$\Sigma V = 0$

$K_{CB} \sin 45° + 1 = 0$

$K_{CB} = -1.41 \text{ KN}$ (Comp)

$\boxed{K_{CB} = 1.41 \text{ KN}}$ (T)

step 2 : Real Forces (F):-



In △ AB'B :-

$\sin 45° = \frac{BB'}{4.24}$

$\boxed{BB' = 3 \text{ m}}$

$\boxed{AB' = 3\text{m}}$

$\Sigma V = 0$

$V_A + V_D = 0.$

$\Sigma M @ D = 0$

$V_A \times 6 + 60 \times 3 = 0$

$\boxed{V_A = -30 \text{ KN}}$

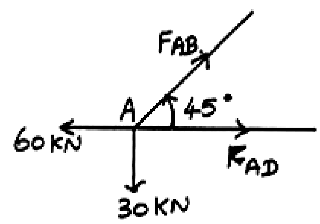
$\boxed{V_D = 30 \text{ KN}}$

$\Sigma H = 0$

$H_A + 60 = 0$

$\boxed{H_A = -60 \text{ KN}}$

At Joint A :-



$\Sigma V = 0$

$F_{AB} \sin 45° = 30$

$\boxed{F_{AB} = 42.42 \text{ KN}}$ (T)

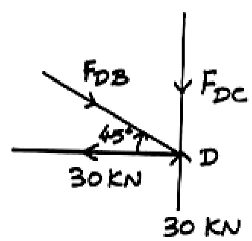
$\Sigma H = 0$

$F_{AD} + F_{AB} \cos 45° = 60$

$F_{AD} + 42.42 \times \cos 45° = 60$

$\boxed{F_{AD} = 30 \text{ KN}}$ (T)

At Joint D :-



$\Sigma H = 0$

$F_{DB} \cos 45° = 30$

$\boxed{F_{DB} = 42.42 \text{ KN}}$ (Comp)

$\Sigma V = 0$

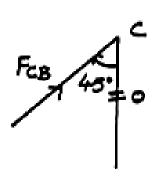
$F_{DC} + F_{DB} \sin 45° = 30$

$F_{DC} + 42.42 \times \sin 45° = 30$

$\boxed{F_{DC} = 0}$

At Joint C :-

$\boxed{F_{CB} = 0}$



Step 3: Virtual Work Equation:-

$$(\Delta_c)_h = \frac{\Sigma KFL}{AE}$$

Members	Length (m)	K_h	F (kN)	ΣKFL (kNm)
AB	4.24	1.41	42.42	253.6
AD	8	0	30	0
DB	4.24	0	-42.42	0
DC	6	-1	0	0
CB	4.24	1.41	0	0
			Sum =	253.6

In the above Table, apply
all compression members are negative.
all Tension members are positive.

$$(\Delta_c)_h = \frac{\Sigma K_h FL}{AE}$$

$$= \frac{253.6 \times 10^6}{600 \times 2 \times 10^5}$$

$$\boxed{(\Delta_c)_h = 2.11 \text{ mm}}$$

$\therefore$ The Horizontal displacement $= 2.11$ mm $(\uparrow)$

6. **Determine the vertical and horizontal deflection of free end E in the frame shown in figure. Take EI = 20000 kNm^2. (AUC Apr/May 2011)**

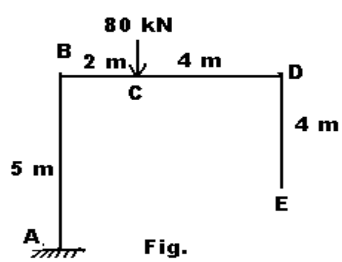
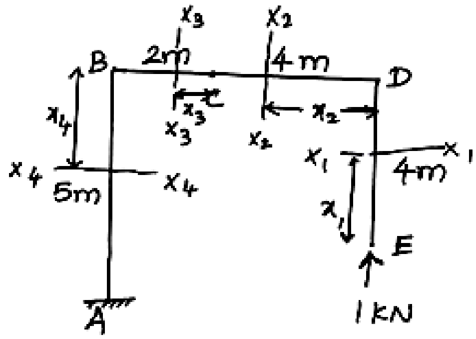


Fig.

Solution:

Step 1 : Virtual Moments (m_v) :-



$m_1 = 0$ (limits 0 to 4)

$m_2 = -x_2$ (limits 0 to 4)

$m_3 = -1(4 + x_3)$ (limits 0 to 2)

$m_4 = -1(6) = -6$ (limits 0 to 5)

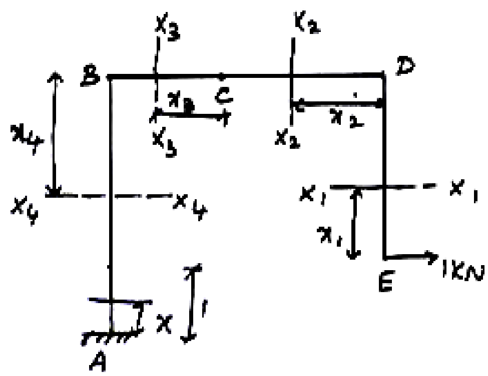
Step 2 : Virtual Moments (m_h) :-

$m_1 = -x$

$m_2 = -4$

$m_3 = -4$

$m_4 = 1(1 - x)$



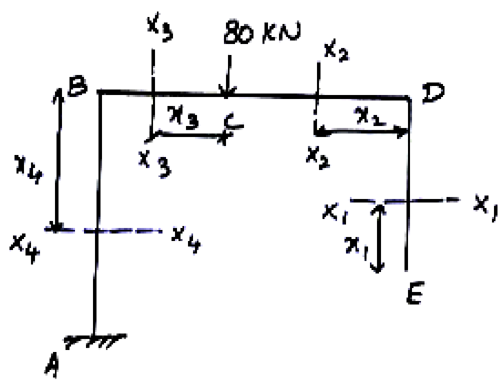
Step 3 : Real Moments (M) :-

$M_1 = 0$

$M_2 = 0$

$M_3 = +80\,x_3$

$M_4 = 80 \times 2 = 160$



Step 4: Virtual Work Equation :-

$$(\Delta_E)_V = \int_0^l \frac{m_v M}{EI}\,dx$$

$$= \frac{1}{EI}\left[\int_0^4 0\,dx + \int_0^4 0(-x_2)\,dx + \int_0^2 -(4+x_3)(80x_3)\,dx + \int_0^5 160(-6)\,dx\right]$$

$$= \frac{1}{EI}\left[\int_0^2 (-320x - 80x^2)\,dx + \int_0^5 (-960)\,dx\right]$$

$$= \frac{1}{EI}\left[-\frac{320x^2}{2} - \frac{80x^3}{3}\right]_0^2 + \frac{1}{EI}\left[-960x\right]_0^5$$

$$= \frac{1}{EI}\left[-853.33\right] - \frac{4800}{EI}$$

$$= -\frac{5653.33}{EI}$$

$$= -\frac{5653.33}{20000}$$

$$= -0.28\,m$$

$$\boxed{(\Delta_E)_V = -280\ mm}$$

$$(\Delta_E)_h = \int_0^l \frac{m_h M}{EI}\,dx$$

$$= \frac{1}{EI}\left[\int_0^4 0\,dx + \int_0^4 0\,dx + \int_0^2 (-4)(80x)\,dx + \int_0^5 (1-x)(160)\,dx\right]$$

$$= \frac{1}{EI}\left[-\frac{320x^2}{2}\right]_0^2 + \frac{1}{EI}\left[160x - \frac{160x^2}{2}\right]_0^5$$

$$= \frac{-640}{EI} - \frac{1200}{EI} = -\frac{1840}{EI} = -\frac{1840}{20000} = -0.092\,m$$

$$\boxed{(\Delta_E)_h = -92\ mm}$$

7. Find the horizontal deflection of joint 'B' in the frame shown in figure. Take E = 2x10^5 MPa and I = 3.5x10^8 mm^4. **(AUC Nov/Dec 2010)**

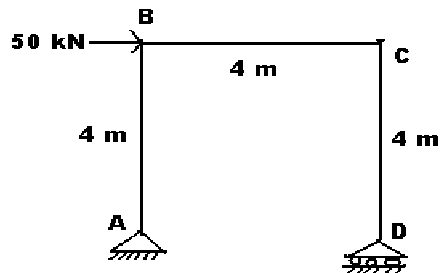
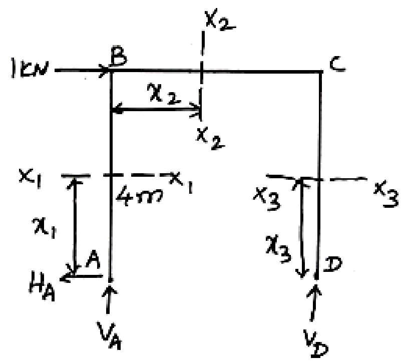


Fig.

Solution:

Step 1: Virtual Moments (m_h):-



$\Sigma H = 0$

$\boxed{H_A = 1\,kN}$

$\Sigma V = 0$

$V_A + V_D = 0$

$\Sigma M @ D = 0$

$V_A \times 4 + (1 \times 4) = 0$

$\boxed{V_A = -01\,kN}$

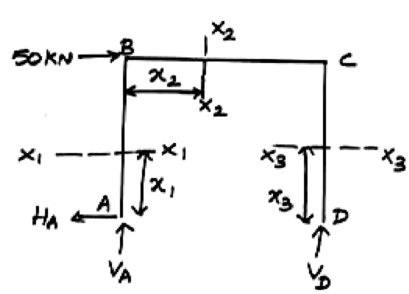
$\boxed{V_D = 01\,kN}$

$m_1 = x_1$ (limits 0 to 4m)

$m_2 = 1 \times 4 - x_2$ (limits 0 to 4m)

$m_3 = 0$ (limits 0 to 4m)

Step 2: Real Moments (M):-



$\Sigma H = 0$

$\boxed{H_A = 50\,kN}$

$\Sigma V = 0$

$V_A + V_D = 0$

$\Sigma M @ D = 0$

$(V_A \times 4) + (H_A \times 0) + (50 \times 4) = 0$

$\boxed{V_A = -50\,kN}$

$\boxed{V_D = 50\,kN}$

$M_1 = 50x_1$ (limits 0 to 4m)

$M_2 = 50\times4 - 50x_2$ (limits 0 to 4m)

$M_3 = 0$ (limits 0 to 4m)

Step 3 : Virtual Work Equation :-

$$(\Delta_B)_h = \int_0^l \frac{mM}{EI}\,dx$$

$$= \frac{1}{EI}\left[\int_0^4 x(50x)\,dx + \int_0^4 (4-x)(200-50x)dx + \int_0^4 0\,dx\right]$$

$$= \frac{1}{EI}\left[\int_0^4 50x^2\,dx + \int_0^4 (800 - 200x - 200x + 50x^2)dx\right]$$

$$= \frac{1}{EI}\left[\left\{\frac{50x^3}{3}\right\}_0^4 + \left\{800x - \frac{200x^2}{2} + \frac{50x^3}{3}\right\}_0^4\right]$$

$$= \frac{1}{EI}\left[(1066.67 - 0) + (3200 - 1600 + 1066.67)\right]$$

$$= \frac{1}{EI}(3733.34)$$

$$= \frac{3733.34}{2\times10^8 \times 3.5\times10^{-4}}$$

$$(\Delta_B)_h = 0.05333\ m$$

$$\boxed{(\Delta_B)_h = 53.33\,mm}$$

$\therefore$ The horizontal deflection at B is 53.33 mm.

8. Determine the vertical deflection of joint E for the Warren truss shown in figure. Take A = 645 mm^2 and E = 200 kN/mm^2 for all the members. (AUC Nov/Dec 2010)

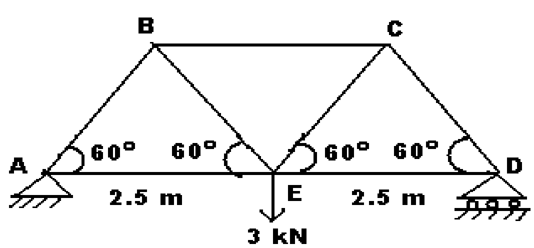
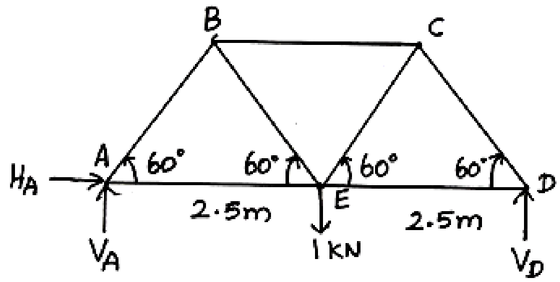


Fig.

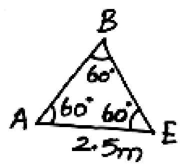
Solution:

Step 1 : Virtual Forces (K_E) :-



Using sine rule, ΔABE

$$\frac{2.5}{\sin 60^\circ} = \frac{AB}{\sin 60^\circ} = \frac{BE}{\sin 60^\circ}$$



$$\boxed{AB = 2.5\,m}$$

$$\boxed{BE = 2.5\,m}$$

$\Sigma V = 0$

$$V_A + V_D = 1$$

$\Sigma M @ A = 0$

$$-V_D \times 5 + 1 \times 2.5 = 0$$

$$\boxed{V_D = 0.5\,KN}$$

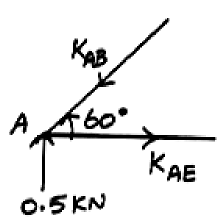
$$\boxed{V_A = 0.5\,KN}$$

$\Sigma H = 0$

$$\boxed{H_A = 0}$$

At Joint A :-

$\Sigma V = 0$



$$K_{AB} \sin 60^\circ = 0.5$$

$$\boxed{K_{AB} = 0.58\,KN} \text{ (Comp)}$$

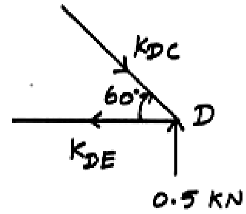
$\Sigma H = 0$

$$K_{AE} = K_{AB} \cos 60^\circ$$

$$= 0.58 \times \cos 60^\circ$$

$$\boxed{K_{AE} = 0.29\,KN} \text{ (T)}$$

At Joint D :-



$\Sigma V = 0$

$$K_{DC} \sin 60^\circ = 0.5$$

$$\boxed{K_{DC} = 0.58 \text{ KN}} \text{ (Comp)}$$

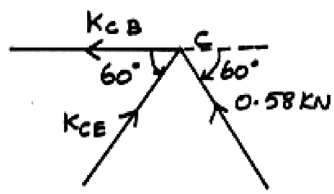
$\Sigma H = 0$

$$K_{DE} = K_{DC} \cos 60^\circ$$

$$= 0.58 \times \cos 60^\circ$$

$$\boxed{K_{DE} = 0.29 \text{ KN}} \text{ (T)}$$

At Joint C :-



$\Sigma V = 0$

$$K_{CE} \sin 60^\circ + 0.58 \times \sin 60^\circ = 0$$

$$K_{CE} = -0.58 \text{ KN (Comp)}$$

$$\boxed{K_{CE} = 0.58 \text{ KN}} \text{ (T)}$$

$\Sigma H = 0$

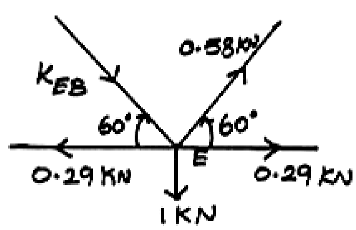
$$K_{CB} = K_{CE} \cos 60^\circ$$

$$= -0.58 \times \cos 60^\circ$$

$$= -0.29 \text{ KN (T)}$$

$$\boxed{K_{CB} = 0.29 \text{ KN}} \text{ (Comp)}$$

At Joint E :-



$\Sigma V = 0$

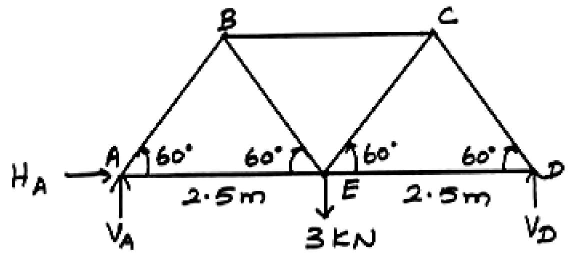
$$1 + K_{EB} \sin 60^\circ = 0.58 \times \sin 60^\circ$$

$$K_{EB} = 0.5 - 1$$

$$= -0.5 \text{ KN (C)}$$

$$\boxed{K_{EB} = 0.5 \text{ KN}} \text{ (T)}$$

Step 2: Real Forces (F):-



Since there is no other external loads except the point C. So, we multiply the virtual member forces into 3 times, we get real forces.

$F_{AB} = 0.58 \times 3 = 1.74\,kN$ (Comp)

$F_{AE} = 0.29 \times 3 = 0.87\,kN$ (T)

$F_{DC} = 0.58 \times 3 = 1.74\,kN$ (Comp)

$F_{DE} = 0.29 \times 3 = 0.87\,kN$ (T)

$F_{CE} = 0.58 \times 3 = 1.74\,kN$ (T)

$F_{CB} = 0.29 \times 3 = 0.87\,kN$ (Comp)

$F_{EB} = 0.5 \times 3 = 1.5\,kN$ (T)

Step 3: Virtual Work Equation:-

$$(\Delta_E)_V = \frac{\Sigma K_V F L}{AE}$$

All compression members are negative.

All Tension members are positive.

$$(\Delta_E)_V = \frac{\Sigma K_V F L}{AE}$$

$$= \frac{11.32 \times 10^6}{645 \times 2 \times 10^5}$$

$$\boxed{(\Delta_E)_V = 0.087\ mm}$$

∴ The vertical displacement at E = 0.087mm (↑)

Members	Length (m)	K_V	F (kN)	$\Sigma K_V F L$ (kNm)
AB	2.5	-0.58	-1.74	2.52
AE	2.5	0.29	0.87	0.63
DC	2.5	-0.58	-1.74	2.52
DE	2.5	0.29	0.87	0.63
CE	2.5	0.58	1.74	2.52
CB	2.5	-0.29	-0.87	0.63
EB	2.5	0.5	1.5	1.87
			$\Sigma KFL =$	11.32

9. Determine the horizontal displacement at the roller support of the rigid jointed frame shown in figure. Take E = 2×10^5 MPa and I_1 = 30×10^8 mm^4.

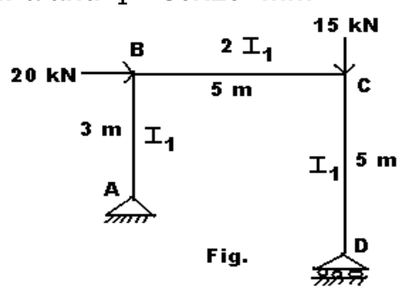


Fig.

Solution:

Step 1: Virtual Moments (m_h) :-

$\Sigma H = 0$

$\boxed{H_A = 1\,kN}$

$\Sigma V = 0$

$V_A + V_D = 0$

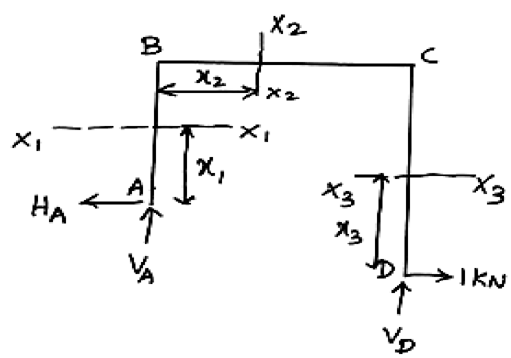
$\Sigma M @ D = 0$

$(V_A \times 5) - (H_A \times 2) = 0$

$5V_A - 2 = 0$

$\boxed{V_A = 0.4\,kN}$

$\boxed{V_D = -0.4\,kN}$



$m_1 = x_1$ (limits 0 to 3m)

$m_2 = 0.4x_2 + 3$ (limits 0 to 5m)

$m_3 = -x_3$ (limits 0 to 5m)

Step 2 : Real Moments (M):-

$\Sigma H = 0$

$\boxed{H_A = 20\,kN}$

$\Sigma V = 0$

$V_A + V_D = 15$

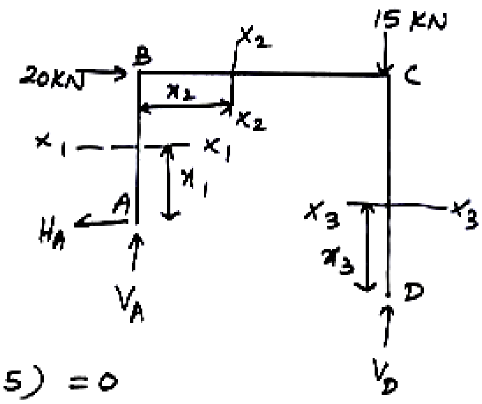
$\Sigma M @ D = 0$

$(V_A \times 5) - (H_A \times 2) + (20 \times 5) = 0$

$5V_A - (20 \times 2) + (20 \times 5) = 0$

$\boxed{V_A = -12\,kN}$

$\boxed{V_D = 27\,kN}$



$M_1 = 20x_1$ (limits 0 to 3m)

$M_2 = (20 \times 3) - 12x_2$ (limits 0 to 5m)

$M_3 = 0$ (limits 0 to 5m)

Step 3 : Virtual Work Equation:-

$$(\Delta_D)_h = \int_0^l \frac{mM}{EI}\,dx$$

$$= \frac{1}{EI}\int_0^3 x(20x)\,dx + \frac{1}{E(2I)}\int_0^5 (0.4x+3)(60-12x)\,dx + \frac{1}{EI}\int_0^5 0(-x_3)\,dx$$

$$= \frac{1}{EI}\int_0^3 20x^2\,dx + \frac{1}{2EI}\int_0^5 (24 - 4.8x + 180 - 36x)\,dx$$

$$= \frac{1}{EI}\left[\frac{20x^3}{3}\right]_0^3 + \frac{1}{2EI}\left[204x - \frac{40.8x^2}{2}\right]_0^5$$

$$= \frac{180}{EI} + \frac{510}{2EI}$$

$$(\Delta_D)_h = \frac{435}{EI}$$

here,

$$E = 2\times10^5\ MPa$$

$$= 2\times10^5\times10^6\ N/m^2$$

$$E = 2\times10^8\ kN/m^2$$

$$I = 30\times10^8\ mm^4$$

$$= 30\times10^8\times10^{-12}\ m^4$$

$$I = 30\times10^{-4}\ m^4$$

$$(\Delta_D)_h = \frac{435}{2\times10^8\times30\times10^{-4}} = 0.00073\ m$$

$$\boxed{(\Delta_D)_h = 0.73\ mm}$$

$\therefore$ The horizontal deflection at D is 0.73mm.

10. Find the vertical deflection of joint 'B' in the frame shown in figure. Take E = 2×10^5 MPa and I = 3.5×10^8 mm^4.

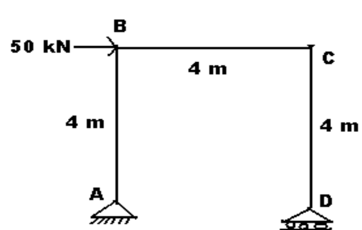


Fig.

Solution:

Step 1: Virtual Moments (m_v) :-

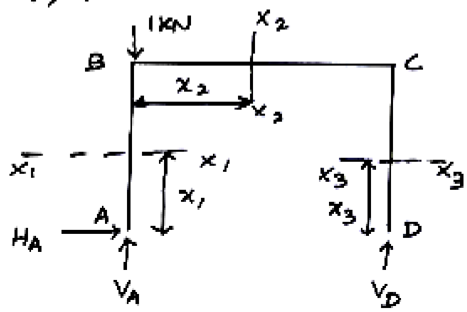
$\Sigma H = 0$

$\boxed{H_A = 0}$

$\Sigma V = 0$

$V_A + V_D = 01$

$\Sigma M @ D = 0$



$V_A \times 4 - 1 \times 4 = 0$

$\boxed{V_A = 1\ KN}$

$\boxed{V_D = -0\ KN}$

$m_1 = 0$ (Limits 0 to 4m)

$m_2 = x - x = 0$ (Limits 0 to 4m)

$m_3 = 0$ (Limits 0 to 4m)

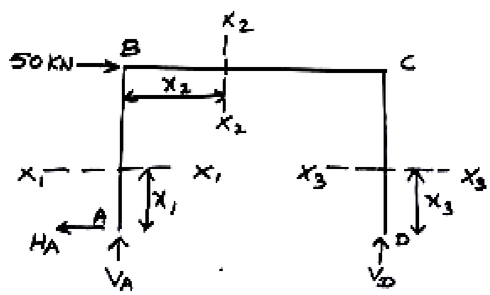
Step 2: Real Moments (M) :-

$\Sigma H = 0$

$\boxed{H_A = 50\ KN}$

$\Sigma V = 0$

$V_A + V_D = 0$



$\Sigma M @ D = 0$

$(V_A \times 4) + (H_A \times 0) + (50 \times 4) = 0$

$\boxed{V_A = -50\ KN}$

$\boxed{V_D = 50\ KN}$

$M_1 = 50x_1$ (limits 0 to 4m)

$M_2 = 50 \times 4 - 50x_2$ (limits 0 to 4m)

$M_3 = 0$ (limits 0 to 4m)

Step 3 : Virtual Work Equation:-

$$(\Delta_B)_V = \int_0^l \frac{mM}{EI}\,dx$$

$$= \frac{1}{EI}\left[\int_0^4 0(50x)\,dx + \int_0^4 0(200-50x)\,dx + \int_0^4 0\,dx\right]$$

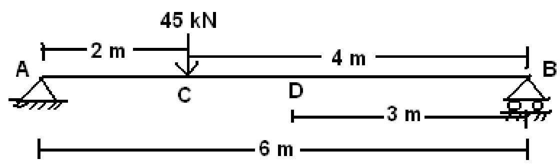
$$= \frac{1}{EI}\left[0\right]$$

$$(\Delta_B)_V = 0$$

$\therefore$ The vertical deflection at B is zero.

11. Determine the deflection under the load point of the beam shown in figure below. Take E = 200 x 10^6 kN/m^2 and I = 14 x 10^{-6} m^4. Use the principle of virtual work.

(AUC Nov/Dec 2013)



Solution:

We know $\quad 1 \,.\, \Delta = \int_0^l \frac{mMdx}{EI}$

Virtual moment, m. Remove the external load. Apply unit vertical load at C.

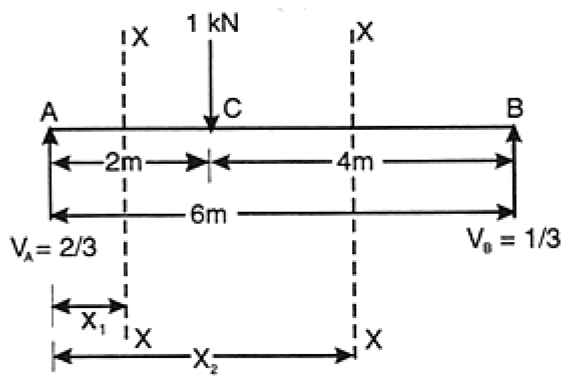


Fig. 1.26

Taking moments about B

$$V_A \times 6 - 1 \times 4 = 0 \qquad V_A = \frac{2}{3}\text{ kN}$$

$$V_B = \text{Total load} - V_A \qquad V_B = \frac{1}{3}\text{ kN}$$

Considering sections xx in AC and CB as shown

$$m_1 = \frac{2}{3} \cdot x_1 \qquad \text{(Limits 0 to 2m)}$$

$$m_2 = \frac{2}{3} x_2 - 1(x_2 - 2) \quad \text{(Limits 2 to 6m)}$$

Real moment, M. Using the above x co-ordinates, the internal moment (due to the given loadng), M is obtained as

$$R_A = \frac{2}{3} \times 45 = 30\text{kN}$$

$$R_B = \frac{1}{3} \times 45 = 15\text{kN}$$

(since there is only one concentrated load of 45kN at C)

$$M_1 = 30 \times x_1$$
$$M_2 = 30x_2 - 45(x_2 - 2)$$

Virtual work equation :

$$1.(\Delta_c)_V = \int_0^l \frac{mMdx}{EI} = \int_0^2 \frac{m_1 M_1 dx_1}{EI} + \int_2^6 \frac{m_2 M_2 dx_2}{EI}$$

$$= \int_0^2 \frac{\left(\frac{2}{3} x_1\right)(30x_1)\, dx_1}{EI} + \int_2^6 \frac{\left(\frac{2}{3} x_2 - (x_2 - 2)\right)[(30x_2 - 45(x_2 - 2)]\, dx_2}{EI}$$

$$= \int_0^2 \frac{20x_1{}^2 dx_1}{EI} + \int_2^6 \frac{[20x_2{}^2 - 30x_2(x_2 - 2) - 30x_2(x_2 - 2) + 45(x_2 - 2)^2]\, dx_2}{EI}$$

$$= \int_0^2 \frac{20x_1{}^2 dx_1}{EI} + \int_2^6 \frac{[20x_2{}^2 - 2 \times 30x_2(x_2 - 2) + 45(x_2{}^2 - 4x_2 + 4)]\, dx_2}{EI}$$

$$= \int_0^2 \frac{20x_1{}^2 dx_1}{EI} + \int_2^6 \frac{(20x_2{}^2 - 60x_2{}^2 + 120x_2 + 45x_2{}^2 - 180x_2 + 180)\, dx_2}{EI}$$

$$= \int_0^2 \frac{20x_1{}^2 dx_1}{EI} + \int_2^6 \frac{(5x_2{}^2 - 60x_2 + 180)\, dx_2}{EI}$$

$$= \left[\frac{20}{EI} \cdot \frac{x_1{}^3}{3}\right]_0^2 + \frac{1}{EI}\left[5 \cdot \frac{x_2{}^3}{3} - 60 \cdot \frac{x_2{}^2}{2} + 180x_2\right]_2^6$$

$$= \frac{20}{EI} \cdot \frac{8}{3} + \frac{1}{EI}\left[\frac{5}{3}(6^3 - 2^3) - \frac{60}{2}(6^2 - 2^2) + 180\,(6 - 2)\right]$$

$$= \frac{1}{EI}[53.33 + 346.67 - 960 + 720] = \frac{160}{EI}$$

$$= \frac{160}{200 \times 10^6 \times 14 \times 10^{-6}} = 0.0571 \text{ m} \quad \text{(or)} \quad 57.1 \text{ mm}$$

$\therefore$ Deflection under the load point = 57.1 mm.

12. Explain the steps involved in the determination of deflections of pin-jointed plane frames and rigid plane frames. (AUC May/June 2014)

The displacement caused in simple pin jointed determinate frames can easily be determined by using the method of virtual forces, as described below in 3 steps.

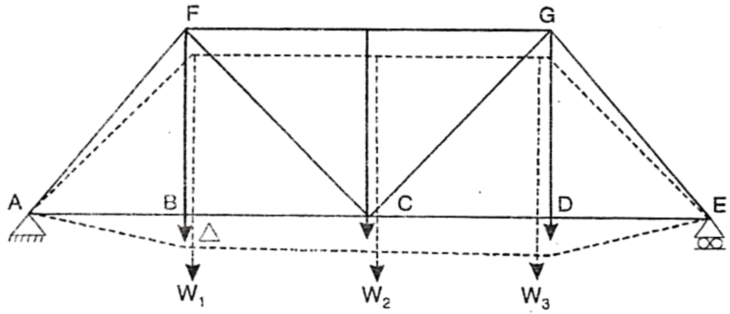


Fig. 1.1

Step 1. Suppose we want to determine the displacement at B of the truss in Fig. 1.1 due to the load system W_1 W_2 W_3 = {W}. We have to first solve for the internal forces F_1 F_2 F_3.....F_n = {F} in the members due to {W}. This is a question of statics. We know that every member will have an additional elongation (or shortening) of F_1l/AE, F_2l/AE, F_3l/AE......due to F_1 F_2 F_3........... . These member displacements will increase from 0 to {Fl/AE} as the internal forces increase from zero to {F}.

Step 2. Now apply only a unit load (virtual force) at B in the direction of the desired deflection Δ. Find the internal forces due to the unit load (k_1 k_2 k_3 .... = {k}). We will forget about the external work done and internal energy stored due to the unit load and proceed to the next step.

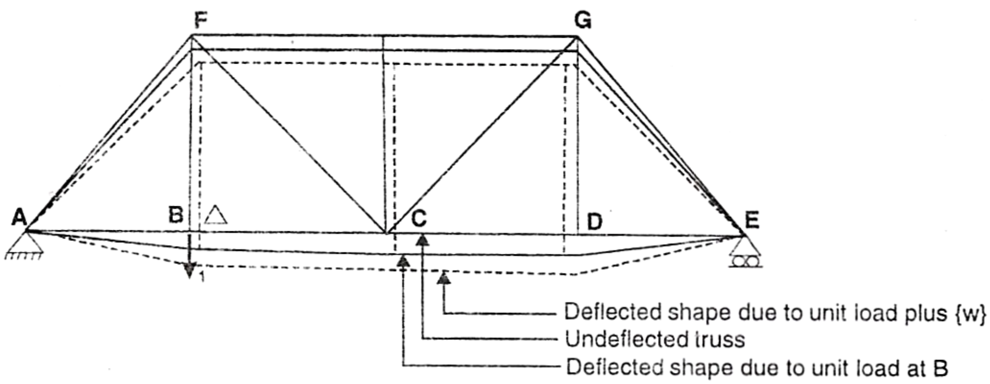


Fig. 1.2

Step 3. Now apply the {W} system in addition to the unit load, which has been applied first.

Energy Equation :

We will now have the deflected truss. The deflections are due to the unit load plus {W} system

The external work done W_e due to the imposition of {W} is in 2 parts .

W_{e1} due to the unit load displacing through Δ at B.

W_{e2} due to the {W} system displacing progressively from zero to Δ_1, Δ_2, Δ_3 ......

And the internal work done is also due to two causes.

W_{i1} due to the pre existing internal force system k_1 k_2 k_3 ......displacing through member displacements F_1l/AE, F_2l/AE ...... caused by {W} system

W_{i2} due to the current force system F_1 F_2 F_3 ...... displacing progressively from 0 to F_1l/AE, F_2l/AE, F_3l/AE ...... etc.

$$W_{e1} + W_{e2} = W_e = 1.\Delta + \frac{W_1\Delta_1}{2} + \frac{W_2\Delta_2}{2} \ldots\ldots$$

$$W_{i1} + W_{i2} = W_i = \Sigma\frac{kFL}{AE} + \Sigma\frac{F^2L}{2AE}$$

If [W] system were alone applied on the structure without the unit load preceeding it we would have got the relation $W_{e2} = W_{i2}$. Therefore we can infer that $1.\Delta = \Sigma\frac{kFL}{AE}$. Thus, we get the deflection Δ at B caused by the {W} system in terms of the internal forces {k} due to a unit load at B and the internal forces {F} due to the {W} system.

Procedure for deflection of pin-jointed plane frames:

Sign convention : Assume that tensile forces are positive and compressive forces are negative.

1. Virtual forces k. Remove all the real loads from the truss. Place a unit load on the truss at the joint and in the direction of the desired displacement. Use the method of joints or the method of sections and calculate the internal forces k in each member of the truss.

2. Real forces F. These forces are caused only by the real loads acting on the truss. Use the method of sections or the method of joints to determine the forces F in each member.

3. Virtual work equation. Apply the equation of virtual work, to determine the desired displacement.

i.e.

$$1.\Delta = \Sigma\frac{kFL}{AE}.$$

Take proper care to retain the algebraic sign for each component k and F. If Δ turns out to be positive, then Δ is in the same direction of the unit load. If a negative value results, Δ is opposite to the direction of the unit load .

Procedure for deflection of rigid jointed plane frames:

1. **Virtual moments (m):**
   Remove all external load and apply a unit load in the horizontal direction (direction of the desired displacement) at D. the support reactions and internal virtual moments are computed.
2. **Real moments(M):**
   These moments are caused only by the real loads acting on the truss. Due to the given loading, the support reactions and the real moments are computed.
3. **Virtual work equation:**
   Apply the equation of virtual work to determine the desired deflection**.**

$$\Delta = \int_0^l \frac{mM}{EI}dx$$

13. Explain the concepts involved in the Williot diagram and its applications.
(AUC May/June 2014)

A truss is made of several members which are in compression or in tension. So from the initial configuration, a truss would take a deviated (or deflected) position when loaded. This is due to the fact that compatible with the altered lengths of members due to strains there would be a different resultant position for each node . The changes in lengths are very small (in the order of 1/1000). Hence a geometric construction taking into account the changes in length, for example BB_a and BB_c in Fig. 1.34 can give the changed position of B (B′) only approximately. Notice that in Fig. 1.35. BB_a and BB_c are shown greatly magnified and not to the same scale as that used for the truss A B C.

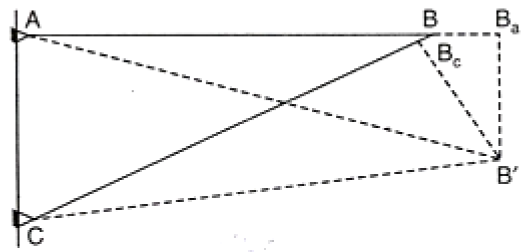


Fig. 1.34. Displacement diagram.

Williot's diagram takes only the displacements in its graphical construction and develops the consequent displacements of the nodes. Hence for the truss in Fig. 1.34, the displacement diagram would be as in Fig. 1.35.

In this diagram all points without displacement would be at the origin. $0 - B_a$ is a vector showing the elongation of AB along AB. $0 - B_c$ is a vector showing the movement of B along BC due to shortening of BC. *a* and *c* are of course points fixed in position. Normals to $0 - B_a$ at B_a and to $0 - B_c$ at B_c intersect at B′ to give the displaced position of B from its original un-displaced position .

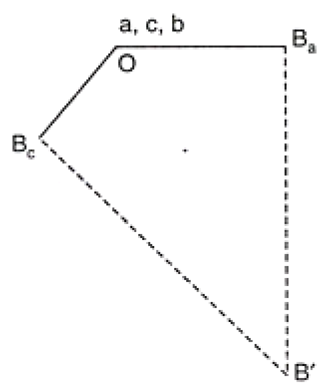


Fig. 1.35. Williot's diagram

Fig. 1.34 and 1.35 differ only in minute details.

1. Fig. 1.34 relegates all un- isplaced points to the origin. This would include the initial positions of all nodes in a truss.

2. The distance from the origin of any newly plotted position of a node indicates its vector displacement from its un-displaced position.

Example 1.11 will further amplify the plotting and interpretation of Williot's displace ment diagram.