Unit – IV

Groups, Algebraic, Structures

(1) Define Semi group and monoid.

Ans :–

Let S be non-empty set with binary operation $*$ on it. Then S is called a semi group w.r.to $*$ if $*$ is associative

$$a*(b*c) = (a*b)*c \quad \forall a,b,c \in S.$$

monoid

A semi group with an identity element is a monoid.

(2) Find All sub groups of the symmetric group $(S_3, \circ)$.

Ans :– $S_3 = \{P_1, P_2, P_3, P_4, P_5, P_6\}$

The sub groups are

$H_1 = \{(P_1, P_2, P_3), \circ\}$

$H_2 = \{\{P_1, P_4\}, \circ)$, $H_3 = (\{P_1, P_5\}, \circ)$

$H_4 = (\{P_1, P_6\}, \circ)$.

(3) Prove that the intersection of two sub groups of a group G is also a sub group.

Proof :–

Let H and K be two subgroups of group $(G, *)$

Let $a, b \in H \cap K$

To prove $a * b^{-1} \in H \cap K$.

$a, b \in H \cap K \Rightarrow a, b \in H$ and $a, b \in K$

$\Rightarrow a * b^{-1} \in H$ and $a * b^{-1} \in K$ [$\because$ H and K are subgroups)

$\Rightarrow a * b^{-1} \in H \cap K$

$\therefore H \cap K$ is sub group of $(G, *)$

(4) Find all the sub groups of $(Z_{12}, +_{12})$

Ans:-

$Z_{12} = \{1, 2, 3, 4 ---- 12\}$

the possible proper subgroups of $(Z_{12} +12)$ are

$H_1 = \{0, 6\}$, $H_2 = \{0, 4, 8\}$, $H_3 = \{0, 3, 6, 9\}$

$H_4 = \{0, 2, 4, 6, 8, 10\}$.

(5) A semi group homomorphism preserves idempotency.

proof:-

Let $a \in S$ be an idempotent element

$\therefore a * a = a$

$\therefore g(a * a) = g(a)$

$\therefore g(a) \circ g(a) = g(a)$ / $\because$ g is semi group homo

$\therefore g(a)$ is an idempotent element

(6) ~~...~~ prove that a monoid homomorphism preserves the property of invertibility.

proof

Let $g : M \rightarrow T$ be monoid homo

to prove If $a^{-1} \in M$ is the inverse of $a \in M$. then $g(a^{-1})$ is the inverse of $g(a)$.

w.k.t $a * a^{-1} = e$

$\Rightarrow g(a * a^{-1}) = g(e)$

Unit – IV
Algebraic structures.

Define ring given an example of a ring which has zero divisors.

Soln

An algebraic system $(S, +, *)$ is called a ring if the binary operation + and • on S satisfy the following properties.

1. $(S, +)$ is an abelian group
2. $(S, *)$ is a semi group
3. The operation * is distributive over + ; that is, for any $a, b, c \in S$

$a \cdot (b+c) = a \cdot b + a \cdot c$ and

$(b+c) \cdot a = b \cdot a + c \cdot a$

Example. The ring $(Z_{10}, +_{10}, \times_{10})$ is not an integral domain.

$5 \times_{10} 2 = 0$, yet $5 \neq 0$, $2 \neq 0$ in Z_{10}

If f is a homomorphism of a group G into a group G' then prove that group homomorphisms preserves identities.

Soln Let $a \in G$, and f is a homo from G into G' then, $f(e) \in G'$

$\Rightarrow f(a) * e' = f(a) = f(a) * f(e)$

$\Rightarrow f(e) = e'$ (by left cancellation law).