UNIT III – APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

PART –A

1. **Classify the partial differential equation 4 $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$**

ANS

Here A=4 ; B=0; C=0

B^2-4AC = 0-0=0

The PDE is parabolic

2. **In the equation of motion of vibrating string $a^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$, what does a^2 stand for?**

ANS

$a^2 = \frac{T}{m}$ where T is the tension of the string and m is the mass per unit length of the string

3. **Write down all possible solutions of one dimensional wave equation**

ANS

- $y(x,t) = [Ae^{px}+Be^{-px}][Ce^{pat}+De^{-pat}]$
- $y(x,t) = [A\cos(px)+B\sin(px)][C\cos(pat)+D\sin(pat)]$
- $y(x,t) = [Ax+B][Ct+D]$

4. **A tightly stretched string with fixed end points x=0 and x=*l* is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released form rest in this position, write the boundary conditions.**

ANS

The boundary conditions are

- $y(0,t) = 0$
- $y(l,t) = 0$
- $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$
- $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$

5. **If ends of a string of length 'l' are fixed and the midpoint of the string is drawn aside through a height 'h' and the string is released from rest, state the initial and boundary conditions.**

ANS

The conditions are

- $y(0,t)= 0$
- $y(l,t) = 0$
- $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

- y(x,0)= $\begin{cases} \frac{2hx}{l}, 0 \le x \le l/2 \\ \frac{2h(l-x)}{l}, l/2 \le x \le l \end{cases}$

6. **Write the initial conditions of the wave equation if the string has an initial displacement f(x) but no initial velocity**

   ANS

   The displacement y(x,t) is from $a^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$

   The conditions are
   - y(0,t) = 0
   - y(l,t) = 0
   - $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$
   - $y(x,0) = f(x)$

7. **Write the initial conditions of the wave equation if the string has an initial velocity g(x) but has no initial displacement.**

   ANS

   The displacement y(x,t) is from $a^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$

   The conditions are
   - y(0,t) = 0
   - y(l,t) = 0
   - $y(x,0) = 0$
   - $\left(\frac{\partial y}{\partial t}\right)_{t=0} = g(x)$

8. **What does a^2 represent in one dimensional heat flow equation $u_t=a^2u_{xx}$?**

   ANS

   $a^2=\frac{k}{pc}$ where k is the thermal conductivity; c is the density and p is the specific heat

9. **In steady state conditions, derive the solution of one dimensional heat flow equation**

   ANS

   The one dimensional heat flow equation is $\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

   In steady state, $\frac{\partial u}{\partial t} = 0$

   $$\Rightarrow \frac{\partial^2 u}{\partial x^2} = 0$$

   $$\Rightarrow \frac{\partial u}{\partial x} = a$$

   $$\Rightarrow u = ax + b$$

10. **Write down the three possible solutions of one dimensional heat equation**

ANS

- ✓ $u(x,t) = [Ae^{px}+Be^{-px}]\, Ce^{\alpha^2 p^2 t}$
- ✓ $u(x,t) = [A\cos(px)+B\sin(px)]\, Ce^{-\alpha^2 p^2 t}$
- ✓ $u(x,t)= [Ax+B]C.$

11. **What is the basic difference between the solution of one dimensional wave equation and one dimensional heat equation?**

ANS

S.NO.	One dimensional wave equation	One dimensional heat equation
1	$a^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$ is hyperbolic	$\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ is parabolic
2	The suitable solution $y(x,t)=[A\cos px+B\sin px][C\cos pat+D\sin pat]$ is periodic w.r.t.time	The suitable solution $u(x,t) = [A\cos(px)+B\sin(px)]C\, e^{-\alpha^2 p^2 t}$ is not periodic with respect to time

12. **What are the possible solutions for Laplace equation $U_{xx}+U_{yy}=0$ by method of separation of variables? (OR) Write all the three possible solutions of steady state two dimensional heat equation**

ANS

- ➢ $u(x,y) = [Ae^{px}+Be^{-px}][C\cos(py)+D\sin(py)]$
- ➢ $u(x,y) = [A\cos(px)+B\sin(px)][Ce^{py}+De^{-py}]$
- ➢ $u(x,y)= [Ax+B][Cy+D]$

PART – B

1. A uniform string is stretched and fastened to two points ‘l’ apart. Motion is started by displacing the string into the form of the curve y=kx (l-x) and then released from this position at time t=0. Derive the expression for the displacement of any point of the string at a distance x from one end at time t.
2. A tightly stretched string with fixed end points x=0 and x=l is initially displaced in the position $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$ and then released form rest. Find the displacement y at any distance x from one end at time t.
3. A tightly stretched string of length ‘l’ has its ends fastened at x=0 and x=l. The midpoint of the string is then taken to height h and released from rest in that position. Find the displacement of a point of the string at time t from the instant of release.
4. A tightly stretched string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $V_0 \sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement y(x,t).
5. A string of length l is initially at rest in its equilibrium position and motion is started by giving each of the points a velocity given by $V = \begin{cases} cx, 0 \le x \le l/2 \\ c(l-x), l/2 \le x \le l \end{cases}$. Find the displacement function y(x,t).

6. A rod 30 cm long has its ends A and B kept at 30^0 C and 80^0 C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0 C and kept so on. Find the resulting temperature function u(x,t) taking x=0 at A.
7. The ends A and B of a rod 40 cm long have their temperature kept at 0^0 C and 80^0 C respectively until steady state condition prevails. The temperature of the end B is then suddenly reduced to 40^0 C and kept so while that of the end A is kept at 0^0 C. Find the subsequent temperature distribution u(x,t) in the rod.
8. A square plate is bounded by the lines x=0; y=0; x=20 and y=20. Its faces are insulated. The temperature along the upper horizontal edge is given by u(x,20) = x(20-x),0<x<20, while the other edges are kept at 0^0C. Find the steady state temperature distribution in the plate.
9. A rectangular plate with insulated surfaces is 20 cm wide and so long compared to its width that it may be considered infinite in length. If the temperature at the short edge x=0 is given by $u=\begin{cases}10y, 0\le y\le 10\\ 10(20-y), 10\le y\le 20\end{cases}$ and the two long edges as well as the other short edge are kept at 0^0C. Find the steady state temperature distribution in the plate.
10. A rectangular plate with insulated surfaces is 10 cm wide and so long compared to its width that it may be considering infinite in length. The temperature at short edge y=0 is given by $u=\begin{cases}20x, 0\le x\le 5\\ 20(10-x), 5\le x\le 10\end{cases}$ and all the other edges are kept at 0^0C. Find the steady state temperature.