EE 6302 – ELECTROMAGNETIC THEORY
UNIT-I ELECTROSTATICS I
PART – A

1. **Mention any two sources of electromagnetic fields (or) What are the sources of various types of EMF?**
   - Current carrying conductors.
   - Mobile phones.
   - Microwave oven.
   - Computer and Television screen.
   - High voltage Power lines.

2. **How can a vector field be expressed as the gradient of a scalar field?**
   The gradient of any scalar function is the maximum space rate of change of that function. If the scalar V represents electric potential, ∇ V represents potential gradient.

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

This operation is called the gradient.

$$\text{grad} V = \nabla V$$

Gadient of a scalar is a vector.

3. **What is the physical significance of the term "divergence of a vector field"?**
   The divergence of the vector field $\vec{A}$ is the outflow from a small closed surface per unit volume as the volume shrinks to zero.

$$div\,\vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta v \to 0} \frac{\oint_s \vec{A} \cdot d\vec{s}}{\Delta v}$$

4. **Define curl of a vector.**
   The curl of a vector A at any point is defined as the limit of its surface integral of its cross product with normal over a closed surface per unit volume as the volume shrinks to zero.
   $\nabla \times A = Curl\ A$

5. **Show that the vectors A and B are parallel**
   $A = 2a_x + 4a_y + a_z$; $B = 4a_x + 8a_y + 2a_z$.
   Solution:

$$A \times B = \begin{vmatrix} a_x & a_y & a_z \\ 2 & 4 & 1 \\ 4 & 8 & 2 \end{vmatrix} = [8-8] - [4-4] + [16-16] = 0$$

Therefore vectors A and B are parallel to each other.

6. **Prove that curl grad Φ = 0**
   Solution:

$$\nabla ¤ = \frac{\partial ¤}{\partial x}\bar{a}_x + \frac{\partial ¤}{\partial y}\bar{a}_y + \frac{\partial ¤}{\partial z}\bar{a}_z$$

$$\nabla \times \nabla \Phi = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial ¤}{\partial x} & \frac{\partial ¤}{\partial x} & \frac{\partial ¤}{\partial x} \end{vmatrix}$$

$$= \left(\frac{\partial^2 \phi}{\partial y\, \partial z} - \frac{\partial^2 \phi}{\partial y\, \partial z}\right) \bar{a}_x - \left(\frac{\partial^2 \phi}{\partial x\, \partial z} - \frac{\partial^2 \phi}{\partial x\, \partial z}\right) \bar{a}_y + \left(\frac{\partial^2 \phi}{\partial x\, \partial y} - \frac{\partial^2 \phi}{\partial x\, \partial y}\right) \bar{a}_z$$

$$= 0$$

Thus proved

7. How the unit vectors are defined in cylindrical Co-ordinate systems?

Unit vectors is having unit magnitude and directed along the co-ordinates axes.

For cylindrical Co-ordinate

$$\bar{A} = \bar{a}_\rho A_\rho + \bar{a}_\phi A_\phi + \bar{a}_z A_z$$

Where

$\bar{a}_\rho$, $\bar{a}_\phi$ & $\bar{a}_z$, are unit vectors in the direction of ρ, ϕ, z respectively.

8. **Given** $\bar{A} = 10\, \bar{a}_y + 3\, \bar{a}_z$ **and** $\bar{B} = 5\, \bar{a}_x + 4\, \bar{a}_y$ **find the angle (or) projection of** $\bar{A}$ **and** $\bar{B}$ **(May/June 2009)**

Solution: The angle Θ_{AB} can be found by either dot or cross product

$\bar{A} \cdot \bar{B} = (0, 10, 3).(5, 4, 0) = 40$

$$|\bar{A}| = \sqrt{10^2 + 3^2} = \sqrt{109}$$

$$|\bar{B}| = \sqrt{5^2 + 4^2} = \sqrt{41}$$

$$\cos \Theta_{AB} = \frac{\bar{A} \cdot \bar{B}}{|\bar{A}|\,|\bar{B}|} = 0.598$$

$$\Theta_{AB} = 53.25$$

9. Calculate the total charge within the volume of the universe, if

$$\rho_v = \frac{e^{-2r}}{r^2} \quad \frac{c}{m^3}$$

Solution:

For Sphere $dv = r^2 \sin\Theta\, dr\, d\Theta\, d\Phi$

$$\text{Total charge } Q = \int_{Vol} \rho_v\, dv$$

$$= \int_{r=0}^{r=1} \frac{e^{-2r}}{r^2} r^2\, dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$Q = 1.732\pi$ coulomb

10. What are the types of co-ordinate systems?

- Cartesian (or) rectangular co- ordinate system
- Cylindrical co- ordinate system
- Spherical co- ordinate system

11. What is the difference between scalar and vector quantity?

S.No.	Scalar	Vector
1.	It is a quantity which is characterized by its magnitude.	It is a quantity which is completely completely characterized by its magnitude and direction.
2.	Example: mass, volume, density.	Example: force, velocity, displacement.

12. Given the two coplanar vector $A=3a_x+4a_y-5a_z$; $B=-6a_x+2a_y+4a_z$; Obtain the unit vector normal to the plane of the vectors A and B.

Solution:

$$\bar{AB} = \bar{B} - \bar{A} = (6\text{-}3)a_x + (2\text{-}4)a_y + (4+5)a_z$$

$$\bar{AB} = -9a_x - 2a_y + 9a_z$$

$$|\bar{AB}| = 12.88$$

$$\text{Unit vector} = \frac{\bar{AB}}{|\bar{AB}|} = -\,0.698a_x - 0.155a_y + 0.6987a_z$$

13. Convert the given point A(2, $\frac{®}{2}$, $\frac{®}{3}$) in spherical co-ordinates into Cartesian co-ordinates

Solution:

Given

$r = 2$

$\theta = \frac{®}{2}$

$\Phi = \frac{®}{3}$

Transform

$x = r \sin\Theta \cos\Phi$

$= 2 \sin(\frac{®}{2}) \cos(\frac{®}{3})$

$= 0.055$

$y = r \sin\Theta \sin\Phi$

$= 2 \sin(\frac{®}{2}) \sin(\frac{®}{3})$

$= 1.002$

$z = r \cos\Theta$

$= 2 \cos(\frac{®}{2})$

$= 1.99$

Cartesian co-ordinates are (0.055,1.002,1.99).

14. State divergence theorem.

The integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field over a volume enclosed by the closed surface.

$$\int_V \nabla \cdot \vec{A}\, dv = \oint_S \vec{A} \cdot d\vec{s}$$

15. State stoke's theorem.

The line integral of a vector around a closed path is equal to the surface integral of the normal component of its curl over any surface bounded by the path.

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S \nabla \times \vec{A} \cdot d\vec{s}$$

16. Express the value of differential length, surface in rectangular co-ordinate system.

The differential length dl = $\sqrt{(dx)^2 + (dy)^2 + (dz)^2}$

The differential area ds = dx dy
= dy dz
= dz dx

17. Express the Gradient of a vector in the three system of orthogonal co-ordination.

For Cartesian coordinates system

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

For cylindrical coordinates system

$$\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}\hat{a}_z$$

For spherical coordinates system

$$\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi$$

18. Show that the two vectors $\bar{A} = 6\,\bar{a}_x + \bar{a}_y - 5\,\bar{a}_z$ **and** $\bar{B} = 3(\bar{a}_x - \bar{a}_y + \bar{a}_z)$ **are perpendicular to each other.**

Solution:

$\bar{A} \cdot \bar{B} = (6 \times 3) + (1 \times (-3)) + ((-5) \times 1)$
$= 10$

19. Find the gradient of the scalar field, A = $\circ^{2} Z\,COS\,2$¤

Solution:

$$\nabla.A = \frac{\partial A}{\partial \circ}\bar{a}_{\circ} + \frac{1}{\circ}\frac{\partial A}{\partial ¤}\bar{a}_{¤} + \frac{\partial A}{\partial z}\bar{a}_{z}$$

$$= \frac{\partial}{\partial \circ}(\circ^{2} z \cos 2¤)\,\bar{a}_{\circ} + \frac{1}{\circ}\frac{\partial}{\partial ¤}(\circ^{2} z \cos 2¤)\,\bar{a}_{¤} + \frac{\partial}{\partial z}(\circ^{2} z \cos 2¤)\,\bar{a}_{z}$$

$$\nabla.A = 2\circ z\,[\sin 2¤ + \cos 2¤] + \circ^{2} \cos 2¤$$

20. Define vector field

If a quantity which is specified in a region to design a field is a vector, then the corresponding field is called vector field.

Ex: The velocity of particles in a moving fluid, wind velocity of atmosphere, displacement of a flying bird in a space.

21. **State Gauss's Law**

The electric flux passing through any closed surface is equal to the total charge enclosed by that surface

· = Q

PART – B

1. **What are the different co-ordinate systems used to represent field vectors? Discuss about them in brief.**
2. **Find the gradient of the following scalar fields:**

   (i) **V = e^{-z} sin 2x cosh y**

   (ii) **U =** $\circ^{2} Z\,COS\,2$¤

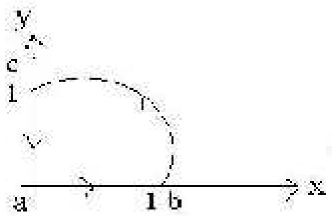
   (iii) **W = 10 r $\sin^{2}\theta$ cosΦ**
3. **Determine the divergence of these vector fields:**

   (i) P = $x^{2}yz\,a_{x} + xz\,a_{z}$

   (ii) Q = $\circ \sin$ ¤ $a_{\circ}$ + $\circ^{2} z\, a_{¤} + z\cos$¤ a_{z}

   (iii) T = $\frac{1}{r^{2}}\cos\bar{\ }\, a_{r} + r\sin\bar{\ }\,\cos$¤ $a_{\bar{\ }} + \cos\bar{\ }\, a_{¤}$

4. **State and Explain Divergence and Stoke's theorems**
5. **Given** $\bar{A} = 2r\cos\ddot{\ }\,\bar{I}_{r} + r\bar{I}_{\ddot{\ }}$ **in cylindrical co-ordinate. For the contour shown verify Stoke's theorem.**

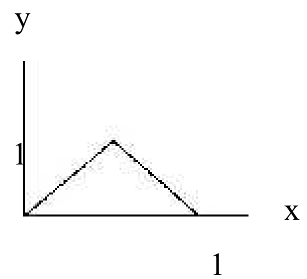


6. Given points P(-2,6,3) and vector A=ya_{x}+(x+z)a_{y}, express P and A in cylindrical and spherical coordinates. Evaluate A at P in the Cartesian, cylindrical and Sphericalsystems.

7. Check validity of the divergence theorem considering the field D=2xy $ax+x^2 ay$ c/m^2 and the rectangular parallelepiped formed by the planes x=0,x=1,y=0,y=2 &z=0,z=3

8. Verify the divergence theorem for the following case $A=XY^2a_x+Y^3a_y+Y^2Za_z$ and the surface is a cuboid defined by 0<x<1, 0<y<1 & 0<z<1

9. Given that $\bar{F}= X^2Ya_x-Ya_y.$ find $\bar{F}.\ dl$ where L is shown in fig. & also verify the stoke's theorem.



10. Drive an expression for the electric field due to a straight and infinite Uniformly charged wire of length 'L' meters and with a charge density of +λ c/m at a point P which lies along the perpendicular bisector of wire.

11. Using Gauss's law calculate the E due to infinitely large uniformly charged plate.

12. Find the Electric potential at a point (4,3)m due to a charge of 10^{-9} clo at the origin in free space.

13. Three point charges $Q_1=10^{-6}$c ,$Q_2=-10^{-6}$c and $Q_3=0.5\text{x}10^{-6}$c are located in air (free space) at the corners of an equilateral triangle of 50cm side. Determine the magnitude and direction of the force on Q_3, Also find Electric field at Q_3.

14. (i) A circular disc of radius 'a', m is charged uniformly with a charge density of σ C/m^2 Find the electric field intensity at a point 'h', m from the disc along its axis.

(ii) A circular disc of 10 cm radius is charged uniformly with a total charge of 10^{-6}c. Find the electric intensity at a point 30 cm away from the disc along the axis.