**UNIT I -LINEAR PROGRAMMING 9**

Principal components of decision problem – Modeling phases – LP Formulation and graphic solution –Resource allocation problems – Simplex method – Sensitivity analysis.

**1. What is linear programming?**

Linear programming is a technique used for determining optimum utilization of limited resources to meet out the given objectives. The objective is to maximize the profit or minimize the resources (men, machine, materials and money)

**2. Write the general mathematical formulation of LPP.**

1. Objective function

Max or Min $Z = C_1x_1 + C_2x_2 + \ldots.. + C_nx_n$

2. Subject to the constraints

$a_{11}x_1+a_{12}x_2+\ldots\ldots\ldots + a_{1n}x_n\ (\le=\ge) b_1$

$a_{21}x_1+a_{22}x_2+\ldots\ldots\ldots + a_{2n}x_n\ (\le=\ge) b_2$

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$a_{m1}x_1+a_{m2}x_2+\ldots\ldots\ldots + a_{mn}x_n\ (\le=\ge) b_m$

3. Non-negative constraints

$x_1, x_2, \ldots. x_m \ge 0$

**3. What are the characteristic of LPP?**

- There must be a well defined objective function.
- There must be alternative course of action to choose.
- Both the objective functions and the constraints must be linear equation or inequalities.

**4. What are the characteristic of standard form of LPP?**

- The objective function is of maximization type.
- All the constraint equation must be of equal type by adding slack or surplus variables
- RHS of the constraint equation must be positive type
- All the decision variables are of positive type

**5. What are the characteristics of canonical form of LPP? (NOV '07)**

In canonical form, if the objective function is of maximization type, then all constraints are of ≤ type. Similarly if the objective function is of minimization type, then all constraints are of ≥ type. But non-negative constraints are ≥type for both cases.

**6. A firm manufactures two types of products A and B and sells them at profit of Rs 2 on type A and Rs 3 on type B. Each product is processed on two machines M1 and M2.Type A requires 1 minute of processing time on M1 and 2 minutes on M2 Type B requires 1 minute of processing time on M1 and 1 minute on M2. Machine M1 is available for not more than 6 hours 40 minutes while machine M2 is available for 10 hours during any working day. Formulate the problem as a LPP so as to maximize the profit. (MAY '07)**

Maximize $z = 2x_1 + 3x_2$

Subject tot the constraints:

$x_1 + x_2 \le 400$

$2x_1 + x_2 \le 600$

$x_1, x_2 \ge 0$

**7. A company sells two different products A and B , making a profit of Rs.40 and Rs. 30 per unit on them,respectively.They are produced in a common production process and are sold in two different markets, the production process has a total capacity of 30,000 man-hours. It takes three hours to produce a unit of A and one hour to produce a unit of B. The market has been surveyed and company official feel that the maximum number of units of A that can be sold is 8,000 units and that of B is 12,000 units. Subject to these limitations, products**

**can be sold in any combination. Formulate the problem as a LPP so as to maximize the profit**

Maximize $z = 40x_1 + 30x_2$

Subject tot the constraints:

$3x_1 + x_2 \leq 30,000$

$x_1 \leq 8000$

$x_2 \leq 12000$

$x_1, x_2 \geq 0$

**8. What is feasibility region? (MAY ’08)**

Collections of all feasible solutions are called a feasible set or region of an optimization model. Or A region in which all the constraints are satisfied is called feasible region**.**

**9. What is feasibility region in an LP problem? Is ti necessary that it should always be a convex set?**

A region in which all the constraints are satisfied is called feasible region. The feasible region of an LPP is always convex set**.**

**10. Define solution**

A set of variables x1,x2….xn which satisfies the constraints of LPP is called a solution.

**11. Define feasible solution? (MAY ’07)**

Any solution to a LPP which satisfies the non negativity restrictions of LPP’s called the feasible solution

**12. Define optimal solution of LPP. (MAY ’09)**

Any feasible solution which optimizes the objective function of the LPP’s called the optimal solution

**13. State the applications of linear programming**

- Work scheduling
- Production planning & production process
- Capital budgeting
- Financial planning
- Blending
- Farm planning
- Distribution
- Multi-period decision problem
  - Inventory model
  - Financial model
  - Work scheduling

**14. State the Limitations of LP.**

- LP treats all functional relations as linear
- LP does not take into account the effect of time and uncertainty
- No guarantee for integer solution. Rounding off may not feasible or optimal solution.
- Deals with single objective, while in real life the situation may be difficult.

**15. What do you understand by redundant constraints?**

In a given LPP any constraint does not affect the feasible region or solution space then the constraint is said to be a redundant constraint**.**

**16. Define Unbounded solution?**

If the feasible solution region does not have a bounded area the maximum value of Z occurs at infinity. Hence the LPP is said to have unbounded solution**.**

**17. Define Multiple Optimal solution?**

A LPP having more than one optimal solution is said to have alternative or multiple optimal solutions.

**18. What is slack variable?**

If the constraint as general LPP be <= type then a non negative variable is introduced to convert the inequalities into equalities are called slack variables. The values of these variables are interpretedas the amount of unused resources.

**19. What are surplus variables?**

If the constraint as general LPP be >= type then a non negative is introduced to convert the inequalities into equalities are called the surplus variables.

**20. Define Basic solution?**

Given a system of m linear equations with n variables(m<n).The solution obtained by setting (n-m) variables equal to zero and solving for the remaining m variables is called a basic solution.

**21. Define non Degenerate Basic feasible solution?**

The basic solution is said to be a non degenerate basic solution if None of the basic variables is zero.

**22. Define degenerate basic solution?**

A basic solution is said to be a degenerate basic solution if one or more of the basic variables are zero.

**23. What is the function of minimum ratio?**

- To determine the basic variable to leave
- To determine the maximum increase in basic variable
- To maintain the feasibility of following solution

**24. From the optimum simplex table how do you identify that LPP has unbounded solution?**

To find the leaving variables the ratio is computed. The ratio is <=0 then there is an unbounded solution to the given LPP.

**25. From the optimum simplex table how do you identify that the LPP has no solution?**

If atleast one artificial variable appears in the basis at zero level with a +ve value in the Xb column and the optimality condition is satisfied
then the original problem has no feasible solution.

**26. How do you identify that LPP has no solution in a two phase method?**

If all $Zj – Cj \leq 0$ & then atleast one artificial variable appears in the optimum basis at non zero level the LPP does not possess any solution.

**27. What do you understand by degeneracy?**

The concept of obtaining a degenerate basic feasible solution in LPP is known as degeneracy. This may occur in the initial stage when atleast one basic variable is zero in the initial basic feasible solution**.**

**28. Write the standard form of LPP in the matrix notation?**

In matrix notation the canonical form of LPP can be expressed as

Maximize Z = CX(obj fn.)
Sub to AX <= b(constraints) and X >= 0 (non negative restrictions)
Where C = (C1,C2,…..Cn),

A = a11 a12 ….. a1n , X = x1 , b = b1
a21 a22….. a2n x2 b2
. . .

. . .
am1 am2…. amn xn bn

**29. Define basic variable and non-basic variable in linear programming.**

A basic solution to the set of constraints is a solution obtained by setting any n variables equal to zero and solving for remaining m variables not equal to zero. Such m variables are called basic variables and remaining n zero variables are called non-basic variables.

**30.Solve the following LP problem by graphical method. (MAY '08)**

Maximize $z = 6x_1 + 4x_2$ Subject tot the constraints:

$x_1 + x_2 \leq 5$

$x_2 \geq 8$

$x_1, x_2 \geq 0$

**31. Define unrestricted variable and artificial variable. (NOV '07)**

- Unrestricted Variable :A variable is unrestricted if it is allowed to take on positive, negative or zero values
- Artificial variable :One type of variable introduced in a linear program model in order to find an initial basic feasible solution; an artificial variable is used for equality constraints and for greater-than or equal inequality constraints

## **PART – B**

1. Use Two – Phase simplex method to solve the following LPP.

Maximize Z = 3X1 + 2 X2

Subject to the constraints

2 X1 + X2 ≤ 2

3 X1 + 4 X2 ≥ 12

X1, X2 ≥ 0.

2. Use Big-M method to solve the following LPP.

Maximize Z = 3X1 - X2

Subject to the constraints

2 X1 + X2 ≥ 2

X1 + 3 X2 ≤ 3

X2 ≤ 4

X1, X2 ≥ 0.

3. Use Big-M method to solve the following LPP.

Minimize Z = 2X1 + X2

Subject to the constraints

3X1 + X2 = 3

4X1 + 3X2 ≥ 6

X1 + 2X2 ≤ 3

X1, X2 ≥ 0.

4. Use Big-M method to solve the following LPP.

Maximize Z = 2X1 + X2 + 3 X3

Subject to the constraints

X1 + X2 + 2X3 ≤ 5

2X1 + 3X2 + 4X3 = 12

X1, X2, X3 ≥ 0.

5. Use Big-M method to solve the following LPP.

Maximize Z = 2X1 +3 X2 + 4X3

Subject to the constraints

3X1 + X2 + 4X3 ≤ 600

2X1 + 4X2 + 2X3 ≥ 480

2X1 + 3X2 + 3X3 = 540

X1, X2, X3 ≥ 0.

6. Use artificial variable technique to solve the LPP.

Maximize Z = X1 +2 X2 + 3X3 – X4

Subject to the constraints

X1 + 2X2 + 3X3 = 15

2X1 + X2 + 5X3 = 20

X1 + 2X2 + X3 + X4 = 10

X1, X2, X3 ,X4 ≥ 0.

7. Solve the following LPP by Big-M method

Minimize Z = 2X1 + 3X2

Subject to the constraints

X1 + X2 ≥ 5

X1 + 2X2 ≥ 6

X1, X2 ≥ 0.

8. Use Simplex method to solve the LPP

Maximize Z = 3X1 + 5X2

Subject to the constraints

3X1 + 2X2 ≤ 18

0 ≤ X1 ≤ 4

0 ≤ X2 ≤ 6

9. Use simplex method to solve the LPP.

Maximize Z = 4X1 + 10X2

Subject to the constraints

2X1 + X2 ≤ 50

2X1 + 5X2 ≤ 100

2X1 + 3X2 ≤ 90 and

X1, X2 ≥ 0.

10. Use Simplex method to solve the LPP.

Maximize Z = 15X1 +6 X2 + 9X3 +2X4

Subject to the constraints

2X1 + X2 + 5X3 + 6X4 ≤ 20

3X1 + X2 + 3X3 + 25X4 ≤ 24

7X1 + X4 ≤ 70

X1, X2, X3 ,X4 ≥ 0.

11. Use graphical method to solve the following LPP.

Maximize Z = 2X1 + X2

Subject to the constraints

X1 +2 X2 ≤ 10

X1 + X2 ≤ 6

$X1 - X2 \leq 2$
$X1 - 2X2 \leq 1$ and
$X1, X2 \geq 0$.

12. Solve the LPP by graphical Method.
Maximize Z = 3X1 + 5X2
Subject to the constraints
$-3X1 + 4X2 \leq 12$
$2X1 - X2 \geq -2$
$2X1 + 3X2 \geq 12$
$X1 \leq 4$
$X2 \geq 2$ and $X1, X2 \geq 0$.

13. Solve by graphically
Maximize Z = 6X1 + 4X2
Subject to the constraints
$X1 + X2 \leq 5$
$X2 \geq 8$
$X1, X2 \geq 0$.

14. Solve by graphically
Maximize Z = 100X1 + 40X2
Subject to the constraints
$5X1 + 2X2 \leq 1000$
$3X1 + 2X2 \leq 900$

$X1 + 2X2 \leq 500$
$X1, X2 \geq 0$

15. A company produces refrigerator in Unit I and heater in Unit II. The two products are produced and sold on a weekly basis. The weekly production cannot exceed 25 in unit I and 36 in Unit II, due to constraints 60 workers are employed. A refrigerator requires 2 man week of labour, while a heater requires 1 man week of labour, the profit available is Rs. 600 per refrigerator and Rs. 400 per heater. Formulate the LPP problem.

16. A firm manufactures two types of products A and B and sells them at profit of Rs 2 on type
A and Rs 3 on type B. Each product is processed on two machines M1 and M2.Type A requires
1 minute of processing time on M1 and 2 minutes on M2 Type B requires 1 minute of processing time on M1 and 1 minute on M2. Machine M1 is available for not more than 6 hours 40 minutes while machine M2 is available for 10 hours during any working day. Formulate the problem as a LPP so as to maximize the profit.

17. A company sells two different products A and B , making a profit of Rs.40 and Rs. 30 per unit on them, respectively. They are produced in a common production process and are sold in two different markets, the production process has a total capacity of 30,000 man-hours. It takes three hours to produce a unit of A and one hour to produce a unit of B. The market has been surveyed and company official feel that the maximum number of units of A that can be sold is
8,000 units and that of B is 12,000 units. Subject to these limitations, products can be sold in any combination. Formulate the problem as a LPP so as to maximize the profit