Unit – I FINITE AUTOMATA

1. Define hypothesis.

The formal proof can be using deductive proof and inductive proof. The deductive proof consists of sequence of statements given with logical reasoning in order to prove the first or initial statement. The initial statement is called hypothesis.

2. Define inductive proof.

It is a recursive kind of proof which consists of sequence of parameterized statements that use the statement itself with lower values of its parameter.

3. Define Set, Infinite and Finite Set.

Set is Collection of various objects. These objects are called the elements of the set.

Eg : A = { a, e, i, o, u }

Infinite Set is a collection of all elements which are infinite in number.

Eg: A = { a | a is always even number }

Finite Set is a collection of finite number of elements.

Eg : A = { a, e, i, o, u }

4. Give some examples for additional forms of proof.

1. Proofs about sets
2. Proofs by contradiction
3. Proofs by counter examples.

5. Prove 1+2+3+………………+n= n(n+1)/2 using induction method.

Consider the two step approach for a proof by method of induction

1. Basis of induction :

Let n = 1 then LHS = 1 and RHS = 1 + 1 / 2 = 1 Hence LHS = RHS.

2. Induction hypothesis :

To prove 1 + 2 + 3 …… + n = n (n + 1) / 2 + (n + 1)

Consider n = n + 1

then 1 + 2 + 3 ……+ n + (n + 1) = n (n + 1) / 2 + (n + 1)

= $n^2 + 3n + 2 / 2$

= (n + 1) (n + 2) /2

Thus it is proved that 1 + 2 + 3 …… + n = n (n + 1) / 2

6. Write down the operations on set.

i) **A U B is Union Operation**

If A = { 1, 2, 3 } B = { 1, 2, 4 } then

A U B = { 1, 2, 3, 4 }

i.e. combination of both the sets.

ii) **A ∩ B is Intersection operation**

If A = { 1, 2, 3 } B = { 1, 2, 4 } then

A U B = { 2, 3 }

i.e. Collection of common elements from both the sets.

iii) **A – B is the difference operation**

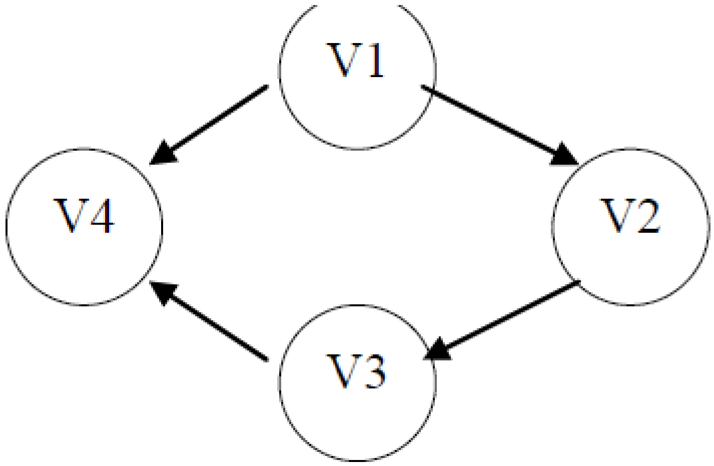
If A = { 1, 2, 3 } B = { 1, 2, 4 } then

A U B = { 1 }

i.e. elements which are there in set A but not in set B.

7. Define Graph, Directed graph and give example.

Graph is consists of finite set of Vertices (Node) V and set of Edges E, edges are nothing but pair of vertices.
It denoted G = (V, E)
E1 is a edge connecting the vertices V1 and V2.
Directed Graph is consists of finite set of Vertices (Node) V and set of Edges E, edges are nothing but pair of vertices.
It denoted G = (V, E)
Eg:



The edge E1 shows the direction to V2 from V1.

8. Write any three applications of Automata Theory.

1. It is base for the formal languages and these formal languages are useful of the programming languages.
2. It plays an important role in complier design.
3. To prove the correctness of the program automata theory is used.
4. In switching theory and design and analysis of digital circuits automata theory is applied.
5. It deals with the design finite state machines.

9. Define Finite Automation.

A finite automata is a collection of 5 tuples (Q, Σ. δ, q_0, F)
where Q is a finite set of states, which is non empty.
Σ is a input alphabet, indicates input set.
δ is a transition function or a function defined for going to next state.
q_0 is an initial state (q_0 in Q)
F is a set of final states.
Two types :
Deterministic Finite Automation (DFA)
Non-Deterministic Finite Automation. (NFA)

10. Define Deterministic Finite Automation.

- The finite automata is called DFA if there is **only one path for a specific input from current state to next state.**
- A finite automata is a collection of 5 tuples (Q, Σ. δ, q_0, F)

where Q is a finite set of states, which is non empty.
Σ is a input alphabet, indicates input set.
δ is a transition function or a function defined for going to next state.
q_0 is an initial state (q_0 in Q)
F is a set of final states.

11. Define Non-Deterministic Finite Automation.

The finite automata is called NFA when there exists **many paths for a specific input from current state to next state.**
A finite automata is a collection of 5 tuples (Q, Σ. δ, q_0, F)
where Q is a finite set of states, which is non empty.
Σ is a input alphabet, indicates input set.
δ is a transition function or a function defined for going to next state.
q_0 is an initial state (q_0 in Q)
F is a set of final states.

12. Define NFA with □ transition.

The □ **is a character** used to indicate null string.
i.e the string which is used simply for transition from **one state to other state without any input.**
A Non Deterministic finite automata is a collection of 5 tuples (Q, Σ. δ, q_0, F)
where Q is a finite set of states, which is non empty.
Σ is a input alphabet, indicates input set.
δ is a transition function or a function defined for going to next state.
q_0 is an initial state (q_0 in Q)
F is a set of final states.

13. Explain the transition function.

The mapping function or transition function denoted by δ. Two parameters are passed to this transition function : (i) current state and (ii) input symbol. The transition function returns a state which can be called as next state.
Eg.:
δ (q0, a) = q1

14. Write short notes on Minimization of DFA?

- Reducing the number of states from given FA
- First find out which two states are equivalent we than replace those two states by one representative state.
- For finding the equivalent states we will apply the following rule
- The two states S1 & S2 are equivalent if and only if both the states are final or non-final states.

15.State regular expression.

Let Σ be an alphabet. The regular expressions over Σ and the sets that they denote are defined recursively as follows
a. Ø is a regular expression and denotes the empty set.
b. □ is a regular expression and denotes the set {□ }

c. For each 'a' □ Σ, 'a' is a regular expression and denotes the set {a}.
d. If 'r' and 's' are regular expressions denoting the languages L_1 and L_2 respectively then
r + s is equivalent to $L_1 \cup L_2$ i.e. union
rs is equivalent to L_1L_2 i.e. concatenation
r^* is equivalent to L_1^* i.e. closure

16. How the kleen's closure or closure of L can be denoted?

$L^* = \bigcup_{i=0}^{n} L^i$ (e.g. a^* ={□ ,a,aa,aaa,……})

17. How do you represent positive closure of L?

$L^+ = \bigcup_{i=1}^{n} L^i$ (e.g. a^+ ={a,aa,aaa,……})

17. Write the regular expression for the language accepting all combinations of a's over the set □ = {a}.

L = { a,aa,aaa,………………….}
R= a^* (i.e. kleen closure)

18. Write regular expression for the language accepting the strings which are starting with 1 and ending with 0, over the set □ = {0,1}.

L = { 10,1100,1010,100010…………………}
R= $1(0+1)^*0$19**6. Show that (0*1*)* = (0+1)*.**

LHS : (0*1*)* = { □ , 0,1,00,11,0011,011,0011110………………}
RHS : (0+1)* = { □ , 0,1,00,11,0011,011,0011110………………}
Hence
LHS = RHS is proved

19. Show that (r+s)* □ r* + s*.

LHS : (r+s)* = { □ , r,s,rs,rr,ss,rrrsssr,………………}
RHS : r* + s* = { □ , r,rr,rrr…………}U { □ , s,ss,sss,…………}
= { □ , r,rr,rrr,s,ss,ssss……………..}
Hence
LHS ≠ RHS is proved

20.What do you mean by homomorphism?

A string homomorphism is a function on strings that works by substring a particular sting for each symbol.
Eg. h(0) = ab
h(1) = □ is a homomorphism, where replace all 0's by ab and replace all 1's by □ .
Let w = 0011
h(w) = abab

21. Explain the application of the pumping lemma.

Pumping Lemma is used to prove the language is not regular.

22. Describe the following by regular expression

a. L1 = the set of all strings of 0's and 1's ending in 00.

b. L2 = the set of all strings of 0's and 1's beginning with 0 and ending with 1.

r1 = (0+1)*00
r2 = 0(0+1)*1

23. Show that (r*)* = r* for a regular expression r.

LHS = r* = { ε, r,rr,rrr, …………….)
(r*)* = { ε, r,rr,rrr, …………….)*
(r*)* = { ε, r,rr,rrr, …………….) = r*
LHS = RHS

24. Write down the closure properties of regular language.

The regular languages are closed under
1. Union
2. Intersection
3. Complement
4. Difference
5. Reversal
6. Closure
7. Concatenation
8. Homomorphism
9. Inverse Homomorphism

2MARKS

1. What is Computation? and Write short notes on TOC.
2. Define Automaton
3. Define Inductive and Deductive proof
4. Define hypothesis.
5. What is the principle of Mathematical Induction?
6. List any four ways of theorem proving.
7. What is structural Induction?
8. Write the central concepts of Automata Theory
9. Define Language and Give example.
10. Define transition diagram.
11. What is Finite Automata and explain the applications of Finite automata.
12. Define the languages described by NFA and DFA.
13. Give the DFA accepting the language over the alphabet 0,1 that have the set of
14. All strings beginning with 101.
15. Give the DFA accepting the language over the alphabet 0,1 that have the set of
16. all strings containing 1101 as a substring.
17. Give the DFA accepting the language over the alphabet 0,1 that have the set of
18. all strings ending in 00.
19. Give the DFA accepting the language over the alphabet 0,1 that have the set of
20. all strings with three consecutive 0's.
21. Give the DFA accepting the language over the alphabet 0,1 that have the set of
22. all strings with 011 as a substring.
23. Give the DFA accepting the language over the alphabet 0,1 that have the set of
24. all strings whose 10_{th} symbol from the right end is 1.
25. Construct a DFA for the following
    a) All stings that contain exactly 4 zeros

b) All strings that don't contain the substring 110.

26. Give the DFA accepting the language over the alphabet 0,1 that have the set of
27. all strings that either begins or end(or both) with 01.
28. Give the DFA accepting the language over the alphabet 0,1 that have the set of
29. all strings such that the no of zero's is divisible by 5 and the no of 1's is
30. divisible by 3.
31. Difference between DFA and NFA
32. Define NFA.
33. Define the language of NFA.
34. . Is it true that the language accepted by any NFA is different from the regular
35. language? Justify your Answer.
36. . Define ε-NFA.
37. Define ε closure.
38. Find the εclosure for each state from the following automata.

Part B

a) If L is accepted by an NFA with ε-transition then show that L is accepted by an
NFA without ε-transition.

b) Construct a DFA equivalent to the NFA.

M=({p,q,r},{0,1}, δ,p,{q,s})

Where δis defined in the following table.

δ	0	1
p	{q,s}	{q}
q	{r}	{q,r}
r	{s}	{p}
s	-	{p}

a)Show that the set $L=\{a^n b^n/n>=1\}$ is not a regular. (6) b)Construct a DFA equivalent to the NFA given below: (10)

	0	1
p	{p,q}	P
q	r	R
r	s	-
s	s	S

a)Check whether the language $L=(0^n 1^n/n>=1)$ is regular or not? Justify your answer.

b) Let L be a set accepted by a NFA then show that there exists aDFA that accepts L.

Define NFA with ε-transition. Prove that if L is accepted by an NFA with ε-transition then L is also accepted by a NFA without ε-transition.

a) Construct a NDFA accepting all string in $\{a,b\}^+$ with either two consecutive a's or two consecutive b's.

b) Give the DFA accepting the following language:set of all strings beginning with a 1 that when interpretedas a binary integer is a multiple of 5.

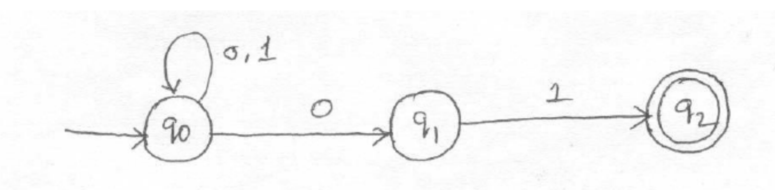
Draw the NFA to accept the following languages.

Set of Strings over alphabet {0,1,…….9} such that the final digit has appeared before. (8)

(ii)Set of strings of 0's and 1's such that there are two 0's separated by a number of positions that is a multiple of 4.

7.a)Let L be a set accepted by an NFA.Then prove that there exists a deterministic finite automaton that accepts L.Is the converse true? Justify your answer. (10)

b)Construct DFA equivalent to the NFA given below: (6)



8.a) Prove that a language L is accepted by some ε–NFA if and only if L is accepted by some DFA. (8)

b) Consider the following ε–NFA.Compute the ε–closure of each state and find it's equivalent DFA. (8)

	ε	A	b	C
p	{q}	{p}	Φ	Φ

q	{r}	ϕ	{q}	Φ
*r	Φ	ϕ	ϕ	{r}

9.a) Prove that a language L is accepted by some DFA if L is accepted by some NFA.

b) Convert the following NFA to it's equivalent DFA

	0	1
p	{p,q}	{p}
q	{r}	{r}
r	{s}	ϕ
*s	{s}	{s}

10.a) Explain the construction of NFA with εtransition from any given regular expression.

b) Let A=(Q,∑, δ, q0 ,{qf) be a DFA and suppose that for all a in ∑wehave δ(q0, a)= δ(qf ,a). Show that if x is a non empty string in L(A),then for all k>0,x^k is also in L(A).