Ma2265 – Discrete Mathematics

Question Bank

Part – A

1. Prove that $2^n < n!$ for $n \geq 4$
2. Define Pigeon Hole Principle.
3. Prove that $n < 2^n$, for $n \geq 1$.
4. Find the value of n if nP_{13}: $(n+1)P_{12}$=3:4.
5. If $nC_5 = 20nC_4$, find 'n'.
6. Find the recurrence relation for the sequence $a_n = 2n+9$, $n \geq 1$.
7. Find the recurrence relation which satisfies $y_n = A\,3^n + B\,(-4)^n$.
8. How many positive integers not exceeding 1000 are divisible by 7 or 11?
9. A survey of 500 from a school produced the following information. 200 play volleyball, 120 play hockey. 60 play both volleyball and hockey. How many are not playing either volleyball or hockey?
10. Define complete bipartite graph with example.
11. State the Handshaking Theorem.
12. For the following degree sequences, 4,4,4,3,2 find if there exist a graph or not.
13. Define mixed graph with example.
14. Define Subgraph.
15. Write the Definition of Adjecency Matrix of a simple graph.
16. Define Incidence Matrix of a simple graph.
17. Write the Definition of Path Matrix.
18. Define Graph Isomorphism.
19. Define Unilaterally Connected.
20. Write the Definition of Euler Graph.
21. Describe the Hamiltonian Graph.
22. Define Group and Abelian Group.
23. Define Semigroup and Monoid.
24. Define Cyclic monoid.
25. Prove that the identity element of a group is unique.
26. Prove that the inverse element of a group is unique.
27. Prove that a group (G,*) is an abelian group iff $(a*b)^2 = a^2 * b^2$ for all a,b $\in$ G.
28. Define Subgroup with example.
29. Define morphism of Groups.

30. Describe Kernel of a Homomorphism.
31. Define Cosets.
32. Define normal subgroups.
33. Define Ring.
34. What is Equialence Relation?
35. Define POSET.
36. Draw the Hasse Diagram for$(\rho(A), \subseteq)$ where A={a,b,c}
37. If X= {2,3,6,12,24,36} and the relation R defined on X by R= {<a,b> / a/b}. Draw the Hasse Diagram for (X,R).
38. Define LUB and GLB.
39. Describe Lattices.
40. Describe Modular Lattice.
41. Prove that $a+\bar{a}b=a+b$
42. Reduce the expression of $ab+abc+ab\bar{c}+\bar{a}bc$
43. Let a,b,c $\in$B then S.T. $a\cdot 0=0$ and $a+1=1$

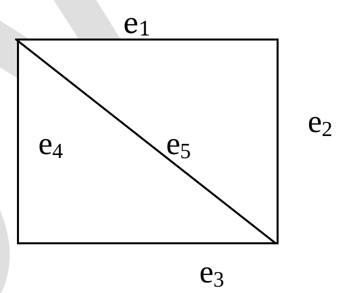
Part – B

1. Prove by Mathematical induction method the following $1+2+3+.......=\frac{n(n+1)}{2}$
2. Prove that 8^n-3^n is divisible by 5.
3. Prove that $n^3+(n+1)^3+(n+2)^3$ is divisible by 9
4. Prove by Mathematical induction method the following $1^3+2^3+3^3+.......=\left[\frac{n(n+1)}{2}\right]^2$
5. P.T. by induction method the following$1^2+2^2+3^2+.......=\left[\frac{n(n+1)(2n+1)}{6}\right]$
6. How many bit strings of length 10 contain
   - Exactly 4 1's
   - Atmost 4 1's
   - Atleast 4 1's
   - An equal number of 0's and 1's
7. Find the explicit formula for the Fibonacci sequence.
8. Solve the recurrence relation D(k)-7D(k-2)+6 D(k-3)=0 with D(0)=8, D(1)=6 and D(2)=22.
9. Solve the recurrence relation S(n)-4 S(n-1)-11 S(n-2)+30 S(n-3)=0 with S(0)=0, S(1)=-35and S(2)=-85.

10. Find all the solution of the recurrence relation $a_{n+1}-a_n=3n^2-n$, $n\geq0$ and $a_0=3$.
11. Solve the recurrence relation $S(n)-3\,S(n-1)-4\,S(n-2)=4^n$.
12. Find all the solution of the recurrence relation $a_n=5a_{n-1}-6a_{n-2}+7^n$.
13. Solve the recurrence relation $a_n-7a_{n-1}+10a_{n-2}=0$ for $n\geq2$ given that $a_0=10$, $a_1=41$ using generating functions.
14. Solve the recurrence relation $S(n+1)-2\,S(n)=4^n$ with $S(0)=1$ for $n\geq0$
15. Identify the sequence having the expression $(5+2x/1-4x^2)$ as a generating function.
16. Identify the sequence having the expression $(6-29x/30x^2-11x+1)$ as a generating function.
17. Find the number of integers between 1 to 250 that are not divisible by any of the integers 2,3,5 and 7.
18. How many positive integers between 1 to 100 that are

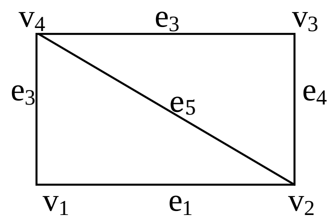
    i)Not divisible by 7,11 or 13 ii)Divisible by 3 but not by 7.
19. Find the number of integers between 1 to 100 that are divisible by

    i) 2,3,5 or 7 ii) 2,3,5 but not by 7.
20. How many prime numbers not exceeding 100 are there?
21. How many solutions does $x_1+x_2+x_3=11$ have, where x_1,x_2 and x_3 are non negative integers with $x_1\leq3$, $x_2\leq4$ and $x_3\leq6$.
22. Prove that In a undirected graph, the number of odd degree vertices are even.
23. Prove that the maximum number of edges in a simple graph with 'n' vertices is $\dfrac{n(n-1)}{2}$
24. Find the adjacency matrix of the following graph G. Hence find degree of each vertex. Also find A^2 and A^3. what is your observations regarding the entries in A^2 and A^3.

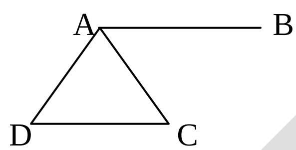


25. Prove that a simple graph with n vertices must be connected if it has more than $\dfrac{(n-1)(n-2)}{2}$ edges.

26. Find the adjacency matrix of the following graph G. Hence find degree of each vertex. Also find A^2 and A^3 what is your observations regarding the entries in A^2 and A^3.



27. For the graph given below find all possible paths of length 4 from vertex B to D.



28. Let G be a simple graph with n vertices. S.T. if $\delta(G) \geq \left[\frac{n}{2}\right]$, then G is connected where $\delta(G)$ is minimum degree of the graph G.

29. P.T. a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

30. Give an example of a graph which is

1. Eulerian but not Hamiltonian. .
2. Hamiltonian but not Eulerian.
3. Both Eulerian and Hamiltonian.
4. Non Eulerian and Non Hamiltonian

31. Show that $(Q^+,*)$ is an abelian group where * is defined by $a*b=\frac{ab}{2}$, for all a, b $\in Q^+$.

32. S.T. (R-{1},*) is an abelian group, where * is defined by $a*b=a+b+ab$, for all a, b $\in$ R.

33. * on R defined by $x*y=x+y+2xy$, for all x, y $\in$ R. Check
   i. (R,*) is a monoid or not. ii. Is it commutative
   iii. Which elements have inverses and what are they?

34. Let S=Q×Q be the set of all ordered pair of rational numbers and given by $(a,b)*(x,y)=(ax,ay+b)$. Check whether (S,*) is a Semigroup. Is it commutative? Also find the identity element of S.

35. Let G denote the set of all matrices of the form $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$ where x $\in$ R*. P.T. G is a Abelian group under matrix multiplication.

36. If (G,*) is an abelian group, S.T. $(a*b)^n = a^n * b^n$, for all a,b $\in$G where n is a +ve integer.

37. Prove that the necessary and sufficient condition that a nonempty subset H of a group G to be a subgroup is a,b$\in$H$\Rightarrow a*b^{-1}\in$H.

38. Prove that the intersection and union of two subgroups is also a subgroup.

39. The kernel of a homomorphism f from a group (G,*) to (G',*) is a subgroup of G.

40. STATE AND PROVE LAGRANGE'S THEOREM.

41. State and Prove Fundamental theorem on Homomorphism of Groups.

42. Let (L, $\wedge,\vee$) be a lattice in which $\wedge,\vee$ are denote the operation of meet and join respectively. For any a, b $\in$L, $a \le b \Leftrightarrow a \vee b = b \Leftrightarrow a \wedge b = a$

43. State and prove the following Laws in lattices
    - ✓ Idempotent law
    - ✓ Commutative Law
    - ✓ Associative law
    - ✓ Absorption Law

44. State and prove Isotonicity property of lattice.

45. State and prove Distributive inequality property of lattice.

46. Prove that the Diamond lattice or M_5 Lattice is a Modular Lattice.

47. Prove that the product of 2 lattices is also a lattice.(or)

    If $(L,\wedge,\vee)$ and $(M,*,\oplus)$ are two lattices, then prove that $(L\times M,\cdot,+)$ is also a lattice.

48. State and prove Demorgan's Law of lattice.

49. In a complemented, distributive lattice, S.T. the following are equivalent.

    $a \le b \Leftrightarrow a \wedge b' = 0 \Leftrightarrow a' \vee b = 1 \Leftrightarrow b' \le a'$

50. In a Boolean algebra show that a=b iff $a\bar{b} + \bar{a}b = 0$