UNIT-IV

CLASSICAL OPTIMISATION THEORY

Unconstrained external problems, Newton – Ralphson method – Equality constraints – Jacobean methods – Lagrangian method – Kuhn – Tucker conditions – Simple problems.

PART-A

1. **Discuss the different types of nonlinear programming problems.**
   - Price elasticity
   - Product-mix problem
   - Graphical nillustration
   - Global and local optimum.

2. **Explain the application areas of nonlinear programming problems.**
   - Transportation problem
   - Product mix problem
   - NP Problems.

3. **State the Lagrangean model.**

The Lagrangian method usually tracks transiently a large amount of particles. The method starts from solving the transient momentum equation for each particle:

$$\frac{d\vec{u}_p}{dt} = F_D\left(\vec{u} - \vec{u}_p\right) + \frac{\vec{g}\left(\rho_p - \rho\right)}{\rho_p} + \vec{F}_a \qquad (4)$$

4. **What is Newton Ralphson method?**

Newton and Joseph Raphson, is a method for finding successively better approximations to the roots (or zeroes) of a real-valued function.

5. **State the equality constraints.**

Consider the *equality constrained convex quadratic* minimization problem:

$$\begin{aligned} \text{minimize} \quad & \frac{1}{2}x^\top Px + q^\top x + r \\ \text{subject to} \quad & Ax = b\,, \end{aligned}$$

where $P \in^{n\times n}, P \succeq 0$ and $A \in \mathbb{R}^{p\times n}$. The optimality conditions are:

$$\begin{cases} Ax^* & = b\,, \\ \nabla f(x^*) + A^\top \lambda^* & = 0\,. \end{cases}$$

6. Define Jacobean method.

4.4 Jacobean methods

Optimization problem

One of the well known method to solve this system of equations is a Newton – Raphson method, which is one of so called Householder's methods in numerical analysis.

For the function of one variable it is based on the fact that for a differentiable function f(x) we have the following approximation:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Similarly, for the system of n functions of n variables:

$$X_{n+1} = X_n - [F'(x_n)]^{-1} F(X_n)$$

$F'(x_n)$,often called Jacobean matrix, is a matrix of first order partial derivatives of all the functions.

7. State the Kuhn-Tucker conditions.

1.Linearity constraint qualification.
2.Linear independence constraint qualification (LICQ):
3.Mangasarian–Fromovitz constraint qualification (MFCQ):
3.Constant rank constraint qualification (CRCQ):
4.Constant positive linear dependence constraint qualification (CPLD):

8. Define nonlinear programming.

Nonlinear programming is the process of solving an optimization problem defined by a system of equalities and inequalities, collectively termed constraints, over a set of unknown real variables, along with an objective function to be maximized or minimized, where some of the constraints or the objective function are nonlinear.

9**. Write the general format of non linear programming**

Let n, m, and p be positive integers. Let X be a subset of Rn, let f, gi, and hj be real-valued functions on X for each i in {1, …, m} and each j in {1, …, p}.

A nonlinear minimization problem is an optimization problem of the form

$$\text{Maximize } f(x_1, x_2, \ldots, x_n),$$

subject to:

$$\begin{aligned} g_1(x_1, x_2, \ldots, x_n) &\leq b_1, \\ &\vdots \\ g_m(x_1, x_2, \ldots, x_n) &\leq b_m, \end{aligned}$$

where each of the constraint functions g_1 through g_m is given. A special case is the linear program that has been treated previously. The obvious association for this case is

$$f(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} c_j x_j,$$

10.Write the format of Newton Ralphson method.

The Newton Raphson method is for solving equations of the form $f(x) = 0$. We make an initial guess for the root we are trying to find, and we call this initial guess x_0. The sequence $x_0, x_1, x_2, x_3, \ldots$ generated in the manner described below should converge to the exact root.
To implement it analytically we need a formula for each approximation in terms of the previous one, i.e. we need x_{n+1} in terms of x_n.
The equation of the tangent line to the graph $y = f(x)$ at the point $(x_0, f(x_0))$ is

$$y - f(x_0) = f'(x_0)(x - x_0)$$

The tangent line intersects the x-axis when $y = 0$ and $x = x_1$, so

$$-f(x_0) = f'(x_0)(x_1 - x_0)$$

Solving this for x_1 gives

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

and, more generally,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad (1)$$

11. What re the optimisation problems?

- Constrained optimisation problems
- Un Constrained optimisation problems.

PART-B

1. Solve the following non linear programming problem using Langrangean multipliers method.
   Minimize $Z=4X_1^2+2X_2^2+X_3^2-4X_1X_2$
   Subject to
   $X_1+X_2+X_3=15$
   $2X_1-X_2+2X_3=20$
   X_1, X_2 AND $X_3 \geq 0$

2. Solve the following non linear programming problem using Kuhn-Tucker conditions.
   Maximize $Z=8X_1+10X_2- X_1^2-X_2^2$
   Subject to
   $3X_1+2X_2 \leq 6$
   X_1 and $X_2 \geq 0$.

3. State and explain the Lagrangean method and steps involved in it with an example.

4. Explain the Kuhn-Tucker method and steps involved in it with an example.

5. Explain the Newton-Raphson method in detail and justify how it is used to solve the non linear equations.

6. What is Jacobian method? Explain the steps how Jacobian matrix is generated?