MA6351 TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS APR/MAY 2015

PART – A

1. Form the partial differential equation by eliminating the arbitrary constants a and b from log(az-1)=x+ay+b.
2. Find the complete solution of q=2px.
3. The instantaneous current I at time t of an alternating current wave is given by $I = I_1 \sin(\omega t + \alpha_1) + I_3 \sin(3\omega t + \alpha_3) + I_5 \sin(5\omega t + \alpha_5) + \cdots$. Find the effective value of the current 'I'.
4. If the fourier series of the function f(x) = x, -π<x<π with period 2π is given by $f(x) = 2\left(\sin x - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} - \frac{\sin(4x)}{4} + \cdots\right)$, then find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$
5. Classify the partial differential equation $(1-x^2)Z_{xx} - 2xyZ_{xy} + (1-y^2)Z_{yy} + xZ_x + 3x^2yZ_y - 2z = 0$.
6. A rod 30 cm long has its ends A and B kept at 20^0C and 80^0C respectively until steady state conditions prevail. Find this steady state temperature in the rod.
7. If the Fourier transform of f(x) is F[f(x)]=F(s), then show that $F[f(x-a)]=e^{ias}.F(s)$.
8. Find the fourier sine transform of 1/x.
9. If Z[X(n)]=X(z), then show that $Z[a^n.x(n)]=X(z/a)$.
10. State the convolution theorem on Z-Transforms.

PART- B

11. (a) (I) Solve $(x^2-yz)p+(y^2-zx)q=(z^2-xy)$.

(ii) Solve $(D^2-3DD'+2D'^2)z=(2+4x)e^{x+2y}$.

(OR)

(b) (i) Obtain the complete solution of $p^2+x^2y^2q^2=x^2z^2$.

(ii) Solve $z=px+qy+p^2q^2$ and obtain its singular solution.

12. (a) (i) Find the half range sine series of $f(x) = \begin{cases} x, 0 < x < \pi/2 \\ \pi - x, \pi/2 < x < \pi \end{cases}$

(ii) Find the complex form of the Fourier series of $f(x) = e^{-x}$ in -1<x<1.

(OR)

(b) (i) Find the Fourier series of f(x) =|sinx| in –π<x<π of periodicity 2π.

(ii) Compute upto the first three harmonics of the Fourier series of f(x) given by the following table:

x	0	π/3	2π/3	π	4π/3	5π/3	2π
y=f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

13. (a) Solve $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = 0 = u(l,t), t \geq 0; u(x,0) = \begin{cases} x, 0 \leq x \leq l/2 \\ l - x, l/2 \leq x \leq l \end{cases}$.

(OR)

(b) A uniform string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string into the form of the curve y=kx (l-x) and then released from this position at time t=0. Derive the expression for the displacement of any point of the string at a distance x from one end at time t.

14. (a) Find the Fourier transform of $f(x)=\begin{cases}1, |x|<a\\ 0, |x|\geq a\end{cases}$. Hence deduce that (i) $\int_0^{\infty}\frac{\sin t}{t}dt=\frac{\pi}{2}$. (ii) $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$.

(OR)

(b) (i) Show that $e^{-x^2/2}$ is self reciprocal under Fourier Transform by finding the Fourier transform of $e^{-a^2x^2}$.

(ii) Find the Fourier cosine transform of x^{n-1}.

15. (a) (i) Find $Z[r^n\cos(n\theta)]$ and $Z^{-1}[(1-az^{-1})^{-2}]$.

(ii) Using convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z-1/2)(z-1/4)}\right]$

(OR)

(b) (i) Using Z-transform, solve the difference equation x(n+2)-3x(n+1)+2x(n)=0 given that x(0) =0 , x(1)=1.

(ii) Using residue method, find $Z^{-1}\left[\frac{z}{z^2-2z+2}\right]$.