UNIT II – FOURIER SERIES

PART –A

1. **State Dirichlet's condition (OR) State the sufficient condition for a function f(x) to be expressed as a Fourier series.**

ANS

- f(x) is single valued, finite and periodic.
- f(x) has a finite number of finite discontinuities.
- f(x) has a finite number of maxima and minima.
- f(x) has no infinite discontinuity.

2. **State whether y=tanx can be expressed as a Fourier series. If so how? If not why?**

ANS

y=tanx cannot be expressed as a Fourier series, since it has infinite number of infinite discontinuity.

3. **Obtain the sum at x=1 of the Fourier series of** $f(x) = \begin{cases} x, 0 < x < 1 \\ 2, 1 < x < 2 \end{cases}$.

ANS

Here x=1 is a discontinuous and middle point.

Sum at x=1 = $\frac{LHL + RHL}{2} = \frac{1+2}{2} = \frac{3}{2}$

4. **Find the sum of the Fourier series of** $f(x)=x+x^2$ **in** $(-\pi,\pi)$ **at** $x=\pi$.

ANS

Here $x=\pi$ is a discontinuous and end point.

Sum at $x=\pi$= $\frac{f(-\pi)+f(\pi)}{2} = \frac{-\pi+\pi^2+\pi+\pi^2}{2} = \frac{2\pi^2}{2} = \pi^2$

5. **Find the constant term in the expansion of** $\cos^2x$ **as a Fourier series in the interval** $(-\pi,\pi)$.

ANS

Given $f(x) = \cos^2x$

$f(-x) = \cos^2(-x) = \cos^2(x)$.

$f(x) + f(-x) = \cos^2x + \cos^2x = 2\cos^2x \neq 0$.

$f(x) - f(-x) = \cos^2x - \cos^2x = 0$

$\rightarrow f(x)$ is an even function $\rightarrow b_n = 0$.

$$a_0 = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{2}{\pi}\int_{0}^{\pi}\cos^2 x dx$$
$$= \frac{2}{\pi}\int_0^{\pi}\frac{(1+\cos 2x)}{2}dx$$
$$= \frac{1}{\pi}\int_0^{\pi}(1+\cos 2x)dx$$
$$= \frac{1}{\pi}\left[x+\frac{\sin 2x}{2}\right]_0^{\pi} = \frac{1}{\pi}[\pi]$$
$$a_0 = 1.$$

$$\text{Constant Term} = \frac{a_0}{2} = \frac{1}{2}.$$

6. Obtain the first term of the Fourier series for the function f(x) = x^2 in –π<x<π.

ANS

Given f(x) = x^2

f (-x) = $(-x)^2 = x^2$

f(x) + f(-x) = $x^2 + x^2 = 2x^2 \neq 0$.

f (x) – f(-x) = $x^2 - x^2 = 0$

→f(x) is an even function →$b_n = 0$.

$$a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(x).\cos(nx)dx$$
$$= \frac{2}{\pi}\int_0^{\pi} x^2.\cos(nx)dx$$
$$= \frac{2}{\pi}\left\{x^2\left[\frac{\sin(nx)}{n}\right] - 2x\left[\frac{-\cos(nx)}{n^2}\right] + 2\left[\frac{-\sin(nx)}{n^3}\right]\right\}_0^{\pi}$$
$$= \frac{2}{\pi}\left\{\frac{x^2\sin(nx)}{n} + \frac{2x\cos(nx)}{n^2} - \frac{2\sin(nx)}{n^3}\right\}_0^{\pi}$$
$$= \frac{2}{\pi}\left\{\frac{2\pi\cos(n\pi)}{n^2}\right\} = \frac{4(-1)^n}{n^2}$$
$$\therefore a_n = \frac{4(-1)^n}{n^2} \Rightarrow a_1 = -4$$

7. **If $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(nx)$ in $(-\pi,\pi)$, deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$**

<u>**ANS**</u>

Put x= π

$$\text{Sum at } (x=\pi) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(n\pi)$$

$x=\pi$ is a discontinuous and end point.

$$\frac{(-\pi)^2+\pi^2}{2} = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}(-1)^n$$

$$\frac{2\pi^2}{2} = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$4\sum_{n=1}^{\infty}\frac{1}{n^2} = \pi^2 - \frac{\pi^2}{3} = \frac{2\pi^2}{3}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{2\pi^2}{12} = \frac{\pi^2}{6}$$

8. **Give the expression for the Fourier series coefficient b_n for the function f(x) defined in (-2,2).**

<u>**ANS**</u>

Here L=2.

The expression for b_n in (-l,l) is given by $b_n = \frac{2}{2L}\int_{-L}^{L} f(x).\sin\left(\frac{n\pi x}{L}\right)dx.$ Put L=2.

$$\therefore b_n = \frac{1}{2}\int_{-2}^{2} f(x).\sin\left(\frac{n\pi x}{2}\right)dx.$$

9. **Define Harmonic Analysis**

<u>**ANS**</u>

The process of finding the Fourier series of a tabular function is called Harmonic Analysis.

10. **Find the Root Mean square value of the function f(x) = x in (0,L).**

<u>**ANS**</u>

$$\text{RMS} = \sqrt{\frac{1}{b-a}\int_a^b f(x)^2 dx}$$

$$= \sqrt{\frac{1}{L}\int_0^L x^2 dx}$$

$$= \sqrt{\frac{1}{L}\left[\frac{x^3}{3}\right]_{x=0}^{L}} = \sqrt{\frac{1}{L}\left[\frac{L^3}{3}\right]} = \sqrt{\frac{L^2}{3}}.$$

PART – B

1. Find the Fourier Series of $f(x)=x^2$ in $(0,2\pi)$. Hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$.
2. Expand $f(x)=x(2\pi-x)$ as a Fourier Series in $(0,2\pi)$ and hence deduce that the sum of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$.
3. Obtain the Fourier series of the function $f(x)=\begin{cases}1-x, -\pi<x<0\\ 1+x, 0<x<\pi\end{cases}$. Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$.
4. Obtain the Fourier Series of $f(x)=x+x^2$ in $(-\pi,\pi)$. Deduce that $\sum_{n=1}^{\infty}\frac{1}{n^1}=\frac{\pi^2}{6}$
5. Obtain the Fourier series of the function $f(x)=\begin{cases}-\pi, -\pi<x<0\\ x, 0<x<\pi\end{cases}$. Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$.
6. Find the Fourier series for $f(x)=x^2$ in $(-\pi,\pi)$. Hence deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots=\frac{\pi^4}{90}$
7. Find the Half range cosine series of $f(x)=x(\pi-x)$ in $0<x<\pi$. Hence deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots=\frac{\pi^4}{90}$
8. Find the half range cosine series of $f(x)=(x-1)^2$ in $0<x<1$.
9. Find the Half range sine series for $f(x) = x$ in $(0,\pi)$. Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}$.
10. Find the Half range sine series of a function $f(x)=x(\pi-x)$ in $0<x<\pi$. Hence deduce that $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^3}=\frac{\pi^3}{32}$.
11. Find the Fourier series up to second harmonic for y=f(x) from the following table.

x	0	π/3	2π/3	π	4π/3	5π/3	2π
y=f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

12. Find the Fourier series up to second harmonic for the following function

x	0	1	2	3	4	5
y=f(x)	9	18	24	28	26	20

13. Compute up to second harmonic of the Fourier series of f(x) given by the following table.

x	0	T/6	T/3	T/2	2T/3	5T/6	T
y=f(x)	1.98	1.30	1.05	1.30	-0.88	-.25	1.98