UNIT 1 – PARTIAL DIFFERENTIAL EQUATIONS

PART -A

1. **Form the PDE from $(x-a)^2+(y-b)^2+z^2=r^2$.**

ANS

Given $(x-a)^2+(y-b)^2+z^2=r^2$.

Diff with respect to x,

$$2(x-a)+2z\frac{\partial z}{\partial x}=0 \Rightarrow (x-a)+zp=0 \Rightarrow (x-a)=-zp$$

Diff with respect to y,

$$2(y-b)+2z\frac{\partial z}{\partial y}=0 \Rightarrow (y-b)+zq=0 \Rightarrow (y-b)=-zq$$

The pde is $(-zp)^2+(-zq)^2+z^2=r^2 \rightarrow z^2p^2+z^2q^2+z^2=r^2$.

2. **Form the partial differential equation by eliminating the constants a and b from $z=(x^2+a^2)(y^2+b^2)$**

ANS

Given $z=(x^2+a^2)(y^2+b^2)$

Diff with respect to x partially

$$\frac{\partial z}{\partial x}=2x(y^2+b^2) \Rightarrow p=2x(y^2+b^2) \Rightarrow (y^2+b^2)=\frac{p}{2x}$$

Diff with respect to y partially,

$$\frac{\partial z}{\partial y}=2y(x^2+a^2) \Rightarrow q=2y(x^2+a^2) \Rightarrow (x^2+a^2)=\frac{q}{2y}$$

$$z=\frac{q}{2y}.\frac{p}{2x} \Rightarrow z=\frac{pq}{4xy} \text{ is the pde.}$$

3. **Find the pde of the family of spheres having their centers on the Z-axis.**

ANS

The equation of sphere having center at (a,b,c) with radius r is given by $(x-a)^2+(y-b)^2+(z-c)^2=r^2$.

Since center (a,b,c) lies on Z-axis, a=0 and b=0.

(1) becomes $x^2+y^2+(z-c)^2=r^2$

Diff with respect to x,

$$2x+2(z-c)\frac{\partial z}{\partial x}=0$$

$$\Rightarrow 2x+2(z-c)p=0 \Rightarrow x+(z-c)p=0 \Rightarrow (z-c)=\frac{-x}{p}$$

Diff with respect to y partially,

$$2y+2(z-c)\frac{\partial z}{\partial y}=0$$

$$\Rightarrow 2y+2(z-c)q=0 \Rightarrow y+(z-c)q=0 \Rightarrow (z-c)=\frac{-y}{q}$$

The pde is $\frac{-x}{p}=\frac{-y}{q} \Rightarrow qx=py$

4. **Find the partial differential equation of all planes cutting equal intercepts from the X and Y-axis.**

ANS

WKT the plane equation is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (INTERCEPT FORM)

Since the required plane having equal intercepts from X and Y-axis, we have a=b.

(1) Becomes, $\frac{x}{a}+\frac{y}{a}+\frac{z}{c}=1$

Diff with respect to x,

$$\frac{1}{a}+\frac{1}{c}\frac{\partial z}{\partial x}=0\Rightarrow\frac{1}{a}+\frac{p}{c}=0\Rightarrow\frac{1}{a}=-\frac{p}{c}$$

Diff with respect to y,

$$\frac{1}{a}+\frac{1}{c}\frac{\partial z}{\partial y}=0\Rightarrow\frac{1}{a}+\frac{q}{c}=0\Rightarrow\frac{1}{a}=-\frac{q}{c}$$

$-\frac{p}{c}=-\frac{q}{c}\Rightarrow p=q.$ is the required pde.

5. **Eliminate the arbitrary function f from z=** $f\left(\frac{y}{x}\right)$

ANS

Given z= $f\left(\frac{y}{x}\right)$

Take u= z, $v=\frac{y}{x}$

Diff w.r.t. x partially, Diff w.r.t. x partially,

$$\frac{\partial u}{\partial x}=\frac{\partial z}{\partial x}=p \qquad \frac{\partial v}{\partial x}=\frac{-y}{x^2}$$

Diff w.r.t y partially, Diff w.r.t y partially,

$$\frac{\partial u}{\partial y}=\frac{\partial z}{\partial y}=q \qquad \frac{\partial v}{\partial y}=\frac{1}{x}$$

The required pde is $\frac{p}{x}+\frac{qy}{x^2}=0\Rightarrow px+qy=0$

6. **Form the partial differential equation by eliminating the arbitrary function from z^2-xy =** $f\left(\frac{x}{z}\right)$

ANS

Given z^2-xy = $f\left(\frac{x}{z}\right)$

Take u= z^2-xy, $v=\frac{x}{z}$

Diff w.r.t. x partially, Diff w.r.t. x partially,

$$\frac{\partial u}{\partial x} = 2z\frac{\partial z}{\partial x} - y = 2zp - y \qquad \frac{\partial v}{\partial x} = \frac{z - x.\frac{\partial z}{\partial x}}{z^2} = \frac{z - xp}{z^2}$$

Diff w.r.t y partially, Diff w.r.t y partially,

$$\frac{\partial u}{\partial y} = 2z\frac{\partial z}{\partial y} - x = 2zq - x \qquad \frac{\partial v}{\partial y} = \frac{-x}{z^2}.\frac{\partial z}{\partial y} = \frac{-xq}{z^2}$$

$$(2zp - y).\frac{-xq}{z^2} - (2zq - x).\left(\frac{z - xp}{z^2}\right) = 0$$

The required pde is $\Rightarrow -xq(2zp - y) - (2zq - x)(z - xp) = 0$

$$\Rightarrow -2xzpq + xyq - 2z^2q + 2zqxp + xz - x^2p = 0$$

$$\Rightarrow xyq - 2z^2q + xz - x^2p = 0$$

7. Find the complete integral of p+q=pq.

ANS

The trial solution is z=ax+by+c where a+b=ab →ab-b=a→ $b(a-1) = a \Rightarrow b = \frac{a}{a-1}$.

The complete solution is $z = ax + \frac{a}{a-1}y + c$

8. Solve the partial differential equation pq=x.

ANS

Given pq=x.

Put q=a

ap=x→ $p = \frac{a}{x}$

$$dz = pdx + qdy \Rightarrow dz = \frac{a}{x}dx + ady$$

Integrate $\Rightarrow \int dz = \int \frac{a}{x}dx + \int ady + b \Rightarrow z = a.\log x + ay + b$ is the complete solution.

9. Solve $(D^2-7DD'+6D'^2)z=0$

ANS

The AE is $(m^2-7m+6)=0$

m=6,1.

CF=$f_1(y+1x)+f_2(y+6x)$.

P.I=0

The solution is z=CF+PI.

10. Solve $(D^3-2D^2D')z=0$.

ANS

The AE is $m^3-2m^2=0 \rightarrow m^2(m-2)=0 \rightarrow m=0,0,2$.

C.F. = $f_1(y+0x)+x.f_2(y+ox)+f_3(y+2x)$.

P.I.=0

The solution is z=C.F.+P.I.

PART - B

1. Eliminate the arbitrary function φ from the equation $\varphi(x^2+y^2+z^2,ax+by+cz)=0$ to form a partial differential equation.
2. Eliminate the arbitrary function f and φ from the equation $z=f(x+ct)+g(x-ct)$ to form a partial differential equation.
3. Solve $x(y-z)p+y(z-x)q=z(x-y)$.
4. Solve $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$.
5. Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$.
6. Solve $(mz-ny)p+(nx-lz)q=(ly-mx)$.
7. Find the singular solution of $z=px+qy+\sqrt{1+p^2+q^2}$
8. Solve $z=px+qy+p^2+pq+q^2$.
9. Solve $z=px+qy+p^2q^2$
10. Solve $(D^2+2DD'+D'^2)z=e^{x+2y}+\sinh(x+y)$.
11. Solve $(D^2+3DD'-4D'^2)z=\cos(2x+y)+\text{Sin}(y)$.
12. Solve $(D^3+D^2D'-4DD'^2-4D'^3)z=\cos(2x+y)$
13. Solve $(D^3-7DD'^2-6D'^3)z=\sin(2x+y)$
14. Solve $(D^2-2DD'+D'^2)z=e^{x+2y}+\sin(2x-3y)$.
15. Solve $(D^2-4DD'+4D'^2)z=xy$.