

# **CE6303 - MECHANICS OF FLUIDS**

(FOR III – SEMESTER)

## **UNIT – II**

### **FLUID STATICS & KINEMATICS**

**PREPARED BY**

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## UNIT – II

### FLUID STATICS & KINEMATICS

Pascal's Law and Hydrostatic equation – Forces on plane and curved surfaces – Buoyancy – Meta centre – Pressure measurement – Fluid mass under relative equilibrium

#### **Fluid Kinematics**

Stream, streak and path lines – Classification of flows – Continuity equation (one, two and three dimensional forms) – Stream and potential functions – flow nets – Velocity measurement (Pilot tube, current meter, Hot wire and hot film anemometer, float technique, Laser Doppler velocimetry)

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## 2 MARKS QUESTIONS AND ANSWERS

**1. Define “Pascal’s Law”:**

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.

**2. What is meant by Absolute pressure and Gauge pressure?**

**Absolute Pressure:**

It is defined as the pressure which is measured with the reference to absolute vacuum pressure.

**Gauge Pressure:**

It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

**3. Define Manometers.**

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing measuring the column of fluid by the same or another column of fluid.

1. Simple M
2. Differential M

**4. A differential manometer is connected at the two points A and B . At B pr is  $9.81 \text{ N/cm}^2$  (abs), find the absolute pr at A.**

$$\text{Pr above } X - X \text{ in right limb} = 1000 \times 9.81 \times 0.6 + p_B$$

$$\begin{aligned} \text{Pr above } X - X \text{ in left limb} &= \\ 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + P_A & \end{aligned}$$

Equating the two pr head

$$\text{Absolute pr at } P_A = 8.887 \text{ N/cm}^2$$

**5. Define Buoyancy.**

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

**6. Define META – CENTRE**

It is defined as the point about which a body starts oscillating when the body is fitted by a small angle. The meta – centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

**7. Write a short notes on “ Differential Manometers”.**

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes/ a differential manometer consists of a U – tube containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are:

1. U – tube differential manometer.
2. Inverted U – tube differential manometers.

**8. Define Centre of pressure.**

Is defined as the point of application of the total pressure on the surface.

The submerged surfaces may be:

1. Vertical plane surface
2. Horizontal plane surface
3. Inclined plane surface
4. Curved surface

**9. Write down the types of fluid flow.**

The fluid flow is classified as :

1. Steady and Unsteady flows.
2. Uniform and Non – uniform flows.
3. Laminar and turbulent flows.
4. Compressible and incompressible flows.
5. Rotational and irrotational flows
6. One, two and three dimensional flows.

**10. Write a short notes on “Laminar flow”.**

Laminar flow is defined as that type of flow in which the fluid particles move along well – defined paths or stream line and all the stream lines are straight and parallel.

Thus the particles move in laminas or layers gliding over the adjacent layer. This type of flow is also called stream – line flow or viscous flow/

**11. Define “ Turbulent flow”.**

Turbulent flow is that type of flow in which the fluid particles move in a zig –zag way. Due to the movement of fluid particles in a zig – zag way.

**12. What is mean by Rate flow or Discharge?**

It is defined as the quantity of a fluid flowing per second through a section of a pipe or channel. For an incompressible fluid( or liquid) the rate of flow or discharge is expressed as volume of fluid flowing across the section per section. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

The discharge (Q) = A X V

Where, A = Cross – sectional area of pipe.

V = Average velocity of fluid across the section.

**13. What do you understand by Continuity Equation?**

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

$$A_1V_1 = A_2V_2..$$

**14. What is mean by Local acceleration?**

Local acceleration is defined as the rate of increase of velocity with respect to time at a given point in a flow field. In equation is given by the expression  $(\partial u / \partial t)$  ,  $(\partial v / \partial t)$  or  $(\partial w / \partial t)$  is known as local acceleration.

**15. What is mean by Convective acceleration?**

It is defined as the rate of change of velocity due to the change of position of fluid particles in a fluid flow. The expressions other than  $(\partial u / \partial t)$  ,  $(\partial v / \partial t)$  and  $(\partial w / \partial t)$  in the equation are known as convective acceleration.

**16. Define Velocity potential function.**

It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is defined by  $\Phi$  (Phi). Mathematically, the velocity, potential is defined as  $\Phi = f ( x,y,z)$  for steady flow such that.

$$u = - (\partial \Phi / \partial x)$$

$$v = - (\partial \Phi / \partial y)$$

$$w = - (\partial \Phi / \partial z)$$

where, u,v and w are the components of velocity in x,y and z directions respectively.

**17. Define Stream function.**

It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. It is denoted by  $\psi$  (Psi) and only for two dimensional flow. Mathematically. For steady flow is defined as  $\psi = f(x,y)$  such that ,

$$\begin{aligned}(\partial \psi / \partial x) &= v \\(\partial \psi / \partial y) &= - u.\end{aligned}$$

**18. What is mean by Flow net?**

A grid obtained by drawing a series of equipotential lines and stream lines is called a flow net. The flow net is an important tool in analyzing the two – dimensional irrotational flow problems.

**19. Write the properties of stream function.**

The properties of stream function ( $\psi$ ) are:

1. If stream function ( $\psi$ ) exists, it is possible case of fluid flow which may be rotational or irrotational.
2. If stream function ( $\psi$ ) satisfies the Laplace equation, it is a possible case of irrotational flow.

**20. What are the types of Motion?**

1. Linear Translation or Pure Translation.
2. Linear Deformation.
3. Angular deformation.
4. Rotation.

**21. Define “Vortex flow”.**

Vortex flow is defined as the flow of a fluid along a curved path or the flow of a rotating mass of a fluid is known ‘ Vortex Flow’. The vortex flow is of two types namely:

1. Forced vortex flow, and
2. Free vortex flow.

**22. Water is flowing through two different pipes, to which an inverted differential manometer having an oil of sp. Gr 0.8 is connected the pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings.**

$$\text{Pr heat at } A = \frac{P_A}{\rho g} = 2m \text{ of water.}$$

$$P_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2$$

$$= 19620 \text{ N/ m}^2$$



$$\text{Pr below X - X in left limb} = P_A - \rho_1 g h_1 = 19620 - 1000 \times 9.81 \times 0.3 = 16677 \text{ N/m}^2$$

$$\text{Pr below X - X in right limb} = P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12 = P_B - 1922.76$$

Equating two pressure, we get,

$$P_B = 16677 + 1922.76 = 18599.76 \text{ N/m}^2 = 1.8599 \text{ N/cm}^2$$

- 23. A differential manometer is connected at the two points A and B . At B pr is 9.81 N/cm<sup>2</sup> (abs), find the absolute pr at A.**

$$\text{Pr above X - X in right limb} = 1000 \times 9.81 \times 0.6 + P_B$$

$$\text{Pr above X - X in left limb} = 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + P_A$$

Equating the two pr head

$$\text{Absolute pr at } P_A = 8.887 \text{ N/cm}^2.$$

- 24. A hydraulic pressure has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic pressure when the force applied at the plunger is 500 N.**

$$\text{Given: Dia of ram, } D = 30 \text{ cm} = 0.3 \text{ m}$$

$$\text{Dia of plunger, } d = 4.5 \text{ cm} = 0.045 \text{ m}$$

$$\text{Force on plunger, } F = 500 \text{ N}$$

**To find :**

$$\text{Weight lifted} = W$$

$$\text{Area of ram, } A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.3)^2 = 0.07068 \text{ m}^2$$

$$\text{Area of plunger, } a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.045)^2 = 0.000159 \text{ m}^2$$

$$\begin{aligned} \text{Pressure intensity due to plunger} &= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{N}{m^2} \\ &= \frac{F}{a} = \frac{500}{0.00159} \text{ N/m}^2 \end{aligned}$$

Due to Pascal law, the intensity of pressure will be equally transmitted in all distance . Hence the pressure intensity at

$$ram = \frac{500}{0.00159} = 314465.4 \text{ N/m}^2$$

$$\text{Pressure intensity at ram} = \frac{\text{weight}}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{0.07068}$$

$$\frac{W}{0.07068} = 314465.4$$

$$\text{Weight} = 314465.4 \times 0.07068 = 22222 \text{ N} = 22.222 \text{ KN}.$$

**25. The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe section 1 is 5 m/s. determine also the velocity at section 2.**

**Solution. Given:**

**At section 1.**  $D_1 = 10 \text{ cm} = 0.1 \text{ m}.$   
 $A_1 = (\pi / 4) \times D_1^2 = (\pi / 4) \times (0.1)^2 = 0.007854 \text{ m}^2.$   
 $V_1 = 5 \text{ m/s}.$

**At section 2.**  $D_2 = 15 \text{ cm} = 0.15 \text{ m}.$   
 $A_2 = (\pi / 4) \times (0.15)^2 = 0.01767 \text{ m}^2.$

1. Discharge through pipe is given by equation

$$\begin{aligned} Q &= A_1 \times V_1 \\ &= 0.007544 \times 5 = 0.03927 \text{ m}^3/\text{s}. \end{aligned}$$

using equation, We have  $A_1 V_1 = A_2 V_2.$

$$V_2 = (A_1 V_1 / A_2) = (0.007854 / 0.01767) \times 5 = 2.22 \text{ m/s}.$$

## 16 MARKS QUESTIONS AND ANSWERS

1. A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs U. U tube is 10 cm and the free surface of mercury is in level with over the centre of the pipe. If the pressure of water in pipe line is reduced to  $9810 \text{ N/m}^2$ , Calculate the new difference in the level of mercury. Sketch the arrangement in both cases.

Given,

$$\text{Difference of mercury} = 10 \text{ cm} = 0.1 \text{ m.}$$

Let  $P_A$  = pr of water in pipe line (ie, at point A)

The point B and C lie on the same horizontal line. Hence pressure at B should be equal to pressure at C.

But pressure at B = Pressure at A and Pressure due to 10 cm (or) 0.1m of water.

$$= P_A + \rho \times g \times h$$

where ,  $\rho = 1000 \text{ kg/m}^3$  and  $h = 0.1 \text{ m}$

$$= P_A + 1000 \times 9.81 \times 0.1$$

$$= P_A + 981 \text{ N / m}^2 \quad \text{(i)}$$

Pressure at C = Pressure at D + pressure due to 10 cm of mercury

$$0 + P_0 \times g \times h_0$$

$$\text{where } p_0 \text{ for mercury} = 13.6 \times 1000 \text{ kg} / \text{m}^3$$

$$h_0 = 10 \text{ cm} = 0.1 \text{ m}$$

$$\begin{aligned} \text{Pressure at C} &= 0 + (13.6 \times 1000) \times 9.81 \times 0.1 \\ &= 13341.6 \text{ N} \end{aligned} \quad (\text{ii})$$

But pressure at B is = to pr @ c. Hence,

equating (i) and (ii)

$$P_A + 981 = 13341.6$$

$$p_A = 13341.6 - 981 = 12360.6 \text{ N} / \text{m}^2$$

II part: Given  $p_A = 9810 \text{ N} / \text{m}^2$

In this case the pressure at A is  $9810 \text{ N/m}^2$  which is less than the  $12360.6 \text{ N/m}^2$ . Hence the mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let,  $x$  = Rise of mercury in left limb in cm

Then fall of mercury in right limb =  $x$  cm.

The points B, C and D show the initial condition.

Whereas points B\*, C\*, and D\* show the final conditions.

The pressure at B\* = pressure at C\*

Pressure at A + pressure due to  $(10-x)$  cm of water.

= pressure at D\* + pressure due to  $(10-2x)$  cm of mercury.

(or)

$$P_A + p_1 g \times h_1 = p D^* + p_2 \times g \times h_2$$

(or)

$$\begin{aligned} \frac{9810}{9.8} + \frac{1000 \times 9.81}{9.81} \left( \frac{10 - x}{100} \right) \\ = 0 + \frac{(13.6 \times 1000) \times 9.81}{9.81} \times \left( \frac{10 - 2x}{100} \right) \end{aligned}$$

Dividing by 9.81 , we get,

$$1000 + 100 - 10x = 1360 - 272x$$

$$272x - 10x = 1360 - 1100$$

$$262 x = 260$$

$$x = \frac{260}{262} = 0.992 \text{ cm}$$

$$\begin{aligned} \therefore \text{ New difference of mercury} &= 10 - 2x \text{ cm} \\ &= 10 - 2 \times 0.992 \\ &= 8.016 \text{ cm.} \end{aligned}$$

2. **A differential manometer is connected at the two points A and B of two pipes a shown in figure. The pipe A contains a liquid of sp. Gr = 1.5 while pipe b contains a liquid of sp . gr = 0.9 . The pressures at A and B are 1 kgf / cm<sup>2</sup> respectively. Find the difference in mercury level in the differential manometer.**

Given,

$$\text{Sp. Gr of liquid at A, } S_1 = 1.5 \quad \therefore p_1 = 1500$$

$$\text{Sp. Gr of liquid at B, } S_2 = 0.9 \quad \therefore p_2 = 900$$

$$\text{Pr at A, } P_A = 1 \text{ kgf / cm}^2 = 1 \times 10^4 \text{ kgf / m}^2$$

$$= 10^4 \times 9.81 \text{ N/m}^2 \quad (1 \text{ kgf} = 9.81 \text{ N})$$

$$\text{Pr at B, } P_B = 1.8 \text{ kgf/cm}^2$$

$$= 1.8 \times 10^4 \times 9.81 \text{ N/m}^2$$

$$\text{Density of mercury} = 13.6 \times 1000 \text{ kg/m}^3$$

$$\text{Pr above X - X in left limb} = 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + P_A$$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 \times 10^4$$

$$\text{Pr above X - X in the right limb} = 900 \times 9.81 \times (h + 2) + P_B$$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating two pressure, we get,

$$13.6 \times 1000 \times 9.81 h + 7500 \times 93.81 + 9.81 \times 10^4$$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by  $1000 \times 9.81$ , we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times 0.9 + 18$$

$$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

$$(13.6 - 0.9)h = 19.8 - 17.5 \quad \text{or} \quad 12.7 h = 2.3$$

$$h = \frac{2.3}{12.7} = 0.181 \text{ m} = 18.1 \text{ cm}$$

- 3, **A vertical sluice gate is used to cover an opening in a dam. The opening is 2m wide and 1.2m high. On the upstream of the gate, the liquid of sp. Gr 1.45, lies upto a height of 1.5m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate and position of centre of pressure. Find also the force acting horizontally at the**

**top of the gate which is capable of opening it. Assume the gate is hinged at the bottom.**

Given,

$$b = 2\text{m} \quad A = b \times d = 2 \times 1.2 = 2.4\text{m}^2$$

$$d = 1.2\text{m} \quad p_1 = 1.45 \times 1000 = 1450\text{kg} / \text{m}^3$$

$F_1$  = force exerted by water on the gate

$$F_1 = p_1 g \overline{Ah_1}$$

$$p_1 = 1.45 \times 1000 = 1450\text{kg} / \text{m}^3$$

$\overline{h}$  = depth of C.G of gate from free surface of liquid.

$$= 1.5 + \frac{1.2}{2} = 2.1\text{m}$$

$$F_1 = 1450 \times 9.81 \times 2.4 \times 2.1$$

$$= 71691\text{N}$$

$$F_2 = p_2 g \overline{Ah_2}$$

**Centre of Pressure ( $h^*$ ):**

Centre of pressure is calculated by using the “Principle of Moments” which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

$$F_2 = p_2 g \overline{Ah_2}$$

$$p_2 = 1000\text{kg} / \text{m}^3$$

$\overline{h_2}$  = Depth of C.G of gate from free surface of water.

$$= \frac{1}{2} \times 1.2 = 0.6\text{m}$$

$$F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126\text{N}$$

i. Resultant force on the gate  $= F_1 - F_2 = 71691 - 14126$   
 $= 57565 \text{ N.}$

ii. Position of centre of pressure of resultant force:

$$h_1^* = \frac{I_G}{Ah_1} + \bar{h}_1$$

$$I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288 \text{ m}^4$$

$$h_1^* = \frac{0.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571 \text{ m}$$

$$\therefore \text{Distance of } F_1 \text{ from hinge} = (1.5 + 1.2) - h_1^* \\ = 2.7 - 2.1571 = 0.5429 \text{ m.}$$

The force  $F_2$  will be acting at a depth of  $h_2^*$  from free surface of water and is given by

$$h_2^* = \frac{I_G}{Ah_2} + \bar{h}_2$$

where  $I_G = 0.288 \text{ m}^4$ ,  $\bar{h}_2 = 0.6 \text{ m}$ ,  $A = 2.4 \text{ m}^2$ ,

$$h_2^* = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8 \text{ m}$$

Distance of  $F_2$  from hinge  $= 1.2 - 0.8 = 0.4 \text{ m.}$

The resultant force  $57565 \text{ N}$  will be acting at a distance given by

$$= \frac{71691 \times 0.5429 - 14126 \times 0.4}{57565} \\ = \frac{38921 - 5650.4}{57565} \text{ m above hinge.}$$



= 0.578 m above the hinge.

iii. Force at the top of gate which is capable of opening the gate:

Taking moment of  $F$ ,  $F_1$  and  $F_2$  about the hing.

$$F \times 1.2 + F_2 \times 0.4 = F_1 \times 0.5429$$

$$F = \frac{F_1 \times 0.5429 - F_2 \times 0.4}{1.2}$$

$$= \frac{71691 \times 0.5429 - 14126 \times 0.4}{1.2}$$

$$= 27725.5 \text{ N.}$$

**4. Find the density of a metallic body which floats at the interface of mercury of sp. Gr 13.6 and water such that 40% of its volume is sub-merged in mercury and 60% in water.**

Solution:

Let the volume of the body =  $V \text{ m}^3$

Then volume of body sub-merged in mercury

$$= \frac{40}{100} V = 0.4Vm^3$$

Volume of body sub-merged I water

$$= \frac{60}{100} \times V = 0.6Vm^3$$

For the equilibrium of the body

Total buoyant force (upward force) = Weight of the body.

But total buoyant force = Force of buoyancy due to water +

Force of buyance due to mercury.

Force of buoyancy due to water = Weight of water displaced by body.

$$= \text{Density of water} \times g \times \text{Volume of water displaced.}$$

$$= 1000 \times g \times \text{volume of body in water.}$$

$$= 1000 \times g \times 0.6 \times V \text{ N}$$

Force of buoyancy due to mercury = Weight of mercury displaced by body.

$$= g \times \text{Density of mercury} \times \text{Volume of mercury displaced.}$$

$$= g \times 13.6 \times 1000 \times \text{Volume of body in mercury}$$

$$= g \times 13.6 \times 1000 \times 0.4V \text{ N.}$$

Weight of the body = Density  $\times$   $g$   $\times$  volume of body

$$= p \times g \times V$$

where  $p$  is the density of the body

For equilibrium,

Total buoyant force = Weight of the body

$$1000 \times g \times 0.6 \times V + 13.6 \times 1000 \times g \times 0.4V = p \times g \times V$$

$$p = 600 + 13600 \times 0.4 = 600 + 54400$$

$$= 6040.00 \text{ kg / m}^3$$

$$\therefore \text{Density of the body} = 6040.00 \text{ kg / m}^3$$

- 5. A solid cylinder of diameter 4.0 m has a height of 3m. Find the meta – centric height of the cylinder when it is floating in water with its axis vertical. The sp gr of the cylinder – 0.6 .**

Given

$$\text{Dia of cylinder , } D = 4.0 \text{ m}$$

$$\text{Height of cylinder, } h = 3.0 \text{ m}$$

$$\text{Sp, gr of cylinder} = 0.6$$

$$\text{Depth of immersion of cylinder} = 0.6 \times 3.0$$

$$= 1.8 \text{ m}$$

$$AB = \frac{1.8}{2} = 0.9m$$

$$AG = \frac{3}{2} = 1.5m$$

$$BG = AG - AB$$

$$= 1.5 - 0.9 = 0.6 \text{ m.}$$

Now the meta – centric height GM is given by equation  $GM = \frac{I}{V} - BG$

I = Moment of Inertia about Y – Y axis of plan of the body

$$= \frac{\pi}{4} D^4 = \frac{\pi}{64} \times (4.0)^4$$

V = volume of cylinder in water.

$$= \frac{\pi}{4} D^2 \times \text{Depth of immersion}$$

$$= \frac{\pi}{4} (4)^2 \times 1.8m^3$$

$$GM = \frac{\frac{\pi}{64} \times (4.0)^4}{\frac{\pi}{4} \times (4.0)^2 \times 1.8} - 0.6$$

$$= \frac{1}{16} \times \frac{4.0^2}{1.8} - 0.6 = \frac{1}{18} - 0.6 = 0.55 - 0.6$$

$$= - 0.05 \text{ m.}$$

- Ve sign indicates meta- centre (M) below the centre of gravity (G)

6. A wooden cylinder of sp. Gr = 0.6 and circular in cross – section is required to float in oil (sp.gr = 0.90) . Find the L/D ratio for the cylinder to float with its longitudinal axis vertical in oil, where L is the height of cylinder and D is its diameters.

Solution:

$$\text{Dia of cylinder} = D$$

$$\text{Height of cylinder} = L$$

$$\text{Sp gr of cylinder, } S_1 = 0.6$$

$$\text{Sp gr of oil, } S_2 = 0.9$$

Let, depth of cylinder immersed in oil = h

Buoyancy principle

Weight of cylinder = Weight of oil dispersed.

$$\frac{\pi}{4} D^2 \times L \times 0.6 \times 1000 \times 9.81 = \frac{\pi}{4} D^2 \times h \times 0.9 \times 1000 \times 9.81$$

$$L \times 0.6 = h \times 0.9$$

$$h = \frac{0.6 \times L}{0.9} = \frac{2}{3} L$$

Dts of centre of gravity C from A,  $AG = \frac{L}{2}$

The distance of centre of buoyancy B from A,  $AB = \frac{h}{2} = \frac{1}{2} \left[ \frac{2}{3} L \right]$

$$BG = AG - AB = \frac{L}{2} - \frac{L}{3} = \frac{3L - 2L}{6} = \frac{L}{6}$$

$$GM = \frac{1}{V} - BG \qquad I = \frac{\pi D^4}{64} \qquad V = \text{Volume of cylinder in oil.}$$

$$= \frac{\pi}{4} D^2 h$$

$$\frac{1}{V} = \left( \frac{\frac{\pi}{64} D^4}{\frac{\pi}{4} D^2 h} \right) = \frac{1}{16} \frac{D^2}{h} = \frac{D^2}{16 \times \frac{2}{3} L} = \frac{3D^2}{32L}$$

$$GM = \frac{3D^3}{32L} - \frac{L}{6}$$

For stable equilibrium, GM should be +Ve or

$$G_M \rightarrow 0 \quad (\text{or}) \quad \frac{3D^2}{32L} - \frac{L}{6} > 0$$

$$\frac{3D^2}{32L} - \frac{L}{6} \quad (\text{or}) \quad \frac{3 \times 6}{32} > \frac{L^2}{D^2}$$

$$\frac{L^2}{D^2} < \frac{18}{32} \quad (\text{or}) \quad \frac{9}{16}$$

$$\frac{L}{D} < \sqrt{\frac{9}{16}} = \frac{3}{4} \Rightarrow \frac{L}{D} < \frac{3}{4}$$

### Experimental Method of Determination of Metacentric Height

W = Weight of vessel including  $w_1$

G = Centre of gravity of the vessel

B = Centre of buoyancy of the vessel

$$GM = \frac{\omega_1 x}{\omega \tan \theta}$$

7. Water flows through a pipe AB 1.2m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter at C, the pipe branches. Branch CD is 0.8m in diameter and carries one third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.

**Solution. Given:**

Diameter of Pipe AB,	$D_{AB} = 1.2 \text{ m.}$
Velocity of flow through AB	$V_{AB} = 3.0 \text{ m/s.}$
Dia. of Pipe BC,	$D_{BC} = 1.5\text{m.}$
Dia. of Branched pipe CD,	$D_{CD} = 0.8\text{m.}$
Velocity of flow in pipe CE,	$V_{CE} = 2.5 \text{ m/s.}$
Let the rate of flow in pipe	$AB = Q \text{ m}^3/\text{s.}$
Velocity of flow in pipe	$BC = V_{BC} \text{ m}^3/\text{s.}$
Velocity of flow in pipe	$CD = V_{CD} \text{ m}^3/\text{s.}$

Diameter of pipe  $CE = D_{CE}$

Then flow rate through  $CD = Q / 3$

And flow rate through  $CE = Q - Q/3 = 2Q/3$

(i). Now the flow rate through  $AB = Q = V_{AB} \times \text{Area of } AB$

$$\begin{aligned} &= 3 \times (\pi / 4) \times (D_{AB})^2 = 3 \times (\pi / 4) \times (1.2)^2 \\ &= 3.393 \text{ m}^3/\text{s.} \end{aligned}$$

(ii). Applying the continuity equation to pipe AB and pipe BC,

$$V_{AB} \times \text{Area of pipe } AB = V_{BC} \times \text{Area of Pipe } BC$$

$$3 \times (\pi / 4) \times (D_{AB})^2 = V_{BC} \times (\pi / 4) \times (D_{BC})^2$$

$$3 \times (1.2)^2 = V_{BC} \times (1.5)^2$$

$$V_{BC} = (3 \times 1.2^2) / 1.5^2 = 1.92 \text{ m/s.}$$

(iii). The flow rate through pipe

$$CD = Q_1 = Q/3 = 3.393 / 3 = 1.131 \text{ m}^3/\text{s.}$$

$$Q_1 = V_{CD} \times \text{Area of pipe } C_D \times (\pi / 4) \times (C_{CD})^2$$

$$1.131 = V_{CD} \times (\pi / 4) \times (0.8)^2$$

$$V_{CD} = 1.131 / 0.5026 = 2.25 \text{ m/s.}$$

(iv). Flow through CE,

$$Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262 \text{ m}^3/\text{s}$$

$$Q_2 = V_{CE} \times \text{Area of pipe CE} = V_{CE} \times (\pi / 4) (D_{CE})^2$$

$$2.263 = 2.5 \times (\pi / 4) (D_{CE})^2$$

$$D_{CE} = \sqrt{(2.263 \times 4) / (2.5 \times \pi)} = 1.0735 \text{ m}$$

Diameter of pipe CE = 1.0735m.

8. In a two – two dimensional incompressible flow, the fluid velocity components are given by  $u = x - 4y$  and  $v = -y - 4x$ . show that velocity potential exists and determine its form. Find also the stream function.

**Solution. Given:**

$$u = x - 4y \text{ and } v = -y - 4x$$

$$(\partial u / \partial x) = 1 \text{ \& } (\partial v / \partial y) = -1.$$

$$(\partial u / \partial x) + (\partial v / \partial y) = 0$$

hence flow is continuous and velocity potential exists.

$$\text{Let } \Phi = \text{Velocity potential.}$$

Let the velocity components in terms of velocity potential is given by

$$\partial \Phi / \partial x = -u = -(x - 4y) = -x + 4y \text{ ----- (1)}$$

$$\partial \Phi / \partial y = -v = -(-y - 4x) = y + 4x. \text{----- (2)}$$

$$\text{Integrating equation(i), we get } \Phi = -(x^2 / 2) + 4xy + C \text{---- (3)}$$

Where C is a constant of Integration, which is independent of 'x'.

This constant can be a function of 'y'.

Differentiating the above equation, i.e., equation (3) with respect to 'y', we get

$$\partial \Phi / \partial y = 0 + 4x + \partial C / \partial y$$

But from equation (3), we have  $\partial \Phi / \partial y = y + 4x$

Equating the two values of  $\partial \Phi / \partial y$ , we get

$$4x + \partial C / \partial y = y + 4x \quad \text{or } \partial C / \partial y = y$$

Integrating the above equation, we get

$$C = (y^2 / 2) + C_1.$$

Where  $C_1$  is a constant of integration, which is independent of 'x' and 'y'.

Taking it equal to zero, we get  $C = y^2/2$ .

Substituting the value of C in equation (3), we get.

$$\Phi = - (x^2/2) + 4xy + y^2/2.$$

### Value of stream functions

Let  $\partial\psi/\partial x = v = -y - 4x$ . ----- (4).

Let  $\partial\psi/\partial y = -u = - (x - 4y) = x + 4y$  -----(5)

Integrating equation (4) w.r.t. 'x' we get

$$\Psi = -yx - (4x^2/2) + k$$
 -----(6)

Where k is a constant of integration which is independent of 'x' but can be a function 'y'.

Differentiating equation (6) w.r.to. 'y' we get,

$$\partial\psi/\partial x = -x - 0 + \partial k/\partial y$$

But from equation (5), we have  $\partial\psi/\partial y = -x + 4y$

Equating the values of  $\partial\psi/\partial y$ , we get  $-x + \partial k/\partial y$  or  $\partial k/\partial y = 4y$ .

Integrating the above equation, we get  $k = 4y^2/2 = 2y^2$ .

Substituting the value of k in equation (6), we get.

$$\Psi = -yx - 2x^2 + 2y^2$$