Reg. No. :

Question Paper Code : 71777

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fifth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 — DISCRETE MATHEMATICS

(Common to B.Tech. Information Technology)

(Regulation 2008/2010)

(Common to PTMA 2265/10144 CS 501 – Discrete Mathematics for B.E. (Part-Time) Third Semester – Computer Science and Engineering – Regulation 2009/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Construct the truth table for the compound proposition $(p \to q) \leftrightarrow (\neg p \to \neg q)$.
2. Given $P = \{2,3,4,5,6\}$, state the truth value of the statement $(\exists x \in P)\ (x + 3 = 10)$.
3. What is the number of arrangements of all the six letters in the word PEPPER?
4. Use mathematical induction to show that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.
5. Give an example of a graph which is Eulerian but not Hamiltonian.
6. Define a connected graph and a disconnected graph with examples.
7. Find the left cosets of $\{[0], [3]\}$ in the group $(Z_6, +_6)$.
8. Define a Field in an algebraic system.
9. Define lattice homomorphism.
10. Prove the Boolean identity : $a.b + a.b' = a$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ is a tautology. (8)

(ii) Show that $(p \rightarrow q) \wedge (r \rightarrow s), (q \rightarrow t) \wedge (s \rightarrow u), \neg(t \wedge u)$ and $(p \rightarrow r) \Rightarrow \neg p$. (8)

Or

(b) (i) Show that $\forall x(p(x) \vee q(x)) \Rightarrow \forall x p(x) \vee \exists x q(x)$ using the indirect method. (8)

(ii) Write the symbolic form and negate the following statements :

(1) Every one who is healthy can do all kinds of work.

(2) Some people are not admired by every one.

(3) Every one should help his neighbors ,or his neighbors will not help him.

(4) Every one agrees with some one and some one agrees with every one. (8)

12. (a) (i) There are three files of identical red, blue and green balls, where each file contains at least 10 balls. In how many ways can 10 balls be selected (1) if there is no restriction (2) if at least 1 red ball must be selected (3)if at least 1 red ball, at least 2 blue balls and at least 3 green balls must be selected (4) if at most 1 red ball is selected. (8)

(ii) Find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2, 3, 5 and 7. (8)

Or

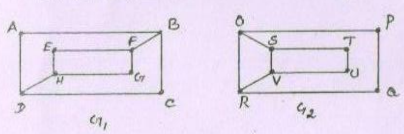
(b) (i) Use the method of generating function to solve the recurrence relation $a_n = 3a_{n-1} + 1; n \geq 1$ given that $a_0 = 1$. (8)

(ii) Prove by mathematical induction that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$ (8)

13. (a) (i) Prove that the number of vertices of odd degree in any graph is even. (8)

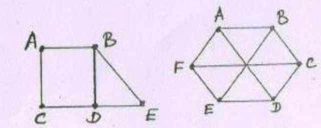
(ii) Examine whether the following pair of graphs are isomorphic or not. Justify your answer. (8)



Or

(b) (i) Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. (8)

(ii) Find an Euler path or an Euler circuit, if it exists in the following graphs. If it does not exist, explain why? (8)



14. (a) (i) Show that M_2, the set of all 2 × 2 non-singular matrices over R is a group under usual matrix multiplication. Is it abelian. (8)

(ii) Show that the union of two subgroups of a group G is a subgroup of G if and only if one is contained in the other. (8)

Or

(b) (i) State and prove Lagrange's theorem. (10)

(ii) If $S = N \times N$, the set of ordered pairs of positive integers with the operation $*$ defined by $(a,b)*(c,d) = (ad + bc, bd)$ and if $f:(S,*) \rightarrow (Q,+)$ is defined by $f(a,b) = \frac{a}{b}$, show that f is a semigroup homomorphism. (6)

15. (a) (i) If S_n is the set of all divisors of the positive integer n and D is the relation of 'division', prove that $\{S_{30}, D\}$ is a lattice. Find also all the sub lattices of $\{S_{30}, D\}$ that contains 6 or more elements. (8)

(ii) Show that every chain is a distributive lattice. (8)

Or

(b) (i) State and prove De Morgan's Law in a complemented distributive lattice. (8)

(ii) If $a,b \in S = \{1, 2, 3, 6\}$ and $a + b = LCM(a,b)$, $a * b = GCD(a,b)$ and $a' = \frac{6}{a}$, show that $\{S, +, ., ', 1, 6\}$ is a Boolean Algebra. (8)

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