

**QUESTION WITH ANSWERS****DEPARTMENT : CIVIL****SEMESTER: IV****SUB.CODE/ NAME: CE 8402 / Strength of Materials****UNIT-5 ADVANCED TOPICS IN BENDING OF BEAMS****PART - A (2 marks)****1. Define Unsymmetrical bending**

The plane of loading (or) that of bending does not lie in (or) a plane that contains the principle centroidal axis of the cross- section; the bending is called Unsymmetrical bending.

**2. State the two reasons for unsymmetrical bending.****(AUC May/June 2012)****(AUC Apr /May 2011)**

- (i) The section is symmetrical (viz. Rectangular, circular, I section) but the load line is inclined to both the principal axes.
- (ii) The section is unsymmetrical (viz. Angle section (or) channel section vertical web) and the load line is along any centroidal axes.

**3. Define shear centre.**

The shear centre (for any transverse section of the beam) is the point of intersection of the bending axis and the plane of the transverse section. Shear centre is also known as "centre of twist"

**4. Write the shear centre equation for channel section.**

$$e = \frac{3b}{6 + \frac{A_w}{A_f}}$$

e = Distance of the shear centre (SC ) from the web along the symmetric axis XX

A<sub>w</sub> = Area of the web

A<sub>f</sub> = Area of the flange

**5. A channel Section has flanges 12 cm x 2 cm and web 16 cm x 1 cm. Determine the shear centre of the channel.****Solution:**

$$b = 12 - 0.5 = 11.5 \text{ cm}$$

$$t_1 = 2 \text{ cm}, t_2 = 1 \text{ cm}, h = 18 \text{ cm}$$

$$A_f = bt_1 = 11.5 \times 2 = 23 \text{ cm}^2$$

$$A_w = ht_2 = 18 \times 1 = 18 \text{ cm}^2$$

$$e = \frac{3b}{6 + \frac{A_w}{A_f}}$$

$$e = \frac{3(11.5)}{6 + \frac{18}{23}} = 5.086 \text{ cm}$$

**6. Write the shear centre equation for unsymmetrical I section.**

$$e = \frac{t_1 h^2 (b_2 - b_1)^2}{4I_{xx}}$$

e = Distance of the shear centre (SC) from the web along the symmetric axis XX

t<sub>1</sub> = thickness of the flange

h = height of the web

b<sub>1</sub> = width of the flange in right portion.

b<sub>2</sub> = width of the flange in left portion.

I<sub>xx</sub> = M.O.I of the section about XX axis.

**7. State the assumptions made in Winkler's Bach Theory.(AUC Nov / Dec 2012)**

**(AUC Nov/Dec 2013) (AUC May/June 2012)**

- (1) Plane sections (transverse) remain plane during bending.
- (2) The material obeys Hooke's law (limit state of proportionality is not exceeded)
- (3) Radial strain is negligible.
- (4) The fibres are free to expand (or) contract without any constraining effect from the adjacent fibres.

**8. State the parallel Axes and Principal Moment of inertia.**

If the two axes about which the product of inertia is found, are such , that the product of inertia becomes zero, the two axes are then called the principle axes. The moment of inertia about a principal axes is called the principal moment of inertia.

**9. Define stress concentration. .**

**(AUC Nov / Dec 2011)**

The term stress gradient is used to indicate the rate of increase of stress as a stress raiser is approached. These localized stresses are called stress concentration.

**10. Define stress – concentration factor.**

It is defined as the ratio of the maximum stress to the nominal stress.

$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{nom}}}$$

$\sigma$

$\sigma$

**11. Define fatigue stress concentration factor.**

The fatigue stress – concentration factor ( $K_f$ ) is defined as the ratio of flange limit of unnotched specimen to the fatigue limit of notched specimen under axial (or) bending loads.

$$K_f = 1 + q(K_t - 1)$$

Value of q ranges from zero to one.

**12. Define shear flow.**

Shear flow is defined as the ratio of horizontal shear force H over length of the beam x. Shear flow is acting along the longitudinal surface located at discharge  $y_1$ . Shear flow is defined by q.

$$q = \frac{H}{x} = V_y \frac{Q_z}{I_z}$$

H = horizontal shear force

**13. Explain the position of shear centre in various sections.**

(i) In case of a beam having two axes of symmetry, the shear centre coincides with the centroid.

(ii) In case of sections having one axis of symmetry, the shear centre does not coincide with the centroid but lies on the axis of symmetry.

**14. State the principles involved in locating the shear centre.**

The principle involved in locating the shear centre for a cross – section of a beam is that the loads acting on the beam must lie in a plane which contains the resultant shear force on each cross-section of the beam as computed from the shearing stresses.

**15. Determine the position of shear centre of the section of the beam shown in fig.**

**Solution:**

$$t_1 = 4 \text{ cm}, b_1 = 6 \text{ cm}, b_2 = 8 \text{ cm}$$

$$h_1 = 30 - 4 = 26 \text{ cm}$$

$$e = \frac{t_1 h^2 (b_2 - b_1)^2}{4I_{xx}}$$

$$I_{xx} = 2 \left[ \frac{14x4^3}{12} + 14x4(13)^3 \right] + \frac{2x22^3}{12} = 20852 \text{ cm}^4$$

$$e = \frac{4x26^2(8-6)^2}{4(20852)} = 0.9077 \text{ cm}$$

**16. State the stresses due to unsymmetrical bending.**

$$\sigma_b = M \left[ \frac{v \cos \theta}{I_{UU}} + \frac{u \sin \theta}{I_{VV}} \right]$$

$\sigma_b$  = bending stress in the curved bar

M = moment due to the load applied

$I_{UU}$  = Principal moment of inertia in the principal axes UU

$I_{VV}$  = Principal moment of inertia in the principal axes VV

**17. Define the term Fatigue.**

Fatigue is defined as the failure of a material under varying loads, well below the ultimate static load, after a finite number of cycles of loading and unloading.

**18. State the types of fatigue stress.**

- (i) Direct stress
- (ii) Plane bending
- (iii) Rotating bending
- (iv) Torsion
- (v) Combined stresses
  - (a) Fluctuating or alternating stress
  - (b) Reversed stress.

**19. State the reasons for stress- concentration.**

When a large stress gradient occurs in a small, localized area of a structure, the high stress is referred to as a stress concentration. The reasons for stress concentration are (i) discontinuities in continuum (ii) contact forces.

**20. Define creep.**

Creep can be defined as the slow and progressive deformation of a material with time under a constant stress.

**21. Define principal moment of inertia.****(AUC Nov/Dec 2013)**

The perpendicular axis about which the product of inertia is zero are called "principal axes" and the moments of inertia with respect to these axes are called as principal moments of inertia.

The maximum moment of inertia is known as Major principal moment of inertia and the minimum moment of inertia is known as Minor principal moment of inertia.

**PART B****1. Explain the stresses induced due to unsymmetrical bending.**

Fig. shows the cross-section of a beam under the action of a bending moment  $M$  acting in plane  $YY$ .

Also  $G$  = centroid of the section,

$XX, YY$  = Co-ordinate axes passing through  $G$ ,

$UU, VV$  = Principal axes inclined at an angle  $\theta$  to  $XX$  and  $YY$  axes respectively

The moment  $M$  in the plane  $YY$  can be resolved into its components in the planes  $UU$  and  $VV$  as follows:

Moment in the plane  $UU$ ,  $M' = M \sin \theta$

Moment in the plane  $VV$ ,  $M'' = M \cos \theta$

The components  $M'$  and  $M''$  have their axes along  $VV$  and  $UU$  respectively.

The resultant bending stress at the point  $(u,v)$  is given by,

$$\sigma_b = \frac{M' u}{I_{VV}} + \frac{M'' v}{I_{UU}} = \frac{M \sin \theta}{I_{VV}} + \frac{M \cos \theta}{I_{UU}}$$

$$\sigma_b = M \left[ \frac{V \cos \theta}{I_{UU}} + \frac{u \sin \theta}{I_{VV}} \right]$$

At any point the nature of  $\sigma_b$  will depend upon the quadrant in which it lies. The equation of the neutral axis (N.A) can be found by finding the locus of the points on which the resultant stress is zero. Thus the points lying on neutral axis satisfy the condition that  $\sigma_b = 0$

$$M \left[ \frac{V \cos \theta}{I_{UU}} + \frac{u \sin \theta}{I_{VV}} \right] = 0$$

$$\frac{V \cos \theta}{I_{UU}} + \frac{u \sin \theta}{I_{VV}} = 0$$

$$v = - \left[ \frac{I_{UU}}{I_{VV}} + \frac{\sin \theta}{\cos \theta} \right] u \quad (\text{or}) \quad v = - \left[ \frac{I_{UU}}{I_{VV}} \tan \theta \right] u$$

This is an equation of a straight line passing through the centroid G of the section and inclined at an angle  $\alpha$  with UU where

$$\tan \alpha = - \left[ \frac{I_{UU}}{I_{VV}} \tan \theta \right]$$

Following points are worth noting:

- i. The maximum stress will occur at a point which is at the greatest distance from the neutral
- ii. All the points of the section on one side of neutral axis will carry stresses of the same nature and on the other side of its axis, of opposite nature.
- iii. In the case where there is direct stress in addition to the bending stress, the neutral axis will still be a straight line but will not pass through G (centroid of section.)

## 2. Derive the equation of Shear centre for channel section. (AUC April/May 2005)

Fig shows a channel section (flanges:  $b \times t_1$  ; Web  $h \times t_2$ ) with XX as the horizontal symmetric axis.

Let  $S$  = Applied shear force. (Vertical downward X)

(Then  $S$  is the shear force in the web in the upward direction)

$S_1$  = Shear force in the top flange (there will be equal and opposite shear force in the bottom flange as shown.)

Now, shear stress ( $\tau$ ) in the flange at a distance of  $x$  from the right hand edge (of the top flange)

$$\tau = \frac{S \bar{A} \bar{y}}{I_{xx} t} \quad \bar{A} \bar{y} = \left( t_1 \cdot x \right) \cdot \frac{h}{2} \quad (\text{where } t = t_1, \text{ thickness of flange})$$

$$\tau = \frac{S t_1 \cdot x}{I_{xx} \cdot t_1} \cdot \frac{h}{2} = \frac{S_x h}{2 I_{xx}}$$

Shear force is elementary area

$$dA = t_1 \cdot dx = \tau \cdot dA = \tau t_1 dz$$

Total shear force in top flange

$$= \int_0^b \tau \cdot t_1 \cdot dx \quad (\text{where } b = \text{breadth of the flange})$$

$$S_1 = \int_0^b \frac{S \times h}{2I_{xx}} \cdot t_1 \cdot dx = \frac{S h t_1}{2I_{xx}} \int_0^b x dx$$

$$(\text{or}) \quad S_1 = \frac{S h t_1}{I_{xx}} \cdot \frac{b^2}{4}$$

Let  $e$  = Distance of the shear centre (sc) from taking moments of shear forces about the centre O of the web, We get

$$S \cdot e = S_1 \cdot h$$

$$= \frac{S h t_1}{I_{xx}} \cdot \frac{b^2}{4} \cdot h = \frac{S \cdot t_1 h^2 b^2}{4 I_{xx}}$$

$$e = \frac{b^2 h^2 t_1}{4 I_{xx}} \quad (1)$$

$$\text{Now, } I_{xx} = 2 \left[ \frac{b \times t_1^3}{12} + b \cdot t_1 \left( \frac{h}{2} \right)^2 \right] + \frac{t_2 h^3}{12} = \frac{b t_1^3}{6} + \frac{b \cdot t_1 h^2}{2} + \frac{t_2 h^3}{12}$$

$$= \frac{b t_1 h^2}{2} + \frac{t_2 h^3}{12} \quad (\text{neglecting the term } \frac{b t_1^3}{6}, \text{ being negligible in comparison to other}$$

$$\text{terms})(\text{or}) \quad I_{xx} = \frac{h^2}{12} [2h + b t_1]$$

Substitute the value of  $I_{xx}$  in equation (1) we get,

$$e = \frac{b^2 h^2 t_1}{4} \times \frac{12}{h^2 [2h + b t_1]} = \frac{3 b^2 t_1}{[2h + b t_1]}$$

Let  $b t_1 = A_f$  (area of the flange)

$h t_2 = A_w$  (area of the web)

Then

$$e = \frac{3bA_f}{A_w + 6A_f} = \frac{3b}{6 + \frac{A_w}{A_f}}$$

i.e

$$e = \frac{3b}{6 + \frac{A_w}{A_f}}$$

### 3. Derive the equation of Shear center for unequal I-section

**Solution:**

Fig. shows an unequal I – section which is symmetrical about XX axis.

Shear stress in any layer,

$$\tau = \frac{SA\bar{y}}{It}$$

$$\text{where } I = I_{XX} = 2 \left[ b_1 + b_2 \frac{t_1^3}{12} + b_1 + b_2 \frac{t_1^3}{12} \right]$$

Shear force  $S_1$  :

$$dA = t_1 dx, A\bar{y} = t_1 x \cdot \frac{h}{2}$$

$$S_1 = \int_0^{b_1} \tau dA = \frac{S \cdot x \cdot t_1}{I_{XX} t_1} \frac{h}{2} x t_1 dx$$

$$= \int_0^{b_1} \frac{S \cdot x}{I_{XX}} \frac{h}{2} t_1 dx = \frac{S h t_1}{2 I_{XX}} \left[ \frac{x^2}{2} \right]_0^{b_1} = \frac{S h t_1 b_1^2}{4 I_{XX}}$$

Similarly the shear force ( $S_2$ ) in the other part of the flange,

$$S_2 = \frac{S h t_1 b_2^2}{4 I_{XX}}$$

Taking moments of the shear forces about the centre of the web O, we get

$$S_2 \cdot h = S_1 \cdot h + S \cdot e \quad (S_3 = S \text{ for equilibrium})$$

(where, e = distance of shear centre from the centre of the web)

$$\text{or, } (S_2 - S_1) h = S \cdot e$$

$$\frac{S h^2 t_1 (b_2^2 - b_1^2)}{4 I_{XX}} = S \cdot e \quad e = \frac{t_1 h^2 (b_2^2 - b_1^2)}{4 I_{XX}}$$

#### 4. Derive the stresses in curved bars using Winkler – Bach Theory.

The simple bending formula, however, is not applicable for deeply curved beams where the neutral and centroidal axes do not coincide. To deal with such cases Winkler – Bach Theory is used.

Fig shows a bar ABCD initially; in its unstrained state. Let AB'CD' be the strained position of the bar.

Let  $R$  = Radius of curvature of the centroidal axis HG.

$Y$  = Distance of the fiber EF from the centroidal layer HG.

$R'$  = Radius of curvature of HG'

$M$  = Uniform bending moment applied to the beam (assumed positive when tending to increase the curvature)

$\theta$  = Original angle subtended by the centroidal axis HG at its centre of curvature O and

$\theta'$  = Angle subtended by HG' (after bending) at the center of curvature  $\theta'$

For finding the strain and stress normal to the section, consider the fibre EF at a distance  $y$  from the centroidal axis.

Let  $\sigma$  be the stress in the strained layer EF' under the bending moment  $M$  and  $e$  is strain in the same layer.

$$\text{Strain, } e = \frac{EF' - EF}{EF} = \frac{(R' + y')\theta' - (R + y)\theta}{(R + y)\theta} \quad \text{or} \quad e = \frac{R' + y'}{R + y} \cdot \frac{\theta'}{\theta} - 1$$

$e_0$  = strain in the centroidal layer i.e. when  $y = 0$

$$= \frac{R'}{R} \cdot \frac{\theta'}{\theta} - 1 \quad \text{or} \quad 1 + e = \frac{R' + y'}{R + y} \cdot \frac{\theta'}{\theta} \quad \text{----- (1)}$$

$$\text{and } 1 + e = \frac{R'}{R} \cdot \frac{\theta'}{\theta} \quad \text{----- (2)}$$

Dividing equation (1) and (2), we get

$$\frac{1 + e}{1 + e_0} = \frac{R' + y'}{R + y} \cdot \frac{R}{R'} \quad \text{or} \quad e = \frac{e_0 \cdot \frac{y'}{R'} + \frac{y'}{R'} + e_0 - \frac{y}{R}}{1 + \frac{y}{R}}$$

According to assumption (3), radial strain is zero i.e.  $y = y'$

$$\text{Strain, } e = \frac{e_0 \cdot \frac{y}{R'} + \frac{y}{R'} + e_0 - \frac{y}{R}}{1 + \frac{y}{R}}$$

Adding and subtracting the term  $e_0 \cdot y/R$ , we get

$$e = \frac{e_0 \cdot \frac{y}{R'} + \frac{y}{R'} + e_0 - \frac{y}{R} + e_0 \frac{y}{R} - e_0 \cdot \frac{y}{R}}{1 + \frac{y}{R}}$$



$$e = e_0 + \frac{(1+e_0)(\frac{1}{R'} - \frac{1}{R})y}{1 + \frac{y}{R}} \quad \text{----- (3)}$$

From the fig. the layers above the centroidal layer is in tension and the layers below the centroidal layer is in compression.

$$\text{Stress , } \sigma = Ee = E(e_0 + \frac{(1+e_0)(\frac{1}{R'} - \frac{1}{R})y}{1 + \frac{y}{R}}) \quad \text{----- (4)}$$

Total force on the section,  $F = \int \sigma \cdot dA$

Considering a small strip of elementary area  $dA$ , at a distance of  $y$  from the centroidal layer HG, we have

$$F = E \int e_0 \cdot dA + E \int \frac{(1+e_0)(\frac{1}{R'} - \frac{1}{R})y}{1 + \frac{y}{R}} dA \quad F = E \int e_0 \cdot dA + E (1+e_0) (\frac{1}{R'} - \frac{1}{R}) \int \frac{y}{1 + \frac{y}{R}} dA$$

$$F = E \int e_0 \cdot A + E (1+e_0) (\frac{1}{R'} - \frac{1}{R}) \int \frac{y}{1 + \frac{y}{R}} dA \quad \text{----- (5)}$$

where  $A$  = cross section of the bar

The total resisting moment is given given by

$$M = \int \sigma \cdot y \cdot dA = E \int e_0 \cdot y \cdot dA + E \int \frac{(1+e_0)(\frac{1}{R'} - \frac{1}{R})y^2}{1 + \frac{y}{R}} dA$$

$$M = E \int e_0 \cdot 0 + E (1+e_0) (\frac{1}{R'} - \frac{1}{R}) \int \frac{y^2}{1 + \frac{y}{R}} dA \quad (\text{since } \int y \cdot dA = 0)$$

$$M = E (1+e_0) \left[ \frac{1}{R'} - \frac{1}{R} \right] \int \frac{y^2}{1 + \frac{y}{R}} dA \quad \text{Let } \int \frac{y^2}{1 + \frac{y}{R}} dA = Ah^2$$

Where  $h^2$  = a constant for the cross section of the bar

$$M = E (1+e_0) \left[ \frac{1}{R'} - \frac{1}{R} \right] Ah^2 \quad \text{----- (6)}$$

$$\text{Now, } \int \frac{y}{1 + \frac{y}{R}} dA = \int \frac{Ry}{R+y} dA = \int \left[ y - \frac{y^2}{R+y} \right] dA = \int y \cdot dA - \int \frac{y^2}{R+y} dA$$

$$\int \frac{y}{1 + \frac{y}{R}} dA = 0 - \frac{1}{R} \int \frac{y^2}{1 + \frac{y}{R}} dA = -\frac{1}{R} Ah^2 \quad \text{----- (7)}$$

Hence equation (5) becomes

$$F = Ee_0 \cdot A - E (1+e_0) \left[ \frac{1}{R'} - \frac{1}{R} \right] \frac{Ah^2}{R}$$

Since transverse plane sections remain plane during bending

$$F = 0$$

$$0 = Ee_0 \cdot A - E (1+e_0) \left[ \frac{1}{R'} - \frac{1}{R} \right] \frac{Ah^2}{R}$$

$$E e_0 \cdot A = E (1+e_0) \left[ \frac{1}{R'} - \frac{1}{R} \right] \frac{Ah^2}{R}$$

$$e_0 = (1+e_0) \left[ \frac{1}{R'} - \frac{1}{R} \right] \frac{Ah^2}{R} \quad (\text{or}) \quad \frac{e_0 R}{h^2} = (1+e_0) \left[ \frac{1}{R'} - \frac{1}{R} \right]$$

Substituting the value of  $\frac{e_0 R}{h^2} = (1+e_0) \left[ \frac{1}{R'} - \frac{1}{R} \right]$  in the equation (6)

$$M = E \frac{e_0 R}{h^2} Ah^2 = e_0 EAR$$

Or  $e_0 = \frac{M}{EAR}$  substituting the value of  $e_0$  in equation (4)

$$\sigma = \frac{M}{AR} + E * \frac{y}{1 + \frac{y}{R}} * \frac{e_0 R}{h^2} \quad (\text{or}) \quad \sigma = \frac{M}{AR} + E * \frac{y}{1 + \frac{y}{R}} * \frac{R}{h^2} * \frac{M}{EAR}$$

$$\sigma = \frac{M}{AR} + \frac{M}{AR} * \frac{Ry}{1 + \frac{y}{R}} * \frac{1}{h^2}$$

$$\sigma = \frac{M}{AR} \left[ 1 + \frac{R^2}{h^2} \left[ \frac{y}{R+y} \right] \right] \quad (\text{Tensile})$$

$$\sigma = \frac{M}{AR} \left[ 1 - \frac{R^2}{h^2} \left[ \frac{y}{R-y} \right] \right] \quad (\text{Compressive})$$

5. The curved member shown in fig. has a solid circular cross –section 0.01 m in diameter. If the maximum tensile and compressive stresses in the member are not to exceed 150 MPa and 200 MPa. Determine the value of load P that can safely be carried by the member.

**Solution:**

**Given,**

$$d = 0.01 \text{ m}; R = 0.10 \text{ m}; G = 150 \text{ MPa} = 150 \text{ MN} / \text{m}^2 \text{ (tensile)}$$

$$\sigma_2 = 200 \text{ MPa} = 200 \text{ MN} / \text{m}^2 \text{ (Compressive)}$$

**Load P:**

Refer to the fig . Area of cross section,

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (0.10)^2 = 7.854 \times 10^{-3} \text{ m}^2$$

Bending moment,  $m = P (0.15 + 0.10) = 0.25 P$

$$h^2 = \frac{d^2}{16} + \frac{1}{128} \cdot \frac{(0.10)^4}{(0.10)^2} = 7.031 \times 10^{-4} \text{ m}^2$$

$$\text{Direct stress, } \sigma_d = \frac{P}{A} \text{ (comp)}$$

**Bending stress at point 1 due to M:**

$$\sigma_{b1} = \frac{M}{AR} \left[ 1 + \frac{R^2}{h^2} \times \frac{y}{R+y} \right] \text{ (tensile)}$$

Total stress at point 1,

$$\sigma_1 = \sigma_d + \sigma_{b1}$$

$$150 = \frac{-P}{A} + \frac{M}{AR} \left[ 1 + \frac{R^2}{h^2} \times \frac{y}{R+y} \right] \text{ (tensile)}$$

$$\begin{aligned} 150 &= \frac{-P}{7.854 \times 10^{-3}} + \frac{0.25P}{7.854 \times 10^{-3} \times 0.10} \left[ 1 + \frac{0.10^2}{7.031 \times 10^{-4}} \times \frac{0.05}{0.10 + 0.05} \right] \\ &= -127.32 P + 318.31 P \times 5.74 \\ &= 1699.78 P \end{aligned}$$

$$P = \frac{150 \times 10^3}{1699.78} = 88.25 \text{ KN} \quad (i)$$

**Bending stress at point 2 due to M:**

$$\sigma_{b2} = \frac{M}{AR} \left[ \frac{R^2}{h^2} \times \frac{y}{R-y} - 1 \right] \text{ (comp)}$$

Total stress at point 2,

$$\sigma_2 = \sigma_d + \sigma_{b2}$$

$$\begin{aligned} 200 &= \frac{P}{A} + \frac{M}{AR} \left[ \frac{R^2}{h^2} \times \frac{y}{R-y} - 1 \right] \\ &= \frac{P}{7.854 \times 10^{-3}} + \frac{0.25P}{7.854 \times 10^{-3} \times 0.10} \left[ \frac{0.10^2}{7.031 \times 10^{-4}} \times \frac{0.05}{0.10 - 0.05} - 1 \right] \\ &= 127.32 P + 318.31 P \times 13.22 \\ &= 4335.38 P \end{aligned}$$

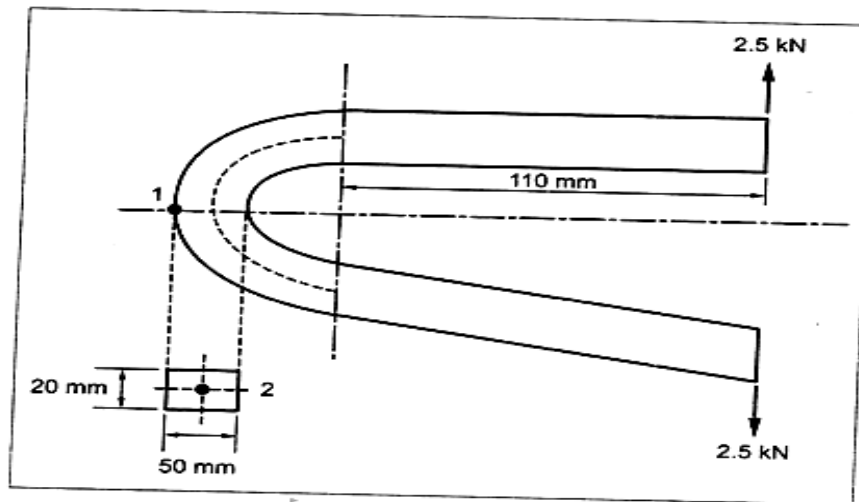
$$P = \frac{200}{4335.38} MN$$

$$P = \frac{200 \times 10^3}{4335.38} = 46.13 KN \quad (ii)$$

By comparing (i) & (ii) the safe load P will be lesser of two values

∴ Safe load = 46.13 KN.

6. Fig. shows a frame subjected to a load of 2.4 kN. Find (i) The resultant stresses at a point 1 and 2; (ii) Position of neutral axis. (April/May 2003)



**Solution:**

Area of section 1-2,

$$A = 48 * 18 * 10^{-6} = 8.64 * 10^{-4} m^2$$

Bending moment,

$$M = -2.4 * 10^3 * (120 + 48) = -403.2 Nm$$

M is taken as -ve because it tends to decrease the curvature.

**(i) Direct stress:**

$$\text{Direct stress } \sigma_d = \frac{P}{A} = \frac{2.4 * 10^3}{8.64 * 10^{-4}} * 10^{-6} = 2.77 MN/m^2$$

$$h^2 = \frac{R^3}{D} \log_e \left( \frac{2R+D}{2R-D} \right) - R^2$$

Here  $R = 48 \text{ mm} = 0.048 \text{ m}$ ,  $D = 48 \text{ mm} = 0.048 \text{ m}$

$$\begin{aligned} h^2 &= \frac{0.048^3}{0.048} \log_e \left( \frac{2(0.048) + 0.048}{2(0.048) - 0.048} \right) - (0.048)^2 \\ &= 0.048^2 (\log_e 3 - 1) = 2.27 * 10^{-4} m^2 \end{aligned}$$

**(ii) Bending stress due to M at point 2:**

$$\sigma_{b2} = \frac{M}{AR} \left[ 1 - \frac{R^2}{h^2} \left[ \frac{y}{R-y} \right] \right] \quad ;$$

$$\sigma = \frac{-403.2}{8.64 * 10^{-4} * 0.048} \left[ 1 - \frac{0.048^2}{2.27 * 10^{-4}} \left[ \frac{0.024}{0.048 - 0.024} \right] \right] * 10^{-6} \text{ MN / m}^2$$

$$= -9.722 (1 - 10.149) = 88.95 \text{ MN/m}^2 \text{ (tensile)}$$

**(iii) Bending stress due to M at point 1:**

$$\sigma_{b1} = \frac{M}{AR} \left[ 1 + \frac{R^2}{h^2} \left[ \frac{y}{R+y} \right] \right]$$

$$\sigma = \frac{-403.2}{8.64 * 10^{-4} * 0.048} \left[ 1 + \frac{0.048^2}{2.27 * 10^{-4}} \left[ \frac{0.024}{0.048 + 0.024} \right] \right] * 10^{-6} \text{ MN / m}^2$$

$$= -42.61 \text{ MN/m}^2 = 42.61 \text{ MN/m}^2 \text{ (comp)}$$

**(iv) Resultant stress:**

Resultant stress at point 2,

$$\sigma_2 = \sigma_d + \sigma_{b2} = 2.77 + 88.95 = 91.72 \text{ MN/m}^2 \text{ (tensile)}$$

Resultant stress at point 1,

$$\sigma_1 = \sigma_d + \sigma_{b1} = 2.77 - 42.61 = 39.84 \text{ MN/m}^2 \text{ (comp)}$$

**(v) Position of the neutral axis:**

$$y = - \left[ \frac{Rh^2}{R^2 - h^2} \right]$$

$$y = - \left[ \frac{0.048 * 2.27 * 10^{-4}}{0.048^2 + 2.27 * 10^{-4}} \right]$$

$$= -0.00435 \text{ m} = -4.35 \text{ mm}$$

Hence, neutral axis is at a radius of 4.35 mm