QUESTION WITH ANSWERS

DEPARTMENT : CIVIL **SEMESTER: V**

SUB.CODE/ NAME: CE8402 **/ Strength of Materials**

UNIT– 3 COULMNS

PART - A (2 marks)

1. Define columns

If the member of the structure is vertical and both of its ends are fixed rigidly while subjected to axial compressive load, the member is known as column.

Example: A vertical pillar between the roof and floor.

2. Define struts.

If the member of the structure is not vertical and one (or) both of its ends is Linged (or) pin jointed, the bar is known as strut.

Example: Connecting rods, piston rods etc,

3. Mention the stresses which are responsible for column failure.

i. Direct compressive stresses
ii. Buckling stresses
iii. Combined of direct compressive and buckling stresses

4. State the assumptions made in the Euler's column theory. . (AUC Nov/Dec 2011)

1. The column is initially perfectly straight and the load is applied axially.
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic and obeys Hooke's law.
4. The self weight of column is negligible.

5. What are the important end conditions of columns?

1. Both the ends of the column are hinged (or pinned)
2. One end is fixed and the other end is free.
3. Both the ends of the column are fixed.
4. One end is fixed and the other is pinned.

6. Write the expression for crippling load when the both ends of the column are hinged.

$$P = \frac{\pi^2 EI}{l^2}$$

P = Crippling load
E = Young's Modulus
I = Moment of inertia
l = Length of column

7. Write the expression for buckling load (or) Crippling load when both ends of the column are fixed?

$$P = \frac{4\pi^2 EI}{L^2}$$

P = Crippling load
E = Young's Modulus
I = Moment of inertia
l = Length of column

8. Write the expression for crippling load when column with one end fixed and other end linged.

$$P = \frac{2\pi^2 EI}{l^2}$$

P = Crippling load
E = Young's Modulus
I = Moment of inertia
l = Length of column

9. Write the expression for buckling load for the column with one fixed and other end free.

$$P = \frac{\pi^2 EI}{4l^2}$$

P = Crippling load
E = Young's Modulus
I = Moment of inertia
l = Length of column

10. Explain equivalent length (or) Effective length.

If l is actual length of a column, then its equivalent length (or) effective length L may be obtained by multiplying it with some constant factor C, which depends on the end fixation of the column (ie) L = C x l.

11. Write the Equivalent length (L) of the column in which both ends hinged and write the crippling load.

Crippling Load $P = \frac{\pi^2 EI}{L^2}$

Equivalent length (L) = Actual length (l)

P = Crippling load
E = Young's Modulus
I = Moment of inertia
L= Length of column

12. Write the relation between Equivalent length and actual length for all end conditions of column.

Both ends linged	L = l	Constant = 1
Both ends fixed	$L = \frac{l}{2}$	Constant = $\frac{1}{2}$
One end fixed and other end hinged	$L = \frac{l}{\sqrt{2}}$	Constant = $\frac{1}{\sqrt{2}}$
One end fixed and other end free	$L = 2l$	Constant = 2

13. Define core (or) Kernel of a section. . (ACU Nov/Dec 2011) (ACU May/June 2012)

When a load acts in such a way on a region around the CG of the section So that in that region stress everywhere is compressive and no tension is developed anywhere, then that area is called the core (or) Kernal of a section. The kernel of the section is the area within which the line of action of the eccentric load P must cut the cross-section if the stress is not to become tensile.

14. Derive the expression for core of a rectangular section. (ACU Nov/Dec 2003)

The limit of eccentricity of a rectangular section b x d on either side of XX axis (or) YY axis is d/6 to avoid tension at the base core of the rectangular section.

Core of the rectangular section = Area of the shaded portion

$$= 2 \times \frac{1}{2} \times \frac{b}{3} \times \frac{d}{6}$$

$$= \frac{bd}{18}$$

15. Derive the expression for core of a solid circular section of diameter D. (ACU April /May 2010) (AUC Nov/Dec 2011)

The limit of eccentricity on either side of both XX (or) YY axis = D/8 to avoid tension of the base.

Core of the circular section = Area of the shaded portion

$$= \pi (D/8)^2$$

$$= \frac{\pi D^2}{64}$$

16. A steel column is of length 8m and diameter 600 mm with both ends hinged. Determine the crippling load by Euler's formula. Take $E = 2.1 \times 10^5$ N/mm².

$$I = \frac{\pi}{64}(d)^4 = \frac{\pi}{64}(600)^4 = 6.36 \times 10^9 \, mm^4$$

Since the column is hinged at the both ends,

$\therefore$ Equivalent length L = l

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2.1 \times 10^5 \times 6.36 \times 10^9}{(8000)^2}$$

$$= 2.06 \times 10^8 \ N$$

17. Define Slenderness ratio. **(AUC Nov/Dec 2013)**

It is defined as the ratio of the effective length of the column (L) to the least radius of gyration of its cross –section (K) (i.e) the ratio of $\frac{L}{K}$ is known as slenderness ratio.

Slenderness ratio = $\frac{L}{K}$

18. State the Limitations of Euler's formula. **(AUC April /May 2005)**

a. Euler's formula is applicable when the slenderness ratio is greater than or equal to 80
b. Euler's formula is applicable only for long column
c. Euler's formula is thus unsuitable when the slenderness ratio is less than a certain value.

19. Write the Rankine's formula for columns. **(AUC April /May 2010)**

$$P = \frac{f_c \times A}{1 + \alpha \left(\frac{L}{K}\right)^2}$$

K = Least radius of gyration $= \sqrt{\frac{I}{A}}$

P = Crippling load

A = Area of the column

f_c = Constant value depends upon the material.

α = Rankine's constant $= \frac{f_c}{\pi^2 E}$

20. Write the Rankine's formula for eccentric column.

$$P = \frac{f_c \times A}{\left(1 + \frac{ey_c}{k^2}\right)\left[1 + \alpha \left(\frac{L}{k}\right)^2\right]}$$

K = Least radius of gyration $= \sqrt{\frac{I}{A}}$

P = Crippling load

A = Area of the column

f_c = Constant value depends upon the material.

α = Rankine's constant $= \frac{f_c}{\pi^2 E}$

21. Define thick cylinder. . **(ACU April /May 2011)**

If the ratio of thickness of the internal diameter of a cylindrical or spherical shell exceeds 1/20, it is termed as a thick shell.

The hoop stress developed in a thick shell varies from a maximum value at the inner circumference to a minimum value at the outer circumference.

Thickness > 1/20

22. State the assumptions involved in Lame's Theory **(AUC Nov/Dec 2013)**

i. The material of the shell is Homogeneous and isotropic.
ii. Plane section normal to the longitudinal axis of the cylinder remains plane after the application of internal pressure.
iii. All the fibers of the material expand (or) contact independently without being constrained by there adjacent fibers.

23. What is the middle third rule? **(ACU Nov/Dec 2003)**

In rectangular sections, the eccentricity 'e' must be less than or equal to b/6. Hence the greatest eccentricity of the load is b/6 form the axis Y-Y and with respect to axis X $-X_1$ the eccentricity does not exceed d/6. Hence the load may be applied with in the middle third of the base (or) Middle d/3.

24. What are the limitations of the Euler's formula?

1. It is not valid for mild steel column. The slenderness ratio of mild steel column is less than 80.
2. It does not take the direct stress. But in excess of load it can withstand under direct compression only.

25. Write the Euler's formula for different end conditions. **(ACU Nov/Dec 2012)**

1. Both ends fixed.
PE = л 2 EI / (0.5L)2
2. Both ends hinged
PE = л 2 EI / (L)2
3. One end fixed ,other end hinged.
PE = л 2 EI / (0.7L)2
4. One end fixed, other end free.
PE = л 2 EI / (2L)2
L = Length of the column

26. How many types will you determine the hoop stress in a thick compound cylinder? **(AUC May/June 2012)**

1. Radial stress
2. Circumferential or hoop stress
3. Maximum at the inner circumference
4. Minimum at the outer circumference of a thick cylinder

27. How columns are differentiate the type of column depending upon the Slenderness ratio. . **(ACU April /May 2011)**

Slenderness ratio is used to differentiate the type of column. Strength of the column depends upon the slenderness ratio, it is increased the compressive strength of the column decrease as the tendency to buckle is increased.

PART -B (13 marks)

1. *A 1.5 m long cast iron column has a circular cross-section of 50 mm diameter. One end of the column is fixed in direction and position and the other end is free. Taking factor of safety as 3, calculate the load using Rankine Gordon formula. Take yield stress as 560 MPa and 'a' = $\frac{1}{1600}$.* ***(Apr 2011)***

Solution:

$$\text{Length, } l = 1.5 \text{ m} = 1500 \text{ mm}$$

$$d = 50 \text{ mm}$$

$$\text{Area} = \frac{\pi}{4} \times 50^2 = 19.63 \times 10^2 \text{ mm}^2$$

$$\text{Moment of inertia, I} = \frac{\pi}{64} \times 50^4 = 30.7 \times 10^4 \text{ mm}^4$$

$$\text{Least radius of gyration, K} = \sqrt{\frac{I}{K}}$$

$$= \sqrt{\frac{30.7 \times 10^4}{19.63 \times 10^2}}$$

$$= 12.5 \text{ mm}$$

End condition: One end is fixed and other is free.

$$\therefore \text{ Effective length, } l_e = 2\,l = 2 \times 1500 = 3000 \text{ mm}$$

$$P_L = \frac{\sigma_C \times A}{1 + a\left[\frac{l_e}{K}\right]^2}$$

$$= \frac{560 \times 1963.5}{1 + \frac{1}{1600}\left[\frac{3000}{12.5}\right]^2}$$

$$= 29708.1 \text{ N}$$

$$\text{Safe load} = \frac{\text{Crippling Load}}{\text{FoS}}$$

$$= \frac{29708.1}{3} = \mathbf{9902.7\ N}$$

2. *A hollow C.I column whose outside diameter is 200 mm has a thickness of 20 mm. It is 4.5 m long and is fixed at both ends. Calculate the safe load by Rankine's formula using a factor of safety 4. Calculate the slenderness ratio and the ratio of Euler's and Rankine's critical loads. For cast iron f_e = 550 N/mm². $\alpha = \frac{1}{1600}$ and $E = 8 \times 10^4$ N/mm².*

Solution: External diameter = 200 mm = D

Thickness, t = 20 mm

$\therefore$ Internal diameter, $d = D - 2t$

$= 200 - (2 \times 20) = 160$ mm

$$\text{Area} = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(200^2 - 160^2)$$

$$= 11309.734 \text{ mm}^2$$

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(200^4 - 160^4)$$

$$= 46369907.57 \text{ mm}^4$$

$$\text{Radius of gyration, } r = K = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{46369907.57}{11309.734}}$$

$$= \sqrt{4100}$$

$$= 64.03 \text{ mm}$$

Since the column is fixed at both ends,

$$l_e = \frac{l}{2} = \frac{4500}{2} = 2250 \text{ mm}$$

$$\text{Slenderness ratio} = \frac{l_e}{K} = \frac{4500}{64.03} = \mathbf{70.30}$$

$$\text{Rankine critical load, } P_R = \frac{\sigma_C \cdot A}{1 + \alpha\left[\frac{l_e}{K}\right]^2}$$

$$= \frac{550 \times 11309.73}{1 + \frac{1}{1600}\left[\frac{2250}{64.03}\right]^2}$$

$$= 351100 \text{ N}$$

$$\therefore \quad \text{Safe load} = \frac{\text{Crippling load}}{\text{FoS}} = \frac{351100}{4}$$

$$= 8777 \text{ N}$$

Ratio of Euler's and Rankine's Critical Load

$$\text{Euler's critical load} = \frac{\pi^2 E I}{l_e^2}$$

$$= \frac{\pi^2 \times 8 \times 10^4 \times 46369907}{(2250)^2}$$

$$= 7232041$$

$$\frac{\text{Euler's critical load}}{\text{Rankine's critical load}} = \frac{7232041}{351100}$$

$$= \mathbf{2.06} \quad \textbf{Ans.}$$

3. *Find Euler's crippling load for a hollow cylindrical steel column of 38 mm external diameter and 2.5 mm thickness. The length of column is 2.3 m hinged at both ends. Take E = 205 GPa. Also find crippling load by Rankine's formula. Using constant 335 kN/mm², $\alpha = \frac{1}{7500}$.* ***(Nov 2012)***

☺ ***Solution:***

$$\text{External diameter} = 38 \text{ mm}$$

$$\text{Internal diameter} = 38 - 2 \times 2.5 = 33 \text{ mm}$$

$$\text{Moment of inertia, } I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}[38^4 - 33^4] = 44117.7 \text{ mm}^4$$

$$\text{Area of column} = \frac{\pi}{4} \times [38^2 - 33^2]$$

$$= 278.67 \text{ mm}^2$$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{44117.7}{278.67}} = 12.58 \text{ r.}$$

$$l_e = l = 2.3 \text{ m} = 2.3 \times 10^3 \text{ mm}$$

$$P_E = \frac{\pi^2 E I}{l_e^2}$$

$$= \frac{\pi^2 \times 205 \times 10^3 \times 44117.7}{(2300)^2}$$

$$= \mathbf{16880\ N}$$

$$P_R = \frac{\sigma_C \cdot A}{1 + \alpha \left[\frac{l_e}{K}\right]^2}$$

$$= \frac{335 \times 10^3 \times 278.67}{1 + \frac{1}{7500\left[\frac{2300}{12.6}\right]^2}}$$

$$= \mathbf{17160\ N}$$

4. *Find the greatest length of a mild steel rod 25 mm × 25 mm which can be used as a compression member with one end fixed and other end free to carrying a working load of 35 kN. Allow a factor of safety of 4. Take 'a' = $\frac{1}{7500}$ and $f_C = 320\ N/mm^2$.*

(Apr/May 2010)

Solution:

Area of the rod, $A = 25 \times 25 = 625\ mm^2$

Moment of inertia of section, $I_{xx} = I_{yy} = \frac{b\,d^3}{12}$

$$= \frac{25 \times 25^3}{12} = 32552.08\ mm^4$$

$$K = \sqrt{\frac{I}{A}}$$

$$K^2 = \frac{I}{A} = \frac{32552.0833}{625} = 52.083$$

$$\text{Crippling load, } P = \frac{f_C \cdot A}{1 + a\left(\frac{l_e}{K}\right)^2}$$

$$35 \times 4 \times 10^3 = \frac{320.625}{1 + \frac{1}{7500} \cdot \frac{l_e^2}{(52.08)^2}}$$

$$1{,}40{,}000 = \frac{2{,}00{,}000}{1 + \frac{l_e^2}{390622.5}}$$

$$\therefore\ l_e = 409.157\ mm$$

Since the rod is fixed at one end and the other end free,

Effective length, $l_e = 2\,l$

$$l = \frac{l_e}{2} = \frac{409.157}{2} = \mathbf{204.57\ mm}$$

5. *A hollow cylindrical cast iron column is 4 m long both the ends being fixed. Design the column to carry an axial load of 250 kN. Use Rankine's formula and adopt a factor of safety 5. The internal diameter may be taken as 0.8 times the external diameter.* ***(Nov/Dec 2011)***

Solution: Let the external diameter be D mm.

$\therefore$ Internal diameter, $d = 0.8\,D$

$$\text{Area of the section} = \frac{\pi}{4} \times (D^2 - d^2)$$

$$= \frac{\pi}{4} \times (D^2 - 0.8^2\,D^2)$$

$$= \frac{\pi}{4} \times (D^2 - 0.64\,D^2)$$

$$= 0.09\,\pi\,D^2$$

$$\text{Moment of inertia} = \frac{\pi}{64} \times (D^4 - d^4)$$

$$K = r = \sqrt{\frac{I}{A}}, \quad K^2 = \frac{I}{A}$$

$$K^2 = \frac{\frac{\pi}{64}(D^4 - d^4)}{\frac{\pi}{4}(D^2 - d^2)} = \frac{\frac{\pi}{64}(D^2 + d^2)(D^2 - d^2)}{\frac{\pi}{4}(D^2 - d^2)}$$

$$= \frac{D^2 + d^2}{16} = \frac{D^2 + 0.64\,D^2}{16} = \frac{1.64\,D^2}{16}$$

$$= 0.1025\,D^2$$

Safe load on column $= 250$ kN

Crippling load = Safe load × Factor of safety

$= 250 \times 5 =$ **1250 kN**

Both end of the column are fixed.

$$\text{Effective length, } l_e = \frac{l}{2}$$

$$= \frac{4000}{2} = \mathbf{2000\ mm}$$

$$P_R = \frac{\sigma_C \cdot A}{1 + a\left(\frac{l_e}{K}\right)^2}$$

$$1250 \times 1000 = \frac{550 \times 0.09\, \pi\, D^2}{1 + \frac{1}{1600} \times \frac{2000^2}{0.1025\, D^2}}$$

$$155.509\, D^4 = 1250000\, D^2 + (1250000 \times 24390.24)$$

$$D^4 - 8038.120^2 - 196051706.8 = 0$$

$$D^2 = \frac{8038.12 \pm \sqrt{8038.12^2 + 4 \times 196051706.8}}{2}$$

$$= \frac{8038.12 \pm 29134.485}{2}$$

D^2 cannot be negative.

$$D^2 = 18586.3$$

$$\therefore D = \mathbf{136.33\ mm}$$

$$d = 0.8 \times 136.33 = \mathbf{109.065\ mm}$$

6. *From the following data of column and circular section, calculate the extreme stresses on the column section. Also find the maximum eccentricity in order that there may be no tension any where on the section.*

External diameter = 20 cm, Internal diameter = 16 cm, Length of the column = 4 m, Load carried by the column = 175 kN. Eccentricity of the load = 2.5 cm (from the axis of the column). End conditions = Both ends fixed. Young's modulus = 94 GN/m^2. ***(Nov/Dec 2010)***

☺ ***Solution:*** Area of the column, $A = \frac{\pi}{4}(20^2 - 16^2)$

$$= 113.1\ cm^2 = 113.1 \times 10^{-4}\ m^2$$

Moment of inertia, $I = \frac{\pi}{64}(D^4 - d^4)$

$$= \frac{\pi}{64}(20^4 - 16^4) = 4637\ cm^4$$

$$= 4637 \times 10^{-8}\ m^4$$

Effective length, $l_e = \frac{l}{2} = \frac{4}{2} = 2\ m$

Maximum bending moment, $M = P \cdot e \cdot \sec \frac{l_e}{2}\sqrt{\frac{P}{EI}}$

$$\frac{l_e}{2}\sqrt{\frac{P}{EI}} = \frac{2}{2}\sqrt{\frac{175 \times 10^3}{94 \times 10^9 \times 4637 \times 10^{-8}}}$$

$$= 0.2 \text{ radians}$$

$$= 11° 28'$$

$$\sec 11° 28' = 1.02$$

Maximum bending moment, $M_{max} = 175 \times (2.5 \times 10^{-2}) \times 1.02$

$$= 4.4625$$

Maximum compressive stress, $\sigma_{max} = \frac{P}{A} + \frac{M}{Z}$

$$= \frac{175}{113.1 \times 10^{-4}} + \frac{4.46}{463.7 \times 10^{-6}}$$

$$\left[\because Z = \frac{I}{y} = \frac{4637 \times 10^{-8}}{10 \times 10^{-2}} = 463.7 \times 10^{-6} \text{ m}^3\right]$$

$$= 25091.32 \text{ kN/m}^2$$

$$= \mathbf{25.09 \text{ MN/m}^2}$$

For no tension, $\sigma_d = \sigma_b$

$$\frac{P}{A} = \frac{M}{Z}$$

$$\frac{P}{A} = \frac{P \cdot e \cdot \sec \frac{l_e}{2}\sqrt{\frac{P}{EI}}}{Z}$$

$$\frac{175}{113.1 \times 10^{-4}} = \frac{175 \times e \times 1.02}{463.7 \times 10^{-6}}$$

$$e = 0.04 \text{ m}$$

$$e = \mathbf{40.02 \text{ mm}}$$

7. *A bar of length 6 m when used as a simply supported beam and subjected to a UDL of 30 kN/m over the whole span, deflects 15 mm at the centre. Determine the crippling loads when it is used as a column with the following end condition:*

(a) Both ends fixed

(b) Both ends pin-jointed

(c) One end fixed and the other end hinged.

Solution:

Length, $l = 4$ m

UDL $= 30$ kN/m

Deflection, $\delta = 15 \text{ mm} = 0.15$ m

We know that, $\delta = \dfrac{5\,w\,l^4}{384\,\text{E I}}$

$$0.015 = \frac{5 \times (30 \times 10^3) \times 4^4}{384\,\text{E I}}$$

$$\text{E I} = 6.66 \times 10^6 \text{ Nm}^2$$

(a) Both ends fixed

For any type of end condition, $P_E = \dfrac{\pi^2 \text{E I}}{l_e^2}$

$$P_{\text{Euler}} = \frac{\pi^2 \text{E I}}{l_e^2} \qquad \left[l_e = \frac{l}{2} = \frac{6}{3} = 3 \text{ m} \right]$$

$$= \frac{\pi^2 \times 6.66 \times 10^6}{3^2} \times 10^{-3} \text{ kN}$$

$$= \mathbf{7296.104 \text{ kN}} \quad \textbf{Ans.}$$

(b) Both ends pin-jointed

$$P_{\text{Euler}} = \frac{\pi^2 \text{E I}}{l_e^2} \qquad [\, l_e = l = 6 \text{ m} \,]$$

$$= \frac{\pi^2 \times 6.66 \times 10^6}{6^2} \times 10^{-3} \text{ kN}$$

$$= \mathbf{1824.026 \text{ kN}} \quad \textbf{Ans.}$$

(c) One end fixed and the other end hinged

$$P_{Euler} = \frac{\pi^2 E I}{l_e^2} \qquad [\, l_e = l/\sqrt{2} = 6/\sqrt{2} \text{ m} \,]$$

$$= \frac{\pi^2 \times 6.66 \times 10^6}{\left(\frac{6}{\sqrt{2}}\right)^2} \times 10^{-3}$$

$$= \frac{65664.936}{\frac{36}{4}} \text{ kN} = \mathbf{7296.104 \ kN}$$

8. *A rectangular strut is 25 cm × 15 cm. It carries a load of 60 kN at an eccentricity of 2 cm in a plane bisecting the thickness. Find the minimum and maximum stresses developed in the section.* ***(Nov/Dec 2011)***

Solution:

$$\text{Load, P} = 60 \text{ kN}$$

$$= 60 \times 1000 \text{ N} = 60000 \text{ N}$$

$$\text{Area, A} = 250 \times 150$$

$$= 37500 \text{ mm}^2$$

$$\text{Eccentricity, } e = 2 \text{ cm} = 20 \text{ mm}$$

$$M = P \cdot e = 60 \times 1000 \times 20$$

$$= 1200000 \text{ N-mm}$$

$$Z = \frac{b\,d^2}{6}$$

$$= \frac{250 \times 150^2}{6}$$

$$= 937500 \text{ mm}^3$$

Direct stress, $\sigma_d = \frac{P}{A}$

$= \frac{60000}{37500}$

$= 1.6 \text{ N/mm}^2$

Bending stress, $\sigma_b = \frac{M}{Z}$

$= \frac{1200000}{937500}$

$= 1.28 \text{ N/mm}^2$

$\sigma_{max} = \sigma_d + \sigma_b$

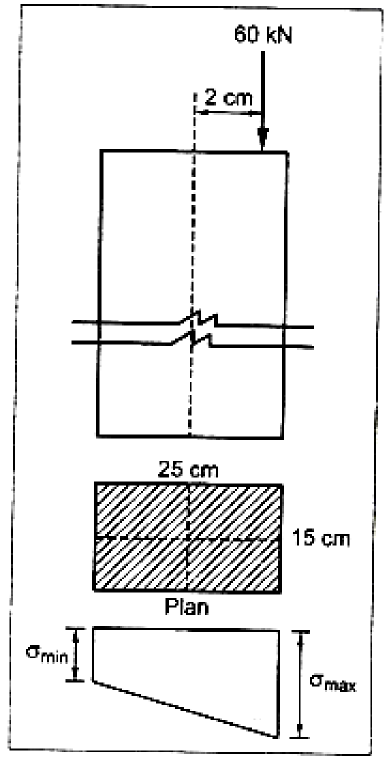
$= 1.6 + 1.28$

$= \mathbf{2.88 \text{ N/mm}^2}$ **Ans.**

$\sigma_{min} = \sigma_d - \sigma_b$

$= 1.6 - 1.28$

$= \mathbf{0.32 \text{ N/mm}^2}$ **Ans.**



9. *Find the safe axial load on a steel stanchion of 8 metres height having one end fixed and the other hinged which build up two 250 mm × 100 mm standard channels placed 10 cm apart back to back with two 300 mm × 10 mm plates riveted to each flange for individual channel section I_{xx} = 3687.9 cm^4, I_{yy} = 298.4 cm^4, A = 35.65 cm^2 distance of C.G from back of web 2.70 cm. Adopt factor of safety as 4. Use Rankine's formula.* ***(Nov/Dec 2010)***

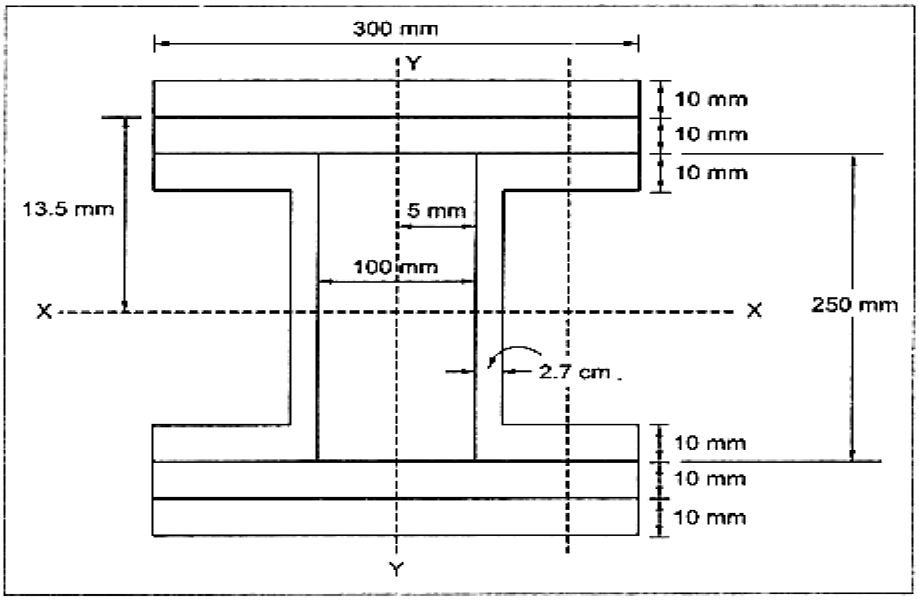
☺ ***Solution:*** Length of column = 8 m

$$L_e = \frac{L}{\sqrt{2}} = \frac{800}{\sqrt{2}} = 565.69$$

Factor of safety = 4

Assume, yield stress, σ_C = 315 N/mm^2

$$a \text{ (or) } \alpha = \frac{1}{7500}$$



$$\text{Area of built up section} = 2 \times 35.65 + 4 \times 30 \times 10$$
$$= 71.30 + 120$$
$$= 191.30 \text{ cm}^2$$

$$I_{xx} = 2 \times 3687.9 + 2\left[\frac{1}{12} \times 30 \times 2^3 + 30 \times 2 \times 13.5^2\right]$$
$$= (7375.8) + 2\,[\,20 + 10935\,]$$
$$= 29285.8$$

$$I_{yy} = 2 \times \frac{1}{12} \times 2 \times 30^3 + 2\,[\,298.4 + 35.65 \times (5 + 2.7)^2\,]$$
$$= 9000 + 786.68$$
$$= 9786.68$$

I_{yy} is the lesser of the two

$$K^2 = \frac{I}{A} = \frac{9786.68}{191.3} = 51.16$$
$$K = 7.15$$

$$\text{Crippling load, } P = \frac{f_C \times A}{1 + a + \left(\frac{l_e}{K}\right)^2}$$
$$= \frac{315 \times 10^2 \times 191.3}{1 + \frac{1}{7500} + \left(\frac{565.69}{7.15}\right)^2}$$
$$= \frac{6025950}{1 + (1.33 \times 10^{-4}) + 6258.98}$$
$$= \mathbf{962.61\ N}$$

10. *A compound cylinder is to be made by shrinking on outer tube of 300 mm external diameter on to an inner tube of 150 mm internal diameter. Determine the common diameter at the junction if the greatest circumferential stress in the inner tube is to be two thirds of the greatest circumferential stress in the outer tube.* ***(Nov/Dec 2012)***

Solution:

External radius of the outer tube, $r_3 = \frac{300}{2} = 150 \text{ mm} = 0.15 \text{ m}$

Inner radius of the inner tube, $r_1 = \frac{150}{2} = 75 \text{ mm} = 0.075 \text{ m}$

Greatest circumferential stress in the inner tube

$$= \frac{2}{3} \times \left\{ \begin{matrix} \text{Greatest circumference} \\ \text{stress in the outer tube} \end{matrix} \right\}$$

Let r_2 = The common radius and

p = Radial pressure at the junction

Common diameter:

$$\text{Outer tube, } (\sigma_C)_{\text{outer tube}} = p\left[\frac{r_3^2 + r_2^2}{r_3^2 - r_2^2}\right] = p\left[\frac{K+1}{K-1}\right]$$

$$\text{where, } K = \frac{r_3^2}{r_2^2} = \frac{0.15^2}{r_2^2}$$

Inner tube:

$$(\sigma_C)_{\text{inner tube}} = p\left[\frac{2\,r_2^2}{r_2^2 - r_1^2}\right] = p\left[\frac{2}{\left(1 - \frac{r_1^2}{r_2^2}\right)}\right]$$

$$= p\left[\frac{2}{1 - \frac{(0.075)^2}{r_2^2}}\right]$$

$$= p\left[\frac{2}{1 - \frac{(0.075)^2}{r_2^2}}\right]$$

$$= p\left[\frac{4}{2 - \frac{(0.15)^2}{r_2^2}}\right] = p\left[\frac{4}{2-K}\right]$$

$$\text{But } (\sigma_C)_{\text{inner tube}} = \frac{2}{3}(\sigma_C)_{\text{outer tube}}$$

$$p\left[\frac{4}{2-K}\right] = \frac{2}{3}p\left[\frac{K+1}{K-1}\right]$$

$$6(K-1) = (2-K)(K+1)$$

$$6K-6 = 2K-K^2+2-K$$

$$K^2+6K-6-2K-2+K = 0$$

$$K^2+5K-8 = 0$$

$$K = \frac{-5\pm\sqrt{5^2+32}}{2} = \frac{-5+\sqrt{57}}{2}$$

[Neglecting –ve value]

$$K = 1.27 = \frac{0.15^2}{r_2^2}$$

$$r_2 = 0.133\text{ m} = 133\text{ mm}$$

$$d_2 = 2\times 133 = 266\text{ mm}$$

Hence the common diameter = **266 mm**

11. A hollow cylindrical cast iron column whose external diameter in 200mm and has a thickness of 20mm is 4.5m long and is fixed at both ends . Calculate the safe load by Rankin's formula using a factor safety of 2.5. Take crushing strength of material as 580N/mm^2 and Rankin's constant as 1/1600 . Find also the ratio of Rulers to Rankin's load E= 150Gpa .
(AUC May/June 2012)

External diameter = 200 mm

Thickness, t = 20 mm

Internal diameter = [200 – 2 × 20] = 160 mm

$$\text{Area} = \frac{\pi}{4}[200^2-160^2]$$

$$= 11310\text{ mm}^2$$

$$\text{Moment of inertia, } I = \frac{\pi}{64}\times(D^4-d^4) = \frac{\pi}{64}[200^4-160^4]$$

$$= 46370000\text{ mm}^4$$

$$\text{Radius of gyration, } K = \sqrt{\frac{I}{A}} = \sqrt{\frac{46370000}{11310}}$$

$$= 64 \text{ mm}$$

$$\text{Length of column, } L = 4500 \text{ mm}$$

End condition = Both the ends are fixed

$$\therefore \text{ Effective length, } l_e = \frac{l}{2} = \frac{4500}{2} = 2250 \text{ mm}$$

Rankine's Formula:

$$\text{Crippling load, } P_R = \frac{f_C \times A}{1 + a\left[\frac{l_e}{K}\right]^2}$$

$$= \frac{550 \times 11310}{1 + \frac{1}{1600}\left(\frac{2250}{64}\right)^2}$$

$$= 3511 \text{ kN}$$

$$\text{Safe load} = \frac{\text{Crippling load}}{\text{FoS}} = \frac{3511}{2.5}$$

$$= \mathbf{1404 \text{ kN}} \quad \textbf{Ans.}$$

P_E = Euler's critical load

$$P_E = \frac{\pi^2 E I}{l_e^2} \text{ (or) } \frac{\pi^2 E I}{\left(\frac{l}{2}\right)^2} \text{ (or) } \frac{4\pi^2 E I}{l^2}$$

$$= \frac{\pi^2 E I}{l_e^2}$$

$$= \frac{\pi^2 \times 150 \times 10^3 \times 46370000}{(2250)^2}$$

$$= 13560 \text{ kN}$$

Ratio of Euler's load to crippling load

$$= \frac{13560}{3511} = \mathbf{3.84 \text{ kN}}$$

12. A pipe of 200mm internal diameter and 50mm thickness carries a fluid at a pressure of a 10Mpa. Calculate the maximum and minimum intensities of circumferential stress distribution and circumferential stress distribution across the section. (AUC Apr2011)

Solution: Internal diameter = 200 mm

$\therefore$ Internal radius = 100 mm

Thickness, t = 50 mm

$\therefore$ External radius, $r_2 = r_1 + t = 100 + 50 = 150$ mm

Fluid pressure, p_x = 10 MPa = 10 N/mm^2

$$\text{The radial pressure, } p_x = \frac{b}{x^2} - a \qquad \ldots(1)$$

Now apply the boundary conditions to the above equation,

1. At $x = r_1 = 100$ mm, $p_x = 10$ N/mm^2
2. At $x = r_2 = 150$ mm, $p_x = 0$

Substituting these boundary conditions in equation (1),

$$10 = \frac{b}{100^2} - a = \frac{b}{10000} - a \qquad \ldots(2)$$

$$0 = \frac{b}{150^2} - a = \frac{b}{22500} - a \qquad \ldots(3)$$

$$(2) - (3) \Rightarrow \qquad 8 = \frac{b}{10000} - \frac{b}{22500}$$

$$8 = (5.555 \times 10^{-5})\, b$$

$$\boxed{b = 144{,}000}$$

Substituting this value in equation (3),

$$0 = \frac{144000}{22500} - a$$

$$a = \frac{144000}{22500}$$

$$a = 64$$

$$\text{Now the hoop stress, } \sigma_n = \frac{b}{x^2} + a = \frac{144000}{x^2} + 6.4$$

$$\sigma_h \text{ at } x = 100 = \frac{144000}{100^2} + 6.4 = \mathbf{20.8\ N/mm^2}$$

$$\sigma_h \text{ at } x = 150 = \frac{144000}{150^2} + 6.4 = \mathbf{12.8\ N/mm^2}$$

13. Explain the failure of long column.

Solution:

A long column of uniform cross-sectional area A and of length l, subjected to an axial compressive load P, as shown in fig. A column is known as long column if the length of the column in comparison to its lateral dimensions is very large. Such columns do not fail y crushing alone, but also by bending (also known buckling)

The load, at which the column just buckles, is known as buckling load and it is less than the crushing load is less than the crushing load for a long column.

Buckling load is also known as critical just (or) crippling load. The value of buckling load for long columns are long columns is low whereas for short columns the value of buckling load is high.

Let

l = length of the long column

p = Load (compressive) at which the column has jus buckled.

A = Cross-sectional area of he column

e = Maximum bending of the column at the centre.

σ_0 = Stress due to direct load $= \frac{P}{A}$

σ_b = Stress due to bending at the centre of the column

$= \frac{P \times e}{Z}$

Where

Z = Section modulus about the axis of bending.

The extreme stresses on the mid-section are given by

Maximum stress = $\sigma_0 + \sigma_b$

Minimum stress = $\sigma_0 - \sigma_b$

The column will fail when maximum stress (i.e) $\sigma_0 + \sigma_b$ is more the crushing stress f_c. In case of long column, the direct compressive stresses are negligible as compared to buckling stresses. Hence very long columns are subjected to buckling stresses.

14. State the assumptions made in the Euler's column Theory. And explain the sign conventions considered in columns. (AUC April/May2003) (AUC May/June2012) (AUC Nov/Dec 2010)

The following are the assumptions made in the Euler's column theory:

1. The column is initially perfectly straight and the load is applied axially
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic and obeys Hooke's law.
4. The length of the column is very large as compared to its lateral dimensions
5. The direct stress is very small as compared to the bending stress
6. The column will fail by buckling alone.
7. The self-weight of column is negligible.

The following are the sign conventions considered in columns:

1. A moment which will tend to bend the column with its convexity towards its initial centre line is taken as positive.
2. A moment which will tend to bend the column with its concavity towards its initial center line is taken as negative.

15. Derive the expression for crippling load when the both ends of the column are hinged. (AUC Nov/Dec 2011)

Solution:

Consider a column AB of length L hinged at both its ends A and B carries an axial crippling load at A.

Consider any section X-X at a distance of x from B.

Let the deflection at X-X is y.

$\therefore$ The bending moment at X-X due to the load P, M $= -P.y$

$$\frac{d^2y}{dx^2} = \frac{-Py}{EI} = -k^2y$$

Where $k^2 = \dfrac{p}{EI}$

$$\therefore \frac{d^2y}{dx^2} + k^2y = 0$$

Solution of this differential equation is

$$y = A\cos\ kx + B\sin\ kx$$

$$\therefore\ y = A\cos x\left(\sqrt{\frac{p}{EI}}\right) + B\sin\ x\left(\sqrt{\frac{p}{EI}}\right)$$

By using Boundary conditions,

At B, x = 0, y = 0 $\Rightarrow$ A = 0
At A, x = l, y = 0

$$\therefore\ 0 = B\sin l\sqrt{\frac{p}{EI}}$$

$$Sin l\sqrt{\frac{p}{EI}} = 0$$

$$l \times \sqrt{\frac{p}{EI}} = 0, \pi, 2\pi, 3\pi......$$

Now taking the lest significant value (i.e) π

$$l\sqrt{\frac{p}{EI}} = \pi\ ; \quad l^2\left(\frac{p}{EI}\right) = \pi^2$$

$$p = \frac{\pi^2 EI}{l^2}$$

$\therefore$`The Euler's crippling load for long column with both ends hinged.

$$p = \frac{\pi^2 EI}{l^2}$$

16. Derive the expression for buckling load (or) crippling load when both ends of the column are fixed. (AUC April/May2010)

Solution:

Consider a column AB of length l fixed at both the ends A and B and caries an axial crippling load P at A due to which buckling occurs. Under the action of the load P the column will deflect as shown in fig.

Consider any section X-X at a distance x from B.Let the deflection at X-X is y.

Due to fixity at the ends, let the moment at A or B is M.

$\therefore$ Total moment at XX = M – P.y

Differential equation of the elastic curve is

$$EI\,\frac{d^2y}{dx^2} = M - Py$$

$$\frac{d^2y}{dx^2} + \frac{py}{EI} = \frac{M}{IE}$$

$$\frac{d^2y}{dx^2} + \frac{py}{EI} = \frac{M}{IE} \times \frac{p}{p}$$

$$\frac{d^2y}{dx^2} + \frac{py}{EI} = \frac{P}{EI} \times \frac{M}{P}$$

The general solution of the above differential equation is

$$y = A\cos x\left(\sqrt{P/EI}\right) + B\sin x\left(\sqrt{P/EI}\right) + \frac{M}{P} \qquad \text{(i)}$$

Where A and B are the integration constant

At, N. x = 0 and y = 0

$\therefore$ From (i)

$$0 = A \times 1 + B \times 0 + \frac{M}{p}$$

$$A = -\frac{M}{p}$$

Differentiating the equation (i) with respect to x,

$$\frac{dy}{dx} = -A\sqrt{\frac{P}{EI}}Sin\left(x.\sqrt{P/EI}\right) + B\sqrt{\frac{P}{EI}}Cos\left(x.\sqrt{\frac{P}{EI}}\right) + 0$$

At the fixed end B, x = 0 and $\frac{dy}{dx} = 0$

$$\therefore B\sqrt{\frac{P}{EI}} = 0$$

Either B = 0 (or) $\sqrt{\frac{P}{EI}} = 0$

Since $\sqrt{\frac{P}{EI}} \neq 0$ as $p \neq 0$

$B = 0$

Subs $A = -\frac{M}{p}$ and B = 0 in equation (i)

$$y = -\frac{M}{P}\cos\left(x.\sqrt{\frac{P}{EI}}\right) + \frac{M}{P}$$

$$y = \frac{M}{P}\left[1 - \cos\left(x..\sqrt{\frac{P}{EI}}\right)\right]$$

Again at the fixed end A, x = l, y = 0

$$0 = \frac{M}{P}\left[1 - Cos\left(l.\sqrt{P/EI}\right)\right]$$

$$l.\sqrt{P/EI} = 0, 2\pi, 4\pi, 6\pi........$$

Now take the least significant value 2π

$$l.\sqrt{\frac{P}{EI}} = 2\pi$$

$$l.^2 \times \frac{P}{EI} = 4\pi^2$$

$$P = \frac{4\pi^2 EI}{l^2}$$

$\therefore$ The crippling load for long column when both the ends of the column are fixed

$$\boxed{P = \frac{4\pi^2 EI}{L^2}}$$

17. Derive the expression for crippling load when column with one end fixed and other end hinged. (ACU April/May 2003) (AUC Nov/Dec 2013)

Solution:

Consider a column AB of length l fixed at B and hinged at A. It carries an axial crippling load P at A for which the column just buckles.

As here the column AB is fixed at B, there will be some fixed end moment at B. Let it be M. To balance this fixing moment M, a horizontal push H will be exerted at A.

Consider any section X-X at a distance x from the fixed end B. Let the deflection at xx is y.

Bending moment at xx = H (l-x) - Py

$\therefore$ Differential equation of the elastic curve is,

$$EI\frac{d^2y}{dx^2} = H(l-x) - Py$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{14(l-x)}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{H(l-x)}{EI} \times \frac{p}{P}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{H(l-x)}{EI} \times \frac{p}{EI}$$

The general solution of the above different equation is

$$y = A\cos\left(x.\sqrt{\frac{p}{EI}}\right) + B\sin\left(x.\sqrt{\frac{p}{EI}}\right) + \frac{H(l-x)}{P}$$

Where A and B are the constants of integration. (i)

At B, x = 0, y = 0

$\therefore$ From (i) $A = \frac{-Hl}{P}$

$$B\sqrt{\frac{P}{EI}} = \frac{H}{P}$$

$$B = \frac{H}{P} \times \sqrt{\frac{EI}{p}}$$

Again at the end A, x = l, y=0. $\therefore$ substitute these values of x, y, A and B in equation (i)

$$0 = -\frac{Hl}{P}Cos\left(l.\sqrt{P/EI}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}}Sin\left(l.\sqrt{P/EI}\right)$$

$$\frac{H}{P}\left(\sqrt{\frac{EI}{p}}Sin.\left(l.\sqrt{P/EI}\right)\right) = \frac{Hl}{P}Cos\left(l.\sqrt{P/EI}\right)$$

$$\tan\left(l.\sqrt{P/EI}.l\right) = \sqrt{P/EI}.l$$

The value of $\tan\left(\sqrt{P/EI}.l\right)$ in radians has to be such that its tangent is equal to itself. The only angle whose tangent is equal to itself, is about 4.49 radians.

$$\sqrt{P/EI}.l = 4.49$$

$$\frac{P}{EI}l^2 = (4.49)^2$$

$$\frac{P}{EI}l^2 = 2\pi^2 \text{ (approx)}$$

$$P = \frac{2\pi^2 EI}{l^2}$$

$\therefore$ The crippling load (or) buckling load for the column with one end fixed and one end hinged.

$$\boxed{P = \frac{2\pi^2 EI}{l^2}}$$

18. Derive the expression for buckling load for the column with one end fixed and other end free. (AUC April/May 2003) (AUC May/June2012)

Solution:

Consider a column AB of length l, fixed at B and free at A, carrying an axial rippling load P at D de to which it just buckles. The deflected form of the column AB is shown in fig. Let the new position of A is A_1.

Let a be the deflection at the free end. Consider any section X-X at a distance x from B.

Let the deflection at xx is y.

Bending moment due to critical load P at xx,

$$M = EI\frac{d^2y}{dx^2} = P(a-y)$$

$$EI\frac{d^2y}{dx^2} = Pa - py$$

$$\frac{d^2y}{dx^2} + \frac{py}{EI} = \frac{pq}{EI}$$

The solution of the above differential equation is,

$y = A\cos\left(x.\sqrt{\frac{P}{EI}}\right) + B\sin\left(x.\sqrt{\frac{P}{EI}}\right) + a$ Where A and B are constants of integration.

At B, x = 0, y = 0

$\therefore$ From (i), A = 0

Differentiating the equation (I w.r. to x

$$\frac{dy}{dx} = -A\sqrt{\frac{P}{EI}} Sin\left(x.\sqrt{\frac{P}{EI}}\right) + B\sqrt{\frac{P}{EI}} Cos\left(x.\sqrt{\frac{P}{EI}}\right)$$

At the fixed end B, x = 0 and $\frac{dy}{dx} = 0$

$$0 = B\sqrt{\frac{P}{EI}}$$

$$As\sqrt{\frac{P}{EI}} \neq 0 \qquad (\therefore p \neq 0)$$

Substitute A = -a and B = 0 in equation (i) we get,

$$y = -a\cos\left(x.\sqrt{\frac{P}{EI}}\right) + a$$

$$y = a\left[1 - \cos\left(x..\sqrt{\frac{P}{EI}}\right)\right] \qquad \text{(ii)}$$

At the free end A, x = l, y = a, substitute these values in equation (ii)

$$a = a\left[1 - \cos\left(1..\sqrt{\frac{P}{EI}}\right)\right]$$

$$\cos\left(1..\sqrt{\frac{P}{EI}}\right) = 0$$

$$1\sqrt{\frac{P}{EI}} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

Now taking the least significant value,

$$1\sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

$$1^2 \frac{P}{EI} = \frac{\pi^2}{4}$$

$$P = \frac{\pi^2 EI}{4l^2}$$

$\therefore$ The crippling load for the columns with one end fixed and other end free.

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

19. A steel column is of length 8 m and diameter 600 mm with both ends hinged. Determine the crippling load by Euler's formula. Take E =2.1 x 10^5 N/mm^2

Solution:

Given,

Actual length of the column, l = 8m = 8000 mm

Diameter of the column d= 600 mm

E = 2.1 x 10^5 N/mm^2

$$I = \frac{\pi}{64}(d)^4$$

$$= \frac{\pi}{64}(600)^4$$

$$I = 6.36 \times 10^9 mm^4$$

Since the column is hinged at the both ends,

$\therefore$ Equivalent length L =l

$\therefore$ Euler's crippling load,

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2 \times 2.1 \times 10^5 \times 6.36 \times 10^9}{(8000)^2}$$

$= 2.06 \times 10^8$ N

20. A mild steel tube 4m long, 3cm internal diameter and 4mm thick is used as a strut with both ends hinged. Find the collapsing load, what will be the crippling load if

i. Both ends are built in?
ii. One end is built –in and one end is free?

Solution:

Given:

Actual length of the mild steel tube, l = 4m = 400 cm
Internal diameter of the tube, d = 3 cm
Thickness of the tube, t = 4mm = 0.4cm.

$\therefore$ External diameter of the tube, D = d + 2t
= 3+2(0.4)
= 3.8 cm.

Assuming E for steel = 2×10^6 Kg/cm^2

M.O.I of the column section,

$$I = \frac{\pi}{64}\left[D^4 - d^4\right]$$

$$= \frac{\pi}{64}\left[(3.8)^4 - (3)^2\right]$$

I = 6.26 cm^4

i. Since the both ends of the tube are hinged, the effective length of the column when both ends are hinged.

L = l = 400 cm

$\therefore$ Euler's crippling load $\Rightarrow$ $P_{cr} = \dfrac{\pi^2 EI}{L^2}$

$$= \frac{\pi^2 \times 2 \times 10^6 \times 6.26}{(400)^2}$$

$$P_{cr} = 772.30 Kg.$$

$\therefore$ The required collapsed load = 772.30 Kg.

ii. When both ends of the column are built –in , then effective length of the column,

$$L = \frac{l}{2} = \frac{400}{2} = 200cm$$

$\therefore$ Euler's crippling load,

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2 \times 10^6 \times 6.26}{(200)^2}$$

P_{cr} = 3089.19 Kg.

iii. When one end of the column is built in and the other end is free, effective length of the column, L = 2l

= 2 x 400

= 800 cm

$\therefore$ Euler's crippling load,

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2 \times 10^6 \times 6.26}{(800)^2}$$

P_{cr} = 193.07 Kg.

21. A column having a T section with a flange 120 mm x 16 mm and web 150 mm x 16 mm is 3m long. Assuming the column to be hinged at both ends, find the crippling load by using Euler's formula. E = 2 x 10^6 Kg/cm^2.

Solution:

Given:

Flange width	=	120 mm = 12 cm
Flange thickness	=	16 mm = 1.6 cm
Length of the web	=	150 mm = 15cm
Width of the web	=	16mm = 1.6cm

E = 2 10^6 Kg/cm^2

Length of the column, l = 3m = 300 cm.

Since the column is hinged at both ends, effective length of the column.

L = l = 300 cm.

From the fig. Y-Y is the axis of symmetry. $\therefore$ The C.G of the whole section lies on Y-Y axis.

Let the distance of the C.G from the 16 mm topmost fiber of the section = $\overline{Y}$

$$\therefore \overline{Y} = \frac{12 \times 1.6 \times \frac{1.6}{2} + 15 \times 1.6\left(1.6 + \frac{15}{2}\right)}{12 \times 1.6 + 15 \times 1.6}$$

$$\overline{Y} = 5.41cm$$

Distance of C.G from bottom fibre = (15+1.6) - 5.41
= 11.19cm

Now M.O.I of the whole section about X-X axis.

$$I_{XX} = \left[\frac{12 \times (1.6)^3}{12} + (12 \times 1.6)\left(5.41 - \frac{1.6}{2}\right)^2\right] + \left[\frac{1.6 \times (15)^3}{12} + (1.6 \times 15)\left(11.19 - \frac{15}{2}\right)^2\right]$$

$$I_{XX} = 1188.92cm^4$$

M.I of the whole section about Y-Y axis

$$I_{yy} = \frac{1.6 \times (12)^3}{12} + \frac{15 \times (106)^3}{12} = 235.52cm^4$$

$$\therefore \ I_{\min} = 235.52cm^4$$

$\therefore$ Euler's Crippling load,

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2 \times 10^6 \times 235.52}{(300)^2} \quad ; \qquad P_{cr} = 51655.32Kg.$$

22. A steel bar of solid circular cross-section is 50 mm in diameter. The bar is pinned at both ends and subjected to axial compression. If the limit of proportionality of the material is 210 MPa and E = 200 GPa, determine the m minimum length to which Euler's formula is valid. Also determine the value of Euler's buckling load if the column has this minimum length.

Solution:

Given,

Dia of solid circular cross-section, d = 50 mm

Stress at proportional limit, f = 210 Mpa
= 210 N/mm^2

Young's Modulus, E = 200 GPa = 200 x 10 ^{3} N/mm^2

Area of cross –section, $A = \frac{\pi}{4} \times (50)^2 = 1963.49 mm^2$

Least moment of inertia of the column section,

$$I = \frac{\pi}{64} \times (50)^4 = 3.6.79 \times 10^3 mm^4$$

Least radius of gyration,

$$k^2 = \frac{I}{A} = \frac{306.79 \times 10^3}{1963.49} \times (50)^4 = 156.25 mm^2$$

$\because$ The bar is pinned at both ends,

$\therefore$ Effective length, L = Actual length, l

$\therefore$ Euler's buckling load,

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/K)^2}$$

For Euler's formula to be valid, value of its minimum effective length L may be found out by equating the buckling stress to f

$$\frac{\pi^2 E}{\left(\frac{L}{K}\right)^2} = 210$$

$$L^2 = \frac{\pi^2 E \times k^2}{210} \qquad L^2 = \frac{\pi^2 \times 2 \times 10^5 \times 156.25}{210}$$

L = 1211.89 mm = 1212 mm = 1.212 m

$\therefore$ The required minimum actual length l =L = 1.212 m

For this value of minimum length,

Euler's buckling load $= \dfrac{\pi^2 EI}{L^2}$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 306.75 \times 10^3}{(1212)^2}$$

= 412254 N = 412.254 KN

Result:

Minimum actual length l = L = 1.212 m
Euler's buckling Load =412.254 KN

23. Explain Rankine's Formula and Derive the Rankine's formula for both short and long column.

Solution:

Rankine's Formula:

Euler's formula gives correct results only for long columns, which fail mainly due to buckling. Whereas Rankine's devised an empirical formula base don practical experiments for determining the crippling or critical load which is applicable to all columns irrespective of whether they a short or long.

If P is the crippling load by Rankine's formula.

P_c is the crushing load of the column material
P_E is the crippling load by Euler's formula.
Then the Empirical formula devised by Rankine known as Rankine's formula stand as:

$$\frac{1}{P} = \frac{1}{P_e} + \frac{1}{P_E}$$

For a short column, if the effective length is small, the value of P_E will be very high and the value of $\dfrac{1}{P_E}$ will be very small as compared to $\dfrac{1}{P_C}$ and is negligible.

For the short column, (i.e) $\boxed{\frac{1}{P} = \frac{1}{P_c}}$ $P = P_C$

Thus for the short column, value of crippling load by Rankine is more or less equal to the value of crushing load:

For long column having higher effective length, the value of P_E is small and $\frac{1}{P_E}$ will be large enough in comparison to $\frac{1}{P_C}$. So $\frac{1}{P_C}$ is ignored.

$\therefore$ For the long column, $\frac{1}{P_C} \approx \frac{1}{P_E}$ (i.e) $p \approx P_E$

Thus for the long column the value of crippling load by Rankine is more or less equal to the value of crippling load by Euler.

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

$$\frac{1}{P} = \frac{P_E \times P_c}{P_c \times P_E}$$

$$p = \frac{P_c \times P_E}{P_E \times P_c} \; ; \; p = \frac{P_c}{1 + \frac{P_c}{P_E}}$$

Substitute the value of $P_c = f_c A$ and $P_E = \frac{\pi^2 EI}{L^2}$ in the above equation,

$$p = \frac{f_c \times A}{1 + \frac{f_c \times A}{\pi^2 EI / L^2}}$$

Where,

f_c = Ultimate crushing stress of the column material.

A = Cross-sectional are of the column

L = Effective length of the column

I = Ak^2

Where k = Least radius of gyration.

$$p = \frac{f_c \times A}{1 + \frac{f_c \times A}{\pi^2 EI / L^2}} = \frac{f_c \times A}{1 + \frac{f_c \times A \times L^2}{\pi^2 EAk^2}}$$

$$p = \frac{f_c \times A}{1 + \alpha \left(\frac{L}{K} \right)^2}$$

where α = Rankine's constant $= \frac{f_c}{\pi^2 E}$

$$P = \frac{Crushing\ Load}{1 + \alpha (L/k)^2}$$

When Rankine's constant is not given then find

$$\alpha = \frac{f_c}{\pi^2 E}$$

The following table shows the value of f_c and α for different materials.

Material	f_c N/mm^2	$\alpha = \frac{f_c}{\pi^2 E}$
Wrought iron	250	$\frac{1}{9000}$
Cast iron	550	$\frac{1}{1600}$
Mild steel	320	$\frac{1}{7500}$
Timber	50	$\frac{1}{750}$

24. A rolled steel joist ISMB 300 is to be used a column of 3 meters length with both ends fixed. Find the safe axial load on the column. Take factor of safety 3, f_c = 320 N/mm^2 and $\alpha = \frac{1}{7500}$. Properties of the column section.

Area = 5626 mm^2, I_{XX} = 8.603 x 10^7 mm^4

I_{yy} =4.539 x 10^7 mm^4

Solution:

Given:

Length of the column, l = 3m = 3000 mm

Factor of safety = 3

f_c = 320 N/mm^2, $\alpha = \frac{1}{7500}$

Area, A = 5626 mm^2

I_{XX} = 8.603 x 10^7 mm^4
I_{yy} =4.539 x 10^7 mm^4

The column is fixed at both the ends,

$\therefore$ Effective length, $L = \frac{l}{2} = \frac{3000}{2} = 1500mm$

Since I_{yy} is less then I_{xx}, $\therefore$ The column section,

$$I = I_{\min} = I_{yy} = 4.539 \times 10^7 \, mm^4$$

$\therefore$ Least radius of gyration of the column section,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{4.539 \times 10^7}{5626}} = 89.82mm$$

Crippling load as given by Rakine's formula,

$$p_{cr} = \frac{f_c \times A}{1 + \alpha\left(\frac{L}{K}\right)^2} = \frac{320 \times 5626}{1 + \frac{1}{7500}\left(\frac{1500}{89.82}\right)^2}$$

P_{cr} = 1343522.38 N

Allowing factor of safety 3,

Safe load = $\frac{Crippling \quad Load}{Factor \ of \ safety}$

$$= \frac{1343522.38}{3} = 447840.79N$$

Result:

i. Crippling Load (P_{cr}) = 1343522.38 N
ii. Safe load =447840.79N

25. A built up column consisting of rolled steel beam ISWB 300 with two plates 200 mm x 10 mm connected at the top and bottom flanges. Calculate the safe load the column carry, if the length is 3m and both ends are fixed. Take factor of safety 3 f_c = 320 N/mm² and $\alpha = \frac{1}{7500}$

Take properties of joist: A = 6133 mm²

I_{XX} = 9821.6 x 10⁴ mm⁴ ; I_{yy} = 990.1 x 10⁴ mm⁴

Solution:

Given:

Length of the built up column, l = 3m = 3000 mm
Factor of safety = 3

f_c =320 N/mm²

$$\alpha = \frac{1}{7500}$$

Sectional area of the built up column,

$$A = 6133 + 2(200 \times 10) = 10133 mm^2$$

Moment of inertia of the built up column section abut xx axis,

$$I_{XX} = 9821.6 \times 10^4 + 2\left[\frac{200 \times 10^3}{12} + (200 \times 10)(155)^2\right]$$

= 1.94 x 10^8 mm⁴

Moment of inertia of the built up column section abut YY axis,

$$I_{YY} = 990.1 \times 10^4 + 2\left(\frac{10 \times 200^3}{12}\right)$$

= 0.23 x 10^8 mm⁴

Since I_{yy} is less than I_{xx} , The column will tend to buckle about Y-Y axis.

Least moment of inertia of the column section,

$$I = I_{\min} = I_{YY} = 0.23 \times 10^8 mm^4$$

The column is fixed at both ends.

$\therefore$ Effective length,

$$L = \frac{l}{2} = \frac{3000}{2} = 1500mm$$

$\therefore$ Least radius of gyration o the column section,

$$K = \sqrt{\frac{J}{A}} = \sqrt{\frac{0.23\times10^{8}}{10133}} = 47.64mm$$

Crippling load as given by Rankine's formula,

$$p_{cr} = \frac{f_c \times A}{1+\alpha\left(\frac{L}{K}\right)^2} = \frac{320\times10133}{1+\frac{1}{7500}\left(\frac{1500}{47.64}\right)^2}$$

= 2864023.3 N

Safe load = $\frac{\text{Crippling load}}{\text{Factor of safety}}$ $= \frac{2864023.3}{3} = 954674.43N$

Result:

i. Crippling load	=	2864023.3 N
ii. Safe load	=	954674.43 N

26. Derive Rankine's and Euler formula for long columns under long columns under Eccentric Loading? (AUC Nov/Dec 2010)

i. Rankine's formula:

Consider a short column subjected to an eccentric load P with an eccentricity e form the axis.

Maximum stress = Direct Stress + Bending stress

$$f_c = \frac{P}{A} + \frac{M}{Z} \qquad Z = \frac{I}{y}$$

$$= \frac{P}{A} + \frac{p.e.y_c}{Ak^2} \qquad I = Ak^2$$

$$k = \sqrt{\frac{I}{A}}$$

where

A = Sectional are of the column
Z = Sectional modulus of the column
y_c = Distance of extreme fibre from N.A
k = Least radius of gyration.

$$f_c = \frac{P}{A}\left(1 + \frac{ey_c}{k^2}\right)$$

$\therefore$

$$\boxed{\text{Eccentric load, } P = \frac{f_c \times A}{1 + \dfrac{ey_c}{k^2}}}$$

Where $\left(1 + \dfrac{ey_c}{k^2}\right)$ is the reduction factor for eccentricity of loading.

For long column, loaded with axial loading, the crippling load,

$$P = \frac{f_c \times A}{1 + \alpha\left(\dfrac{L}{K}\right)^2}$$

Where $\left(1 + \alpha\left(\dfrac{L}{K}\right)^2\right)$ is the reduction factor for buckling of long column.

Hence for a long column loaded with eccentric loading, the safe load,

$$\boxed{P = \frac{f_c \times A}{\left(1 + \dfrac{ey_c}{K^2}\right)\left[1 + \alpha\left(\dfrac{L}{K}\right)^2\right]}}$$

ii. Euler’s formula

Maximum stress n the column = Direct stress + Bending stress

$$= \frac{P}{A} + \frac{P \times e \sec\sqrt{P/EI}\,\dfrac{l}{2}}{Z}$$

Hence, the maximum stress induced in the column having both ends hinged and an eccentricity of e is $\frac{P}{A}+\frac{Pe}{Z}\sec\left(\sqrt{P/EI}\,\frac{l}{2}\right)$

The maximum stress induced in the column with other end conditions are determined by changing the length in terms of effective length.

27. A column of circular section has 150 mm dia and 3m length. Both ends of the column are fixed. The column carries a load of 100 KN at an eccentricity of 15 mm from the geometrical axis of the column. Find the maximum compressive stress in the column section. Find also the maximum permissible eccentricity to avoid tension in the column section. E = 1 x 10^5 N/mm^2

Solution:

Given,

Diameter of the column,	D	=	150 mm
Actual length of the column, l		=	3m = 3000 mm
Load on the column,	P	=	100 KN = 1000 x 10^3 N
	E	=	1 x 10^5 N/mm^2
Eccentricity,	e	=	15 mm

Area of the column section $A=\frac{\pi\times D^2}{4}$

$$=\frac{\pi}{4}(150)^2$$

= 17671 mm^2

Moment of inertia of the column section N.A.,

$$I=\frac{\pi}{64}\times D^4=\frac{\pi}{64}\times(150)^4$$

$$= 24.85 \text{ x } 10^6 \text{ mm}^4$$

Section modulus,

$$Z=\frac{I}{y}=\frac{I}{D/2}$$

$$=\frac{24.85\times10^6}{\frac{150}{2}}=331339mm^3$$

Both the ends of the column 2 are fixed.

Effective length of the column, $L=\frac{l}{2}=\frac{3000}{2}=1500mm$

Now, the angle

$$\sqrt{P/EI} \times \frac{L}{2} = \sqrt{\frac{100 \times 10^3}{1 \times 10^5 \times 24.85 \times 10^6}} \times \frac{1500}{2}$$

$$= 0.1504 \text{ rad} = 8.61^o$$

Maximum compressive stress,

$$= \frac{P}{A} + \frac{P \times e}{Z}\left(\sec \sqrt{P/EI}\,\frac{L}{2}\right)$$

$$= \frac{100 \times 10^3}{17671} + \frac{100 \times 10^3 \times 15 \times \sec 8.61^o}{331339}$$

$$= 10.22 \text{ N/mm}^2$$

To avoid tension we know,

$$\frac{P}{A} = \frac{M}{Z}$$

$$\Rightarrow \quad \frac{P}{A} = \frac{p \times e \times \sec .8.61^o}{Z}$$

$$\frac{100 \times 10^3}{17671} = \frac{100 \times 10^3 \times e \times \sec .8.61^o}{331339}$$

$$e = 18.50 \text{ mm}$$

Result:

i. Maximum compressive stress $= 10.22$ N/mm^2

ii. Maximum eccentricity $= 18.50$ mm

28. State the assumptions and derive Lame's Theory?

1. The assumptions involved in Lame's Theory.

i. The material of the shell is homogenous and isotropic

ii. Plane sections normal to the longitudinal axis of the cylinder remain plane after the application of internal pressure.

iii. All the fibres of the material expand (or) contract independently without being constrained by their adjacent fibres.

2 **Derivation of Lame's Theory**

Consider a thick cylinder

Let

r_c = Inner radius of the cylinder

r_0 = Outer radius of the cylinder

P_i = Internal radial pressure

P_o = External radial pressure

L = Length of the cylinder

f_2 = Longitudinal stress.

Lame's Equation:

$$f_x = p_x + 2_a$$

$$P_x = \frac{b}{x^2} -_a$$

$$\therefore \quad f_x = \frac{b}{x^2} - a + 2a$$

$$f_x = \frac{b}{x^2} + a$$

where

f_x = hoop stress induced in the ring.

p_x = Internal radial pressure in the fig.

$P_x + dP_x$ = External radial pressure in the ring.

The values of the two constants a and to b are found out using the following boundary conditions:

i. Since the internal radial pressure is $P_{i,}$

At $x = r_i$, $P_x = P_i$

ii. Since the external radial pressure is $P^0{}_,$

At $x = r_0$, $P_x = P_0$

29. A thick steel cylinder having an internal diameter of 100 mm an external diameter of 200 mm is subjected to an internal pressure of 55 M pa and an external pressure of 7 Mpa. Find the maximum hoop stress.

Solution:

Given,

Inner radius of the cylinder, $r_i = \frac{100}{2} = 50mm$

Outer radius of the cylinder, $r_o = \frac{200}{2} = 100mm$

Internal pressure, P_i = 55 Mpa

External pressure, P_0 = 7 Mpa

In the hoop stress and radial stress in the cylinder at a distance of x from the centre is f_x and p_x respectively, using Lame's equations,

$$f_x = \frac{b}{x^2} + a \quad \text{(i)}$$

$$P_x = \frac{b}{x^2} - a \quad \text{(ii)}$$

where a and b are constants,

Now by equation, at x = 50 mm, P_x = 55 MPa (Boundary condition)

Using these boundary condition in equation (ii)

$$P_x = \frac{b}{x^2} - a$$

$$55 = \frac{b}{(50)^2} - a \quad \text{(iii)}$$

Then x = 100 mm, p_x = 7 Mpa

Using these boundary condition is equation (ii)

$$7 = \frac{b}{100^2} - a \quad \text{(iv)}$$

Solving (iii) & (iv)

$$b/(100)^2 - a = 7$$

$$b/(50)^2 - a = 55$$

(-) (+)

$$-\frac{3b}{10000} = -48$$

$$\boxed{\begin{array}{l} b = 160000 \\ a = 9 \end{array}}$$

Substitute a & b in equation (i)

$$f_x = \frac{160000}{x^2} + 9$$

The value of f_x is maximum when x is minimum

Thus f_x is maximum for $x = r_i = 50$ mm

$$\therefore \text{ Maximum hoop stress } = \frac{160000}{(50)^2} + 9$$

= 73 Mpa (tensile)

Result:

Maximum hoop stress = 73 MPa (tensile)

30. A cast iron pipe has 200 mm internal diameter and 50 mm metal thickness. It carries water under a pressure of 5 N/mm^2. Find the maximum and minimum intensities of circumferential stress. Also sketch the distribution of circumferential stress and radial stress across the section.

Solution:

Given:

Internal diameter,	d_i	=	200 mm
Wall thickness,	t	=	50 mm
Internal pressure,	P_i	=	5 N/mm^2
External pressure,	P_0	=	0.

$\therefore$ Internal radius $r_i = \frac{di}{2} = \frac{200}{2} = 100mm$

External radius $r_0 = r_i + t = 100 + 50 = 150mm$

Let f_x and P_x be the circumferential stress and radial stress at a distance of x from the centre of the pipe respectively.

$\therefore$ Using Lame's equations,

$$f_x = \frac{b}{x^2} + a \qquad \text{(i)}$$

$$p_x = \frac{b}{x^2} - a \qquad \text{(ii)}$$

where, a & b are arbitrary constants.

Now at x = 100 mm, $P_x = 5$ N/mm^2
At x = 150 mm, $P_x = 0$

Using boundary condition is (ii)

$$5 = \frac{b}{(100)^2} - a \qquad \text{(ii)}$$

$$0 = \frac{b}{(150)^2} - a \qquad \text{(iv)}$$

By solving (iii) & (iv) a = 4 ; b = 90000

$$\therefore \; f_x = \frac{90000}{x^2} + 4, \quad P_x = \frac{90000}{x^2} - 4,$$

Putting x = 100 mm, maxi circumferential stress.

$$f_x = \frac{90000}{(100)^2} + 4 = 13N/mm^2 (tensile)$$

Putting x = 150 mm, mini circumferential stress.

$$f_x = \frac{90000}{(150)^2} + 4 = 8N/mm^2 (tensile)$$

31. Explain the stresses in compound thick cylinders.

Solution:
Consider a compound thick cylinder as shown in fig.
Let,

r_1 = Inner radius of the compound cylinder

r_2 = Radius at the junction of the two cylinders

r_3 = Outer radius of the compound cylinder

When one cylinder is shrunk over the other, thinner cylinder is under compression and the outer cylinder is under tension. Due to fluid pressure inside the cylinder, hoop stress will develop. The resultant hoop stress in the compound stress is that algebraic sum of the hoop stress due to initial shrinkage and that due to fluid pressure.

a. Stresses due to initial shrinkage:
Applying Lame’s Equations for the outer cylinder,

$$P_x = \frac{b_1}{x^2} - a_1$$

$$f_x = \frac{b_1}{x^2} + a_1$$

At $x = r_3$, $P_x = 0$ and at $x = r_2$, $p_x = p$

Applying Lame's Equations for the inner cylinder

$$P_x = \frac{b_2}{x^2} - a_2$$

$$f_x = \frac{b_2}{x^2} + a_2$$

At $x = r_2$, $P_x = p$ and at $x = r_3$, $p_x = 0$

b. Stresses due to Internal fluid pressure.

To find the stress in the compound cylinder due to internal fluid pressure alone, the inner and outer cylinders will be considered together as one thick shell. Now applying Lame's Equation,

$$P_x = \frac{B}{x^2} - A$$

$$f_x = \frac{B}{x^2} + A$$

At $x = r_1$, $P_x = p_f$ (P_f being the internal fluid pressure)

At $x = r_3$, $p_x = 0$

The resultant hoop stress is the algebraic sum of the hoop stress due to shrinking and due internal fluid pressure.

32. A compound cylinder is composed of a tube of 250 mm internal diameter at 25 mm wall thickness. It is shrunk on to a tube of 200 mm internal diameter. The radial pressure at the junction is 8 N/mm$^{2.}$ Find the variation of hoop stress across the wall of the compound cylinder, if it is under an internal fluid pressure of 60 N/mm^2

Solution:

Given:

Internal diameter of the outer tube, d_1 = 250 mm

Wall thickness of the outer tuber , t = 25 mm

Internal diameter of the inner tube , d_2 = 200 mm

Radial pressure at the junction P = 8 N/mm^2

Internal fluid pressure within the cylinder P_f = 60 N/mm^2

$\therefore$ External radius of the compound cylinder,

$$r_2 = \frac{d_1 + 2t}{2}$$

$$= \frac{1}{2}(250 + 2 \times 25) = 150\,mm$$

Internal radius of the compound cylinder,

$$r_1 = \frac{d_2}{2} = \frac{200}{2} = 100\,mm$$

Radius at the junction, $r_1 = \frac{d_1}{2} = \frac{250}{2} = 125\,mm$

Let the radial stress and hoop stress at a distance of x from the centre of the cylinder be p_x and f_x respectively.

i. Hoop stresses due to shrinking of the outer and inner cylinders before fluid pressure is admitted.

a. Four outer cylinder:

Applying Lame's Equation

$$P_x = \frac{b_1}{x^2} - a_1 \qquad \text{(i)}$$

$$f_x = \frac{b_1}{x^2} + a_1 \qquad \text{(ii)}$$

Where a_1 and b_1 are arbitrary constants for the outer cylinder.

Now at x = 150 mm, $P_x = 0$

X = 125 mm, $P_x = 8\ N/mm^2$

$$\therefore o = \frac{b_1}{(150)^2} - a_1 \qquad \text{(iii)}$$

$$8 = \frac{b_1}{(125)^2} + a_1 \qquad \text{(iv)}$$

Solving equation (iii) & (iv) $a_1 = 18$; $b_1 = 409091$

$$f_x = \frac{409091}{x^2} + 18 \qquad \text{(v)}$$

Putting x = 150 mm in the above equation stress at the outer surface,

$$f_x = \frac{409091}{(150)^2} + 18 = 36 N/mm^2 \text{ (Tensile)}$$

Again putting x = 125 mm in equation (v), stress at junction,

$$f_x = \frac{409091}{(125)^2} + 18 = 44 N/mm^2 \text{ (Tensile)}$$

b). For inner cylinder:

Applying Lame's Equation with usual Notations.

$$P_x = \frac{b_2}{x^2} - a_2 \qquad \text{(iv)}$$

$$f_x = \frac{b_2}{x^2} + a_2 \qquad \text{(v)}$$

Now at x = 125 mm, P_x = 8 N/mm^2

x =100 mm, P_x = 0

$$\therefore 8 = \frac{b_2}{(125)^2} - a_2 \qquad \text{(vi)}$$

$$o = \frac{b_2}{(100)^2} - a_2 \qquad \text{(vii)}$$

By solving (vi) & (vii) a_2 = -22

b_2 = -222222

$$\therefore f_x = \frac{-222222}{(100)^2} - 22 = -44.2 N/mm^2 \text{ (Comp)}$$

$$f_x = \frac{-222222}{(125)^2} - 22 = -36.2 N/mm^2 \text{ (Comp)}$$

iii. Hoop stresses due to internal fluid pressure alone for the compound cylinder:

In this case, the two tubes will be taken as a single thick cylinder. Applying Lame's equations with usual notations.

$$P_x = \frac{B}{x^2} - A \qquad \text{(viii)}$$

$$f_x = \frac{B}{x^2} + A \qquad \text{(ix)}$$

At x = 150 mm, $P_x = 0$
x = 100 mm, $P_x = p_f = 60\ N/mm^2$

∴ From Equation (viii)

$$\therefore O = \frac{B}{(150)^2} - A \qquad \text{(x)}$$

$$60 = \frac{B}{(100)^2} - A \qquad \text{(xi)}$$

By solving (x) & (xi)

A = 133, B = 3 x 10^6

$$\therefore f_x = \frac{3\times10^6}{x^2} + 133$$

Putting x = 150 mm, hoop stress at the outer surface

$$f_x = \frac{3\times10^6}{(150)^2} + 133 = 266\,N/mm^2 \text{ (Tensile)}$$

Again putting x = 125 mm, hoop stress at the junction

$$f_x = \frac{3\times10^6}{(125)^2} + 133 = 325\,N/mm^2 (Tensile)$$

Putting x = 100 mm, hoop stress at the inner surface

$$f_x = \frac{3\times10^6}{(100)^2} + 133 = 433\,N/mm^2 (Tensile)$$

iii. Resultant hoop stress (shrinkage +Fluid pressure):

a. Outer cylinder

Resultant hoop stress at the outer surface = 36 + 266
= 302 N/ mm^2 (Tensile)

Resultant hoop stress at the junction = 44 + 325 = 369 N/mm^2 (tensile)

b. Inner cylinder;

Resultant hoop stress at the inner face = - 44.2 + 433

= 388.8 N/mm^2 (Tensile)

Resultant hoop stress at the junction = - 36.2 + 325

= 288.8 N/mm^2 (Tensile)

33. A column with alone end hinged and the other end fixed has a length of 5m and a hollow circular cross section of outer diameter 100 mm and wall thickness 10 mm. If E = 1.60 x 10^5 N/mm^2 and crushing strength $\sigma_0 = 350 N/mm^2$, Find the load that the column may carry with a factor of safety of 2.5 according to Euler theory and Rankine – Gordon theory. If the column is hinged on both ends, find the safe load according to the two theories. (ACU April/May 2003)

Solution:

Given:

L = 5 m = 5000 mm

Outer diameter	D	=	100 mm
Inner diameter	d = D-2t	=	100 – 2 (10) = 80 mm
Thickness		=	10 mm
	I	=	1.60 x 10^5 N/mm^2
	σ_0	=	$350 N/mm^2$
	f	=	2.5

i. Calculation of load by Euler's Theory:

Column with one end fixed and other end hinged.

$$P = \frac{2\pi^2 EI}{L^2} \qquad L = \frac{l}{\sqrt{2}} = \frac{5000}{\sqrt{2}} = 3536.06\,mm$$

$$P = \frac{2 \times (3.14)^2 \times 1.60 \times 10^5 \times I}{(3536.06)^2}$$

$$I = \frac{\pi}{64}\left(D^4 - d^4\right)$$

$$= \frac{\pi}{64}\left(100^4 - 80^4\right)$$

$$= \frac{\pi}{64}(100000000 - 40960000)$$

I = 28.96 x 10^5 mm^4

$$P = \frac{2 \times (3.14)^2 \times 1.60 \times 10^5 \times 28.96 \times 10^5}{12503716.14}$$

p = 73.074 x 10^3 N

ii. Calculation of load by Rankine-Gordon Theory:

Rankine's Constant $a = \frac{1}{7500}$ (assume the column material is mild steel.)

$$\therefore \; p = \frac{f_c \times A}{1 + a\left(\frac{L}{K}\right)^2}$$

K = lest radius of Gyration

$$= \sqrt{\frac{I}{A}} = \sqrt{\frac{28.96 \times 10^5}{2826}} = 32.01$$

$$A = \frac{\pi}{4}\left(100^2 - 80^2\right)$$

$$= \frac{\pi}{4}(10000 - 6400)$$ $f_c = \sigma_c$

= 2826 mm^2

$$P = \frac{350 \times 28.26}{1 + \frac{1}{7500}\left(\frac{3536.06}{32.01}\right)^2}$$

$$P = \frac{989100}{1.33 \times 10^{-4} \times 12203.036}$$

$$P = 60.94 \times 10^4 \, N$$

iii. Both ends are hinged
Euler's theory

$$P = \frac{\pi^2 EI}{L^2}$$ L = l

$$= \frac{(3.14)^2 \times 1.60 \times 10^5 \times 28.96 \times 10^5}{(5000)^2}$$

P = 18.274 x 10^4 N ; Safe Load = $\frac{18.274 \times 10^4}{2.5}$

$= 73096$ N

Rankine's Theory

$$p = \frac{f_c \times A}{1 + a\left(\frac{L}{K}\right)^2}$$

$$= \frac{350 \times 2826}{1 + \frac{1}{7500}\left(\frac{5000}{32.01}\right)^2}$$

$$= \frac{989100}{1.33 \times 10^{-4} \times 24398.81}$$

Safe load $= \frac{30.480 \times 10^4}{2.5} = 121920$ N

P = 30.480 x 10^4

Result:

i. Euler's Theory

One end fixed & one end hinged P = 73.074 x 10^3 N

Both ends hinged P = 18.274 x 10^4 N

ii. Rankine's Theory

One end fixed & one end hinged P = 60.94 x 10^4 N

Both ends hinged P = 30.480 x 10^4 N

iii. Safe Load

Euler's Theory = 73096 N

Rankine's theory = 121920 N

34. A column is made up of two channel ISJC 200 mm and two 25 cm x 1 cm flange plate as shown in fig. Determine by Rankine's formula the safe load, the column of 6m length, with both ends fixed, can carry with a factor of safety 4. The properties of one channel are A = 17.77 cm^2, I_{xx} = 1,161.2 cm^4 and I_{yy} = 84.2 cm^4. Distance of centroid from back of web = 1.97 cm. Take f_c = 0.32 KN/mm^2 and Rankine's Constant $= \frac{1}{7500}$

(ACU April /May 2003)

Solution:

Given:

Length of the column l = 6 m = 600 mm

Factor of safety = 4

Yield stress, f_c = 0.32 KN/mm^2

Rankine's constant, $a = \frac{1}{7500}$

Area of column,

A = 2 (17.77+25 x 1)

A = 85.54 cm^2

A = 8554 mm^2

Moment of inertia of the column about X-X axis

$$I_{XX} = 2 \times 1{,}161.2 + \left(\frac{25 \times 1^3}{12} + 25 \times 1 \times 10.5^2 \right) \quad = 7839.0 \text{ cm}^4$$

$$I_{YY} = 2\left\{ \frac{1 \times 25^3}{12} + 8.42 + 17.77 \times (5 + 1.97)^2 \right\} = 4{,}499.0 \text{ cm}^4$$

$I_{yy} < I_{XX}$ $\therefore$ The column will tend to buckle in yy-direction

$I = I_{yy} = 4499.0 \text{ cm}^4$

Column is fixed at both the ends

$$L = \frac{l}{2} = \frac{6000}{2} = 3000mm$$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{4499 \times 10^4}{855^4}} = 72.5mm$$

$$P = \frac{f_c . A}{1 + a\left(\frac{K}{L}\right)^2} \qquad = \frac{0.32 \times 8554 . A}{1 + \frac{1}{75000}\left(\frac{3000}{72.5}\right)^2} = 2228 \text{ KN}$$

Safe load of column $= \dfrac{P}{F.O.S}$

$= \dfrac{2228}{4}$ =557 KN

Result:

Safe load = 557 KN