CS6702 GRAPH THEORY AND APPLICATIONS
2 MARKS QUESTIONS AND ANSWERS

UNIT I INTRODUCTION

1. Define Graph.

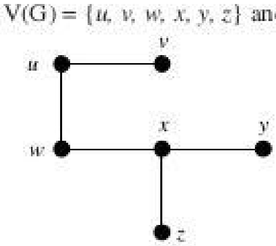
A graph G = (V, E) consists of a set of objects V={v_1, v_2, v_3, … } called **vertices** (also called **points** or **nodes**) and other set E = {e_1, e_2, e_3, ...} whose elements are called **edges** (also called **lines** or **arcs**).

The set V(G) is called the **vertex set** of G and E(G) is the **edge set** of G.

For example :

A graph G is defined by the sets V(G) = {*u, v, w, x, y, z*} and E(G) = {*uv, uw, wx, xy, xz*}.

Graph G:

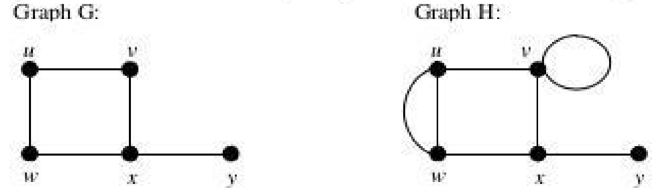


A graph with *p*-vertices and *q*-edges is called a **(*p, q*) graph.** The (1, 0) graph is called **trivial graph.**

2. Define Simple graph.

- An edge having the same vertex as its end vertices is called a self-loop.
- More than one edge associated a given pair of vertices called parallel edges.
- A graph that has neither self-loops nor parallel edges is called simple graph.

Graph G: Graph H:



Simple Graph Pseudo Graph

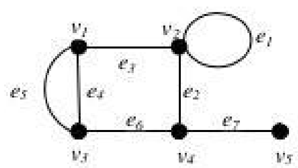
3. Write few problems solved by the applications of graph theory.

Konigsberg bridge problem
Utilities problem
Electrical network problems
Seating problems

4. Define incidence, adjacent and degree.

When a vertex v_i is an end vertex of some edge e_j, v_i and e_j are said to be *incident* with each other. Two non parallel edges are said to be *adjacent* if they are incident on a common vertex. The number of edges incident on a vertex v_i, with self-loops counted twice, is called the *degree* (also called valency), $d(v_i)$, of the vertex v_i. A graph in which all vertices are of equal degree is called *regular graph*.

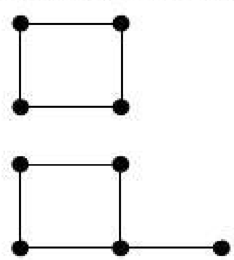
Graph G:



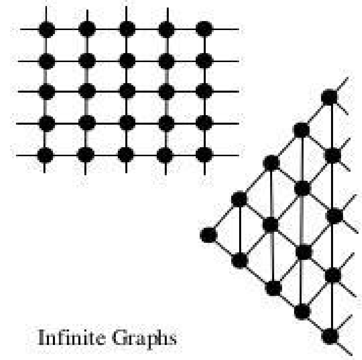
The edges e_2, e_6 and e_7 are incident with vertex v_4.
The edges e_2 and e_7 are adjacent.
The edges e_2 and e_4 are not adjacent.
The vertices v_4 and v_5 are adjacent.
The vertices v_1 and v_5 are not adjacent.
$d(v_1) = d(v_3) = d(v_4) = 3$. $d(v_2) = 4$. $d(v_5) = 1$.

5. What are finite and infinite graphs?

A graph with a finite number off vertices as well as a finite number of edges is called a *finite* graph; otherwise, it is an *infinite* graph.



Finite Graphs

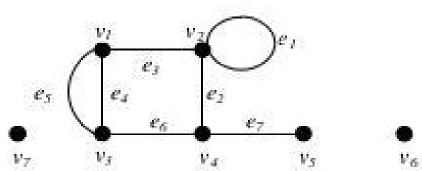


Infinite Graphs

6. Define Isolated and pendent vertex.

A vertex having no incident edge is called an *isolated vertex*. In other words, isolated vertices are vertices with zero degree. A vertex of degree one is called a *pendant vertex* or an *end vertex*.

Graph G:

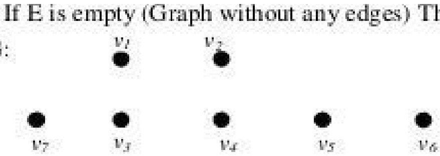


The vertices v_6 and v_7 are *isolated vertices*.
The vertex v_5 is a *pendant vertex*.

7. Define null graph.

In a graph G=(V, E), If E is empty (Graph without any edges) Then G is called a *null graph*.

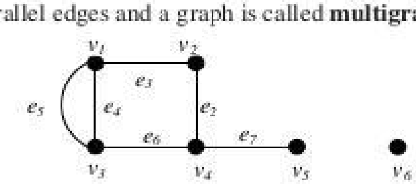
Graph G:



8. Define Multigraph

In a multigraph, no loops are allowed but more than one edge can join two vertices, these edges are called **multiple edges** or parallel edges and a graph is called **multigraph.**

Graph G:



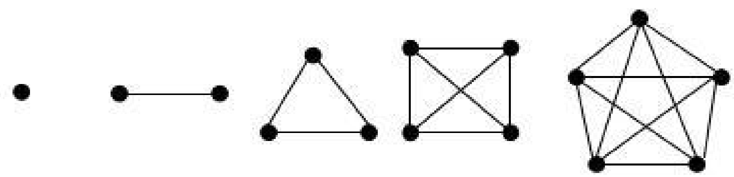
The edges e_5 and e_4 are **multiple** (parallel) edges.

9. Define complete graph

A simple graph G is said to be **complete** if every vertex in G is connected with every other vertex. *i.e.*, if G contains exactly one edge between each pair of distinct vertices.

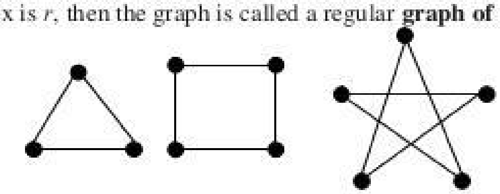
A complete graph is usually denoted by $\mathbf{K}_n$. It should be noted that K*n* has exactly $n(n-1)/2$ edges.

The complete graphs K_n for $n = 1, 2, 3, 4, 5$ are show in the following Figure.



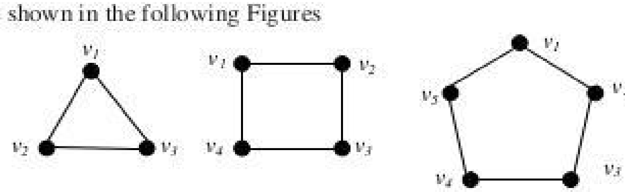
10. Define Regular graph

A graph in which all vertices are of **equal degree,** is called a **regular graph.**
If the degree of each vertex is *r*, then the graph is called a regular **graph of degree *r*.**



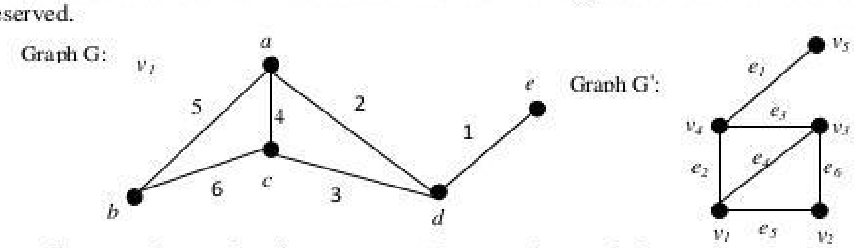
11. Define Cycles

The cycle C_n, $n \geq 3$, consists of *n* vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}$, ……, $\{v_{n-1}, v_n\}$, and $\{v_n, v_1\}$.
The cyles c_3, c_4 and c_5 are shown in the following Figures



12. Define Isomorphism.

Two graphs G and G' are said to be **isomorphic** to each other if there is a one-to-one correspondence between their vertices and between their edges such that the incidence relationship is preserved.



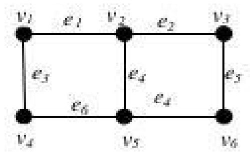
Correspondence of vertices	Correspondence of edges
$f(a) = v_1$	$f(1) = e_1$
$f(b) = v_2$	$f(2) = e_2$
$f(c) = v_3$	$f(3) = e_3$
$f(d) = v_4$	$f(4) = e_4$
$f(e) = v_5$	$f(5) = e_5$

Adjacency also preserved. Therefore G and G' are said to be isomorphic.

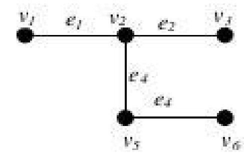
13. What is Subgraph?

A graph G' is said to be a subgraph of a graph G, if all the vertices and all the edges of G' are in G, and each edge of G' has the same end vertices in G' as in G.

Graph G:



Subgraph G' of G:

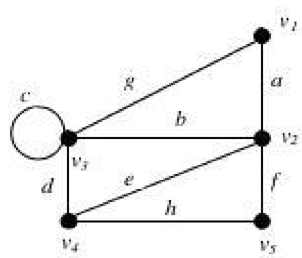


14. Define Walk, Path and Circuit.

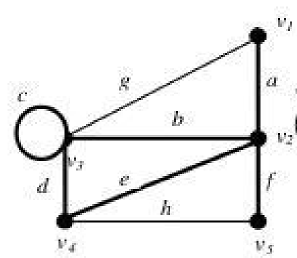
A **walk** is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices. No edge appears more than once. It is also called as an edge train or a chain.

An open walk in which no vertex appears more than once is called **path**. The number of edges in the path is called **length of a path**.

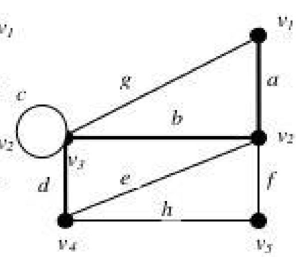
A closed walk in which no vertex (except initial and final vertex) appears more than once is called a circuit. That is, a circuit is a closed, nonintersecting walk.



Graph G:



Open walk



Path of length 3

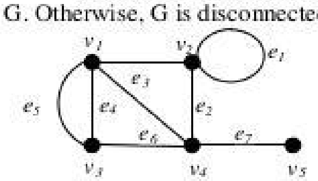
$v_1\ a\ v_2\ b\ v_3\ c\ v_3\ d\ v_4\ e\ v_2\ f\ v_5$ is a walk. v_1 *and* v_5 are terminals of walk.

$v_1\ a\ v_2\ b\ v_3\ d\ v_4$ is a path. $a\ v_2\ b\ v_3\ c\ v_3\ d\ v_4\ e\ v_2\ f\ v_5$ is not a path.

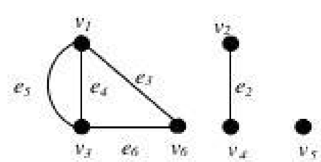
$v_2\ b\ v_3\ d\ v_4\ e\ v_2$ is a circuit.

15. Define connected graph. What is Connectedness?

A graph G is said to be **connected** if there is at least one path between every pair of vertices in G. Otherwise, G is disconnected.



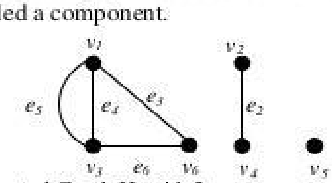
Connected Graph G



Disconnected Graph H

16. Define Components of graph.

A disconnected graph consists of two or more connected graphs. Each of these connected subgraphs is called a component.

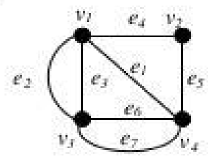


Disconnected Graph H with 3 components

17. Define Euler graph.

A path in a graph G is called Euler path if it includes every edges exactly once. Since the path contains every edge exactly once, it is also called Euler trail / Euler line.

A closed Euler path is called Euler circuit. A graph which contains an Eulerian circuit is called an Eulerian graph.

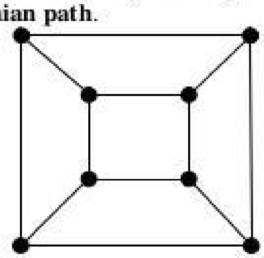


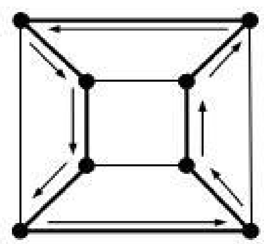
v_4 e_1 v_1 e_2 v_3 e_3 v_1 e_4 v_2 e_5 v_4 e_6 v_3 e_7 v_4 is an Euler circuit. So the above graph is Euler graph.

18. Define Hamiltonian circuits and paths

A **Hamiltonian circuit** in a connected graph is defined as a closed walk that traverses every vertex of graph G exactly once except starting and terminal vertex.

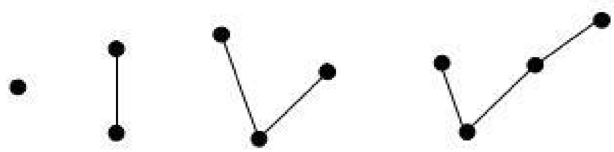
Removal of any one edge from a Hamiltonian circuit generates a path. This path is called **Hamiltonian path**.





19. Define Tree

A tree is a connected graph without any circuits. Trees with 1, 2, 3, and 4 vertices are shown in figure.



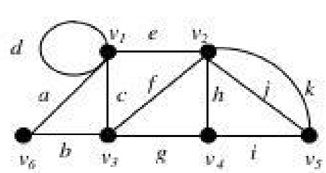
20. List out few Properties of trees.

1. There is one and only one path between every pair of vertices in a tree T.
2. In a graph G there is one and only one path between every pair of vertices, G is a tree.
3. A tree with *n* vertices has *n*-1 edges.
4. Any connected graph with *n* vertices has *n*-1 edges is a tree.
5. A graph is a tree if and only if it is minimally connected.
6. A graph G with *n* vertices has *n*-1 edges and no circuits are connected.

21. What is Distance in a tree?

In a connected graph G, the distance $d(v_i, v_j)$ between two of its vertices v_i and v_j is the length of the shortest path.

Graph G:



Paths between vertices v_6 and v_2 are (a, e), (a, c, f), (b, c, e), (b, f), (b, g, h), and (b, g, i, k).
The shortest paths between vertices v_6 and v_2 are (a, e) and (b, f), each of length two.
Hence $d(v_6, v_2) = 2$

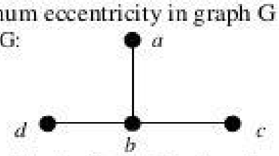
22. Define eccentricity and center.

The eccentricity E(v) of a vertex v in a graph G is the distance from v to the vertex farthest from v in G; that is,

$$E(v) = \max_{v_i \in G} d(v, v_i)$$

A vertex with minimum eccentricity in graph G is called a center of G

Graph G:



Distance d(a, b) = 1, d(a, c) =2, d(c, b)=1, and so on.
Eccentricity E(a) =2, E(b) =1, E(c) =2, and E(d) =2.
Center of G = A vertex with minimum eccentricity in graph G = b.

23. Define distance metric.

The function $f(x, y)$ of two variables defines the distance between them. These function must satisfy certain requirements. They are

1. Non-negativity: $f(x, y) \geq 0$, and $f(x, y) = 0$ if and only if $x = y$.
2. Symmetry: $f(x, y) = f(x, y)$.
3. Triangle inequality: $f(x, y) \leq f(x, z) + f(z, y)$ for any z.

24. What are the Radius and Diameter in a tree.

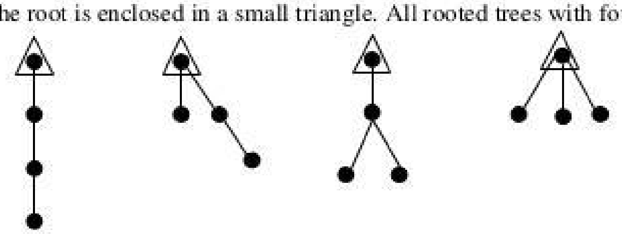
The eccentricity of a center in a tree is defined as the radius of tree.
The length of the longest path in a tree is called the diameter of tree.

25. Define Rooted tree

A tree in which one vertex (called the root) is distinguished from all the others is called a **rooted tree**.

In general tree means without any root. They are sometimes called as **free trees** (non rooted trees).

The root is enclosed in a small triangle. All rooted trees with four vertices are shown below.



26. Define Rooted binary tree

There is exactly one vertex of degree two (root) and each of remaining vertex of degree one or three.

A binary rooted tree is special kind of rooted tree. Thus every binary tree is a rooted tree. A non pendent vertex in a tree is called an internal vertex. Prepared by G. Appasami, Assistant professor, Dr. pauls Engineering College.

BIG QUESTIONS

UNIT I - INTRODUCTION

1. i) In a complete graph having odd number of vertices, how many edge disjoint Hamiltonian circuits exist? Explain.

ii) State the two theorems to check if a connected graph G is Eulerian. Explain with proof.

iii) Find a path of length 9 and a circuit of length 8 in the Peterson graph.

2. i) Illustrate the search algorithm than can be employed to find the components or blocks in a graph, with an example.

ii) Explain the following theorem with proof "In a graph the number of the vertices with odd degree is even".

3. Give the proof for the following theorem

i) If a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

ii) A connected graph is an Euler graph if and only if every vertex has even degree.

iii) A connected graph is an Euler graph if and only if it can be decomposed into circuits.

4. i) Show that the ring-sum of any two cut-sets in a graph is either third cut-set or an edge disjoint union of cut-sets.

ii) Show that a vertex *v* in a connected graph G is a cut-vertex if and only if there exists two vertices *x* and *y* in G such that every path between *x* and *y* passes through *v*.

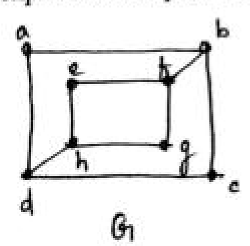
iii) Find |v| for the following graph or multigraphs G.

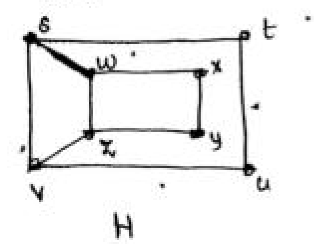
a) G has nine edges and all vertices have degree 3

b) G has ten edges with two vertices of degree 4 and all other of degree 3.

5. i) Give the explanation to prove any undirected graph has an even number of vertices of odd degree.

ii) Give the explanation to prove that the following graphs G and H are not isomorphic.

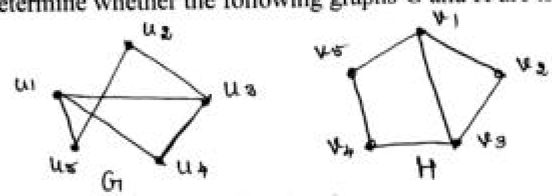




6. Give the explanation to prove that a connected graph G is Eulerian if and only if all the vertices are of even degree.

7. Prove that graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other in V_2.

8. i) Determine whether the following graphs G and H are isomorphic. Give reason

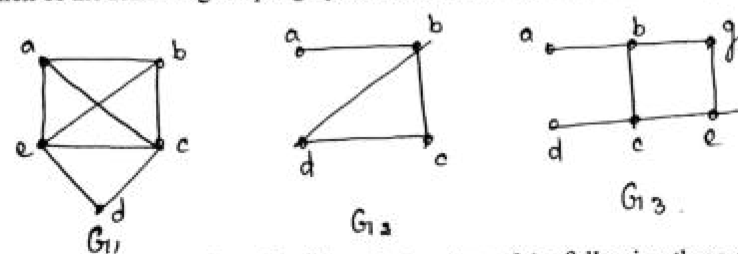


ii) Give the proof for the following theorem :

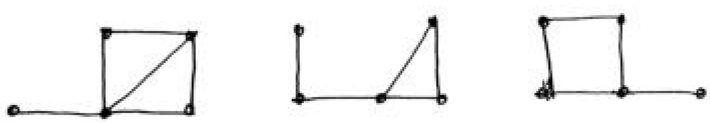
A given connected graph G is an Euler graph if and only if all the vertices of G are of even degree.

9. i) Prove that a simple graph with n vertices and k components cannot have more than $(n-k)(n-k+1)/2$ edges.

ii) Which of the following simple graphs have a Hamilton Circuit or if no, a Hamilton Path?

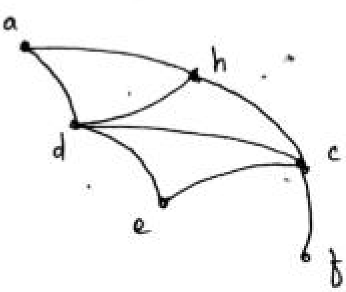


10. i)Define isomorphism of graphs. Show that no two of the following three graphs a shown in figure are isomorphic



ii) Define Euler circuit. Discuss Konigsberg bridge problem.

11. i) In the undirected graph



Find a) An a-a circuit of length 6

b) An a-a cycle of maximum length

ii) Seven students of a class have lunch together at a circular table. Using Hamilton cycles, Predict the minimum number of days required for each of them to sit next to every member of the class.

12. i) If G is an undirected graph with n vertices and e edges, let δ=min {deg(v)} and Δ=max {deg(v)}, then prove that $\delta \leq 2(e/n) \leq \Delta$.

ii) Define Hamilton cycle. How many edge-disoint Hamilton cycles exist in the complete graph with seven vertices? Also, Design the graph to show these Hamilton cycles.

13. i) Let G=(V,E) be the undirected graph in the following figure. How many paths are there in G from a to h? How many of these paths have length 5?

ii) Let G=(V,E) be an undirected graph, where |v| ≥2. If every induced subgraph of G is connected, can we identify the graph G?

14. i) Define a bipartite graph? Show that the complement of a bipartite graph need not to be a bipartite?

ii) Define the following with one example each

a) Infinite graph
b) Hamiltonian path
c) Component of a graph
d) Euler graph
e) Spanning subgraph

13) i)