UNIT IV PERMUTATIONS & COMBINATIONS

1. Define Fundamental principles of counting

The Fundamental Counting Principle is a way to figure out the total number of ways different events can occur.

If the first task can be performed in m ways, while a second task can be performed in n ways, and the two tasks cannot be performed simultaneously, then performing either task can be accomplished in any one of $m + n$ ways.

If a procedure can be broken into first and second stages, and if there are m possible outcomes for the first stage and if, for each of these outcomes, there are n possible outcomes for the second stage, then the total procedure can be carried out, in the designed order, in mn ways.

2. Define rule of sum.

If the first task can be performed in m ways, while a second task can be performed in n ways, and the two tasks cannot be performed simultaneously, then performing either task can be accomplished in any one of $m + n$ ways.

Example: A college library has 40 books on C++ and 50 books on Java. A student at this college can select 40+50=90 books to learn programming language.

3. Define rule of Product

If a procedure can be broken into first and second stages, and if there are m possible outcomes for the first stage and if, for each of these outcomes, there are n possible outcomes for the second stage, then the total procedure can be carried out, in the designed order, in mn ways.

Example: A drama club with six men and eight can select male and female role in 6 x 8 = 48 ways.

4. Define Permutations

For a given collection of n objects, any linear arrangement of these objects is called a permutation of the collection. Counting the linear arrangement of objects can be done by rule of product.

For a given collection of n distinct objects, and r is an integer, with $1 \le r \le n$, then by rule of product, the number of permutations of size r for the n objects is

$$P(n,r) = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1) = \frac{n!}{(n-r)!}, \quad 0 \le r \le n$$

Example: In a class of 10 students, five are to be chosen and seated in a row for a picture.

The total number of arrangements = 10 x 9 x 8 x 7 x 6 = 30240.

5. Define combinations

For a given collection of n objects, each selection, or combination, of r of these objects, with no reference to order, corresponds to $r!$ (Permutations of size r from the n objects). Thus the number of combinations of size r from a collection of size n is

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!\,(n-r)!}, \quad 0 \le r \le n$$

Example: In a test, students are directed to answer 7 questions out of 10. The student can answer the examination in

$$C(n,r) = C(10,7) = \frac{10!}{7!\,(10-7)!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120\ ways$$

6. State Binomial theorem

The Binomial theorem: If x and y are variables and n is a positive integer, then

$$(x+y)^n = \binom{n}{0}x^0y^n + \binom{n}{1}x^1y^{n-1} + \binom{n}{2}x^2y^{n-2} + \cdots$$
$$\binom{n}{n-1}x^{n-1}y^1 + \binom{n}{n}x^ny^0 = \sum_{k=0}^{n}\binom{n}{k}x^ky^{n-k}$$

$\binom{n}{k}$ is referred as Binomial coefficient.

7. Define combinations with repetition

If there is a selection with repetition, r of n distinct objects, then the combinations with of n objects taken r at a time with repetition is $C(n+r-1, r)$.

$$C(n+r-1,r) = \frac{(n+r-1)!}{r!\,(n-1)!} = \binom{n+r-1}{r}$$

Example: A donut shop offers 20 kinds of donuts. Assuming that there are at least a dozen of each kind when we enter the shop. We can select a dozen donuts in $C(20+12-1, 12) = C(31, 12) =$ 141120525 ways

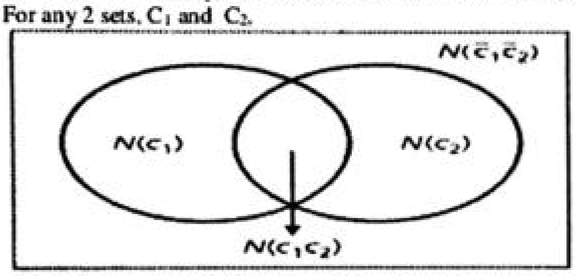
8. Define Catalan numbers

The Catalan numbers form a sequence of natural numbers that occur in various counting problems, often involving recursively-defined objects. They are named after the Belgian mathematician Eugène Charles Catalan. the nth Catalan number is given directly in terms of binomial coefficients by Prepared by G. Appasami, Assistant professor, Dr. pauls Engineering College.

$$C_n = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!\,n!} = \frac{1}{n+1}\binom{2n}{n} \quad \text{for } n \ge 0$$

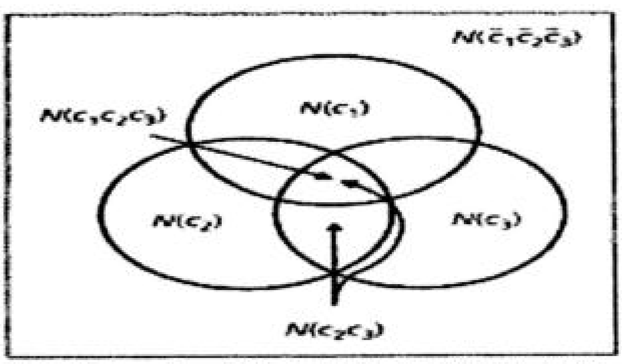
9. Write the Principle of inclusion and exclusion formula.

For any 2 sets, C_1 and C_2.



$$N(\bar{c}_1\bar{c}_2) = N - [N(c_1) + N(c_2)] + N(c_1c_2)$$

For any 3 sets, C_1, C_2 and C_3.



$$N(\bar{c}_1\bar{c}_2\bar{c}_3) = N - [N(c_1) + N(c_2) + N(c_3)] + [N(c_1c_2) + N(c_1c_3) + N(c_2c_3)] - N(c_1c_2c_3).$$

For any 4 sets, C_1, C_2, C_2 and C_4.

$$N(\bar{c}_1\bar{c}_2\bar{c}_3\bar{c}_4) = N - [N(c_1) + N(c_2) + N(c_3) + N(c_4)]$$
$$+ [N(c_1c_2) + N(c_1c_3) + N(c_1c_4) + N(c_2c_3) + N(c_2c_4) + N(c_3c_4)]$$
$$- [N(c_1c_2c_3) + N(c_1c_2c_4) + N(c_1c_3c_4) + N(c_2c_3c_4)]$$
$$+ N(c_1c_2c_3c_4).$$

10. Define Derangements

A derangement is a permutation of the elements of a set, such that no element appears in its original position.

The number of derangements of a set of size n, usually written D_n, d_n, or !n, is called the "derangement number" or "de Montmort number".

Example: The number of derangements of 1, 2, 3, 4 is

d_4 = 4! [1 – 1 + (1/2!)-(1/3!)+(1/4!)] = 9.

11. What is meant by Arrangements with forbidden (banned) positions.

The number of acceptable assignments is equal to the number of ways of placing nontaking rooks on this chessboard so that none of the rooks is in a forbidden position. The key to determining this number of arrangements is the inclusion- exclusion principle.

UNIT IV - PERMUTATIONS & COMBINATIONS

1. i) Give the number of distinct permutations that can be formed from all the letters of each word a. RADAR b. UNUSUAL

   ii) If six people, designated as *A, B, ……,F* are seated about a round table, how many different circular arrangements are possible, if arrangements are considered the same when one can be obtained from the other by rotation?

   iii) In how many can the symbols *a, b, c, d, e, e, e, e, e* be arranged so that no *e* is adjacent to another *e*.
2. i) Over the Internet, data are transmitted in structured block of bits called datagrams

   a) In how many ways can the letters in *DATAGRAM* be arranged?

   b) For the arrangements of part (a) how many have all three *A*'s together?

   ii) Show How many positive integers *n* can be formed using the digits *3, 4, 4, 5, 5, 6, 7* if we want n to exceed 5000000?
3. i) Sixteen people are to be seated at two circular tables, one of which seats 10 while the other seats 6. How many different seating arrangements are possible.

   ii) A committee of 15 members,(9 are women and 6 are men) is to be seated at a circular table (with 15 seats).Show In how many ways can the seats be assigned so that no two men are seated next to each other?
4. i) Show How many numbers of arrangements can be made with the letters *TALLAHASSEE* having no adjacent *A*'s?

   ii) List all the combinations of size 3 that results for letters *m, r, a, f* and *t*?
5. i) Show How many permutations of size 3 can one produce with the letters m, r, a, f and t.

   ii)A father, mother, 2 boys and 3 girls are asked to line up for a photo Give the number of ways they can line up if a) No restriction b) Parents stand together c) Parents do not stand together d) All females stand together.
6. Determine the coefficient of x^9y^3 in the expansions of

   a) $(x+y)^{12}$ b) $(x+2y)^{12}$ and c)$(2x-3y)^{12}$
7. Develop the details in the proof of the multinomial theorem.
8. i) Explain In how many ways can we distribute seven bananas and six oranges among four children so that each child receives at least one banana?

   ii) Explain the linear arrangements of the given n objects. Give an example.
9. i) State and prove the principle of inclusion and exclusion.

ii) Determine the number of positive integers n where $1 \leq n \leq 100$ is not divisible by 2,3 or 5.

10. i) Explain in how many ways can the 26 letters of the alphabet be permuted so that none of the letters car, dog, pun or byte occurs?

ii) Determine and Give the number of positive integers x where $x \leq 9999999$ and the sum of the digits in x equals 31.

11. i) While at the racetrack, Ralph bets on each of the 10 horses in a race to come in according to how they are favored. In how many ways can they reach the finish line so that he loses all of his bets.

ii)Six married couples are to be seated at a circular table. In how many ways can they arrange themselves so that no wife sits next to her husband?

12. Let A = {*1,2,3,4*} and B={*u,v,w,x,y,z*}. How many one-to-one function f:A->B satisfy none of the following conditions.

C1: f(1)= *u* or *v*, C2: f(2)=*w*, C3:f(3)=*w* or *x*, C4:f(4)=*x,y* or *z*

13. i) In a box there are 5 black pens, 3 white pens, and 4 red pens. In how many ways can 2 black pens, 2 white pens and 2 red pens can be chosen?

ii) The Indian cricket team consists of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can a cricket 11 be selected if we have 1 wicket keeper and 4 bowlers?

14. i) Evaluate a) ${}^{6}C_2 + {}^{6}C_1$ b) ${}^{8}C_2 + {}^{8}C_1$ c) ${}^{5}C_3 + {}^{5}C_2$ d) ${}^{10}C_2 + {}^{10}C_3$

ii) A committee of 5 persons is to be formed from 6 men and 4 women. In how many ways can this be done when a)At least 2 women are included b)At most 2 women are included