UNIT IV

RECIPROCATING PUMP (Single Acting)

▸ 20.2 MAIN PARTS OF A RECIPROCATING PUMP

The following are the main parts of a reciprocating pump as shown in Fig. 20.1 :

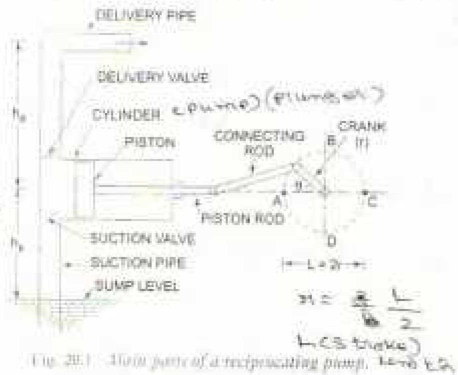

Fig. 20.1 Main parts of a reciprocating pump. length

Discharge Through a Reciprocating pump:

D – Diameter of cylinder

A – cross-sec area of piston or cylinder

$= \frac{\pi}{4} D^2$

r – radius of crank.

N – r.p.m. of crank.

h_s – height of the axis of the cylinder from water surface in sump. (suction head) (below pump axis)

h_d – height of delivery outlet above the cylinder axis. (delivery head)

Volume of water delivered in one revolution or discharge of water in one revolution

$= \text{Area} \times \text{Length of stroke} = A \times L$

Number of revolution per sec. $= \frac{N}{60}$

Therefore.
~~Actual~~ Discharge pump per second.

$Q =$ Discharge in one revolution $\times$ No. of revolution per second.

$= A \times L \times \frac{N}{60} = \frac{ALN}{60}$

Weight of water delivered per second.

$W = \rho \times g \times Q = \frac{\rho g ALN}{60}$ $\quad \therefore g = 9.81$

Work done by reciprocating pump:

Work done per second $= W \times (h_s + h_d)$

$(h_s + h_d) =$ Total height through which water is lifted

$W = \frac{\rho g ALN}{60}$

Work done per second $= \frac{\rho g ALN}{60} \times (h_s + h_d)$. KJ

$\therefore$ Power required to drive the pump, in KW

$P = \frac{\text{Work done per Second}}{1000} = \frac{\rho g \times ALN \times (h_s + h_d)}{60 \times 1000}$

$= \frac{\rho g \times ALN \times (h_s + h_d)}{60{,}000}$ KW.

Discharge, Work Done and Power required to drive a Double-Acting pump:

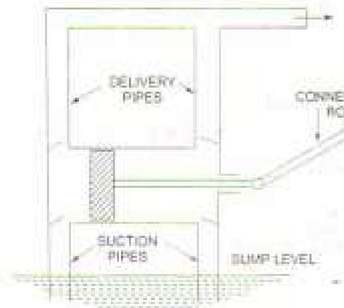

Fig. 20.2

D – diameter of the piston

d – Diameter of the piston rod.

Area on one side of piston

$$A = \frac{\pi}{4} D^2$$

Area on the other side of piston, where piston rod is connected to the piston.

$$A_1 = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2 = \pi/4 (D^2 - d^2)$$

$$Q = \frac{2ALN}{60}.$$

Work done by Double-Acting reciprocating pump.

Work done per second $= 2\rho g \times \frac{ALN}{60} \times (h_s + h_d)$

∴ Power req to drive the double-acting pump in KW

$$P = \frac{2\rho g \times ALN \times (h_s + h_d)}{60{,}000}$$

Slip of Reciprocating pump:

$$Slip = Q_{th} - Q_{act}$$

$$\% \text{ of slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$= (1 - C_d) \times 100$$

where C_d = Co-eff of discharge $\left(\because \frac{Q_{act}}{Q_{th}} = C_d\right)$

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1) **Problem 20.1** *A single-acting reciprocating pump, running at 50 r.p.m., delivers 0.01 m³/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine :*
(i) The theoretical discharge of the pump, (ii) Co-efficient of discharge, and (iii) Slip and the percentage slip of the pump.

G.D:

$N = 50$ r.p.m.

$Q_{act} = 0.01\ m^3/s$

$D = 200mm$

$= \frac{200}{1000} = 0.2m.$

$L = 400mm$

$= \frac{400}{1000} = 0.4m.$

$Area = A = \frac{\pi}{4} D^2$

$= \frac{\pi}{4} (0.2)^2$

$= 0.031416\ m^2$

Soln:

(i) Theoretical discharge of the pump:

$$\boxed{Q_{th} = \frac{ALN}{60}} = \frac{0.031416 \times 0.4 \times 50}{60}$$

$$= 0.01047\ m^3/s.$$

(ii)

(iii)

Re

(ii) Co - eff of Discharge:

$$\boxed{C_d = \frac{Q_{act}}{Q_{th}}} = \frac{0.01}{0.01047} = 0.95511$$

(iii) Slip and the % of slip of pump:

$$\boxed{\text{Slip} = Q_{th} - Q_{act}}$$

$$= 0.01047 - 0.01$$

$$= 4.7 \times 10^{-04} \ m^3/s$$

$$\boxed{\% \text{ of Slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100}$$

$$= \frac{0.01047 - 0.01}{0.01047} \times 100$$

$$= 4.489\%$$

RESULT:

$Q_{th} = 0.01047 \ m^3/s$

$C_d = 0.9511$

$\text{Slip} = 4.7 \times 10^{-04} \ m^3/s$

$\% \text{ of Slip} = 4.489\%$

3) **Problem 20.2** *A double-acting reciprocating pump, running at 40 r.p.m., is discharging 1.0 m³ of water per minute. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.*

Given Data:

$N = 40 \text{ r.p.m.}$

$Q_{act} = 1.0 \text{ m}^3/\text{min}$

$= \frac{1.0}{60} = 0.01666 \text{ m}^3/\text{s}$

$L = 400 \text{ mm}$

$= \frac{400}{1000} = 0.4 \text{ m}$

$D = 200 \text{ mm}$

$= \frac{200}{1000} = 0.2 \text{ m}$

$A = \frac{\pi}{4} D^2$

$= \frac{\pi}{4} (0.2)^2$

$= 0.031416 \text{ m}^2$

$h_d = 20 \text{ m}$

$h_s = 5 \text{ m}$

Soln: Theo discharge for double-acting pump

$\boxed{Q_{th} = \frac{2ALN}{60}} = \frac{2(0.031416)\, 0.4 \times 40}{60}$

$= 0.01675 \text{ m}^3/\text{s}$

$\boxed{\text{Slip} = Q_{th} - Q_{act}}$

$= 0.01675 - 0.01666$

$= 9 \times 10^{-05} \text{ m}^3/\text{s}$

Power Required the double acting pump

$$\boxed{P = \frac{2 \rho g\ ALN \times (h_s + h_d)}{60{,}000}}$$

$$= \frac{2 \times 1000 \times 9.81 \times 0.031416 \times 0.4 \times 40 \times (5+20)}{60{,}000}$$

$= 4.10921$ KW

RESULT:

$Q_{th} = 0.01675\ m^3/s$

$Slip = 9 \times 10^{-05}\ m^3/s$

$P = 4.10921$ KW.

Variation of Velocity And Acceleration in the Suction And Delivery Pipes Due to Acceleration of the piston:

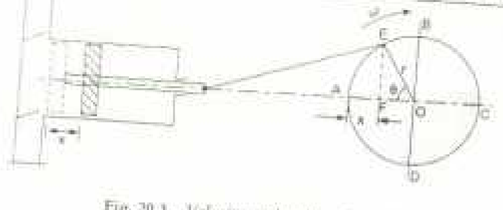


Fig. 20.3 Velocity and acceleration of piston.

ω = Angular speed of the crank in rad/s

A = Area of the cylinder

a = Area of the pipe (suction or delivery)

l = Length of the pipe (suction or delivery)
l_s l_d

r = Radius of the crank.

$$\omega = \frac{2\pi N}{60}$$

Pr. head due to acceleration in suction and delivery pipes.

(Pr head due to acceleration = h_a)

h_a at suction pipe
$$\boxed{h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos\theta}$$

h_a at delivery pipe
$$\boxed{h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos\theta}$$

Diff value of θ are.

$\theta = 0$ $\cos 0^\circ = 1$

$\theta = 90^\circ$ $\cos 90^\circ = 0$

$\theta = 180$ $\cos 180^\circ = -1$

Problem 20.3 *The cylinder bore diameter of a single-acting reciprocating pump is 150 mm and its stroke is 300 mm. The pump runs at 50 r.p.m. and lifts water through a height of 25 m. The delivery pipe is 22 m long and 100 mm in diameter. Find the theoretical discharge and the theoretical power required to run the pump. If the actual discharge is 4.2 litres/s, find the percentage slip. Also determine the acceleration head at the beginning and middle of the delivery stroke.*

G.D:

$D = 150\,mm$

$= \frac{150}{1000} = 0.15\,m$ $A = \frac{\pi}{4} D^2$

$= 0.01767\,m^2$

$L = 300\,mm$

$= \frac{300}{1000} = 0.3\,m.$

$N = 50\,r.p.m.$

$H = 25\,m$

$l_d = 22\,m.$

$d_d = 100\,mm$

$= \frac{100}{1000} = 0.1\,m$

$Q_{act} = 4.2\,l/s.$

$= \frac{4.2}{1000}$

$= 4.2 \times 10^{-3} = 0.0042\,m^3/s$

Soln:

Theoretical discharge

$\boxed{Q_{th} = \frac{ALN}{60}}$ $= \frac{0.01767 \times 0.3 \times 50}{60}$

$Q_{th} =$ ~~0.004417~~ 4.4175×10^{-3}

Theoretical power.

$\boxed{P_t = \frac{\rho g Q_{th} \times H}{1000}}$

$= \frac{1000 \times 9.81 \times 4.4175 \times 10^{-3} \times 25}{1000}$

$P_t = 1.0833\,kW$

% of Slip :

$$\boxed{\% \text{ of Slip} = \frac{Q_{th} - Q_{act}}{Q_{th}}}$$

$$= \frac{4.4175\times10^{-3} - 4.2\times10^{-3}}{4.4175\times10^{-3}}$$

% of Slip = 4.92 %

Acceleration head at the beginning of delivery pipe :

$$\boxed{h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \times \cos\theta}$$

r – crank radius

g – gravity 9.81

l_d – length of delivery pipe

a_d – diameter ,, ,, – $a_d = \frac{\pi}{4} d^2 = 7.854\times10^{-3}$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 50}{60}$$

$$= 5.236$$

$$r = \frac{L}{2} = \frac{0.3}{2} = 0.15\,m$$

$$h_{ad} = \frac{22}{9.81} \times \frac{0.01767}{7.854\times10^{-3}} \times 5.236^2 \times 0.15 \times \cos\theta$$

$$= 20.75 \times \cos\theta$$

At the beginning of the delivery stroke.

$\theta = 0$ and hence $\cos\theta = 1$

$$h_{ad} = 20.75 \times 1 = 20.75\,m$$

Acceleration head at middle of delivery stroke:

$\theta = 90°$ hence $\cos 90 = 0$

$h_{ad} = 20.75 \times 0 = 0$.

RESULT:

$Q_{th} = 4.4175 \times 10^{-3}$

% of Slip = 4.92 %.

$h_{ad} = 20.75$ m. (beginning)

$h_{ad} = 0$. middle

1×10^{-3}

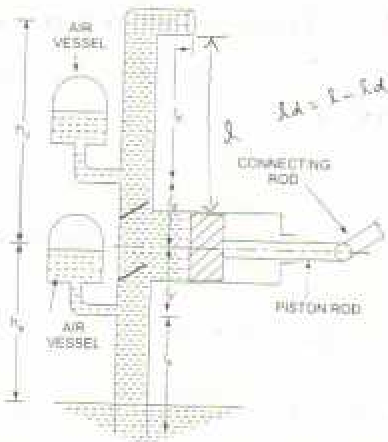


Fig. 20.9 Air vessels fitted to reciprocating pump.

Let A = Cross-sectional area of the cylinder,
a = Cross-sectional area of suction or delivery pipe.
d_d = Diameter of delivery pipe
d_s = " " suction "
l_d = Length of delivery pipe beyond the air vessel,
l_d' = Length of delivery pipe between cylinder and air vessel,
l_s' = Length of suction pipe between cylinder and air vessel,
l_s = Length of suction pipe below air vessel,
h_{ad} = Pressure head due to acceleration in delivery pipe,
h_{as} = Pressure head due to acceleration in suction pipe,
h_{fd} = Loss of head due to friction in delivery pipe beyond the air vessel,
h_{fd}' = Loss of head due to friction in delivery pipe between cylinder and air vessel,
h_{fs} = Loss of head due to friction in suction pipe below the air vessel, and
h_{fs}' = Loss of head due to friction in suction pipe between cylinder and air vessel.

l = total length of delivery pipe ([illegible] water [illegible])

h_s = suction Head
h_d = Discharge of water height (Delivery head)

$$\omega = \frac{2\pi N}{60} \text{ rad/s}$$

$g = 9.81$
$\rho = 1000$

(*i*) At the beginning of the delivery stroke, $\theta = 0°$, $\sin\theta = 0$ and $\cos\theta = 1$ and hence total pressure head

$$= (h_d + h_{ad} + h_{fd}' + h_{fd}) + \text{velocity head at the outlet of delivery}$$

$$= h_d + h_{ad} + h_{fd}' + h_{fd} + \frac{\overline{V}_d^2}{2g} \qquad (\because \text{ Velocity at outlet is equal to mean velocity})$$

$$= h_d + \frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r + 0 + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2}{2g} \qquad \left(\because \overline{V}_d = \frac{A}{a_d} \times \frac{\omega r}{\pi}\right)$$

$$\boxed{= h_d + \frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r + \frac{4fl_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2} \qquad \text{...(20.25)}$$

(*ii*) In the middle of the stroke, $\theta = 90°$, $\sin\theta = 1$ and $\cos\theta = 0$ and hence total pressure head

$$= h_d + h_{ad} + h_{fd}' + h_{fd} + \frac{\overline{V}_d^2}{2g} \text{ above atmospheric pressure head}$$

$$= h_d + 0 + \frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2 + \frac{4f \times l_d}{d_d \times 2g} \times \left[\frac{A}{a_d} \times \frac{\omega r}{\pi}\right]^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2$$

$$\boxed{= h_d + \frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2 + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2} \qquad \text{...(20.26)}$$

(*iii*) At the end of the delivery stroke, $\theta = 180°$, $\sin\theta = 0$ and $\cos\theta = -1$ and hence total pressure head

$$= h_d - \frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 \qquad \text{...(20.27)}$$

In equations (20.25), (20.26) and (20.27), the quantities

$$\left(\frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r\right) \text{ and } \left[\frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2\right]$$

are very small and can be neglected.

Problem 20.14 *The cylinder of a single-acting reciprocating pump is 15 cm in diameter and 30 cm in stroke. The pump is running at 30 r.p.m. and discharge water to a height of 12 m. The* diameter *and length of the delivery pipe are 10 cm and 30 m respectively. If a large air vessel is fitted in the delivery pipe at a distance of 2 m from the centre of the pump, find the pressure head in the cylinder.*

(i) At the beginning of the delivery stroke, and

(ii) In the middle of the delivery stroke. Take f = .01

G.D:

$D = 15\,cm$

$= \frac{15}{100} = 0.15\,m$

$Area = \pi/4\, D^2$

$= 0.01767\,m^2$

Stroke length $L = 30\,cm$

$= \frac{30}{100} = 0.30\,m$

$\therefore$ Crank radius $= r = \frac{L}{2} = \frac{0.30}{2} = 0.15\,m$

$N = 30\,r.p.m. \Rightarrow \omega = \frac{2\pi N}{60} = 3.14\,rad/s$

$h_d = 12\,m$

$d_d = 10\,cm \Rightarrow a_d = \pi/4\, d^2 = 7.854\times10^{-3}$

$= \frac{10}{100} = 0.1\,m$

$l = 30\,m$

$l_d' = 2\,m$

$l_d = l - l_d' = 30 - 2$

$= 28\,m$

Co-efficient of friction $f = 0.01$

(i) At the beginning of delivery stroke:

$$= h_d + \frac{l_d'}{g} \times \frac{A}{a_d}\,\omega^2 r + \frac{4f\,l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2$$

$= 12 + \frac{2}{9.81} \times \frac{0.01767}{7.854\times10^{-3}} \times 3.14^2 \times 0.15$

$+ \frac{4(0.01)\ 28}{0.1 \times 2(9.81)} \times \left[\frac{0.01767}{7.854\times10^{-3}} \times \frac{3.14 \times 0.15}{\pi} \right]^2$

$+ \frac{1}{2\times9.81} \left[\frac{0.01767}{0.1} \times \frac{3.14 \times 0.15}{\pi} \right]^2$

$= 12 + 0.6783 + 0.065 + 0.0058$

$= \underline{12.75 m.}$

(ii) <u>middle of the delivery stroke:</u>

$= hd + \frac{4f \times ld'}{dd \times 2g} \times \left(\frac{A}{ad} \omega r \right)^2 + \frac{4f \times ld}{dd \times 2g} \times$

$\left[\frac{A}{ad} \times \frac{\omega r}{\pi} \right]^2 + \frac{1}{2g} \left[\frac{A}{ad} \times \frac{\omega r}{\pi} \right]^2$

$= 12 + \frac{4(0.01) \times 2}{0.1 \times 2(9.81)} \times \left[\frac{0.01767}{7.854\times10^{-3}} \times 3.14 \times 0.15 \right]^2$

$+ \cancel{4(0.01)} \ 0.065 + 0.0058$

$= 12 + 0.0458 + 0.065 + 0.0058$

$= \underline{12.116 m.}$

Problem 20.15 *A single-acting reciprocating pump is to raise a liquid of density 1200 kg per cubic metre through a vertical height of 11.5 metres, from 2.5 metres below pump axis to 9 metres above it. The plunger, which moves with S.H.M., has diameter 125 mm and stroke 225 mm. The suction and delivery pipes are 75 mm diameter and 3.5 metres and 13.5 metres long respectively. There is a large air vessel placed on the delivery pipe near the pump axis. But there is no air vessel on the suction pipe. If separation takes place at 8.829 N/cm² below atmospheric pressure, find :*

(i) maximum speed, with which the pump can run without separation taking place, and

(ii) power required to drive the pump, if f = 0.02.

Neglect slip for the pump.

Solution. Given :

G.D:

Density $\rho = 1200 kg/m^3$

Total vertical height = 11.5 m

suction head $h_s = 2.5 m$

delivery head $h_d = 9 m$

$D = 125 mm$

$= \frac{125}{1000} = 0.125$

$A = \pi/4 \, D^2$

$= 0.0123 m^2$

Stroke length $L = 225 mm$

$= \frac{225}{1000} = 0.225 m$ $\Rightarrow r = \frac{L}{2} = \frac{.225}{2} = .1125 m$

Dia of suction and delivery pipe $d = 75 mm$

$= \frac{75}{1000} = 0.075 m$

$a = \pi/4 \, d^2$

$= \cancel{0.00442 m^2}$

$= 4.4178 \times 10^{-3} = 0.00442 m^2$

Sep Pr = $8.829 \frac{N}{cm^2}$

$f = 0.02$

length of suction pipe $l_s = 3.5 m$

" " delivery " $l_d = 13.5 m$

Air vessel is placed on the delivery side only. Hence, the velocity in the delivery pipe will be uniform. And there will be no accelerating head on delivery side.

Separation pr $= 8.829 \frac{N}{cm^2} = 8.829 \times 10^4 \frac{N}{m^2}$ below atmp pr.

Sep pr head, $\boxed{h_{sep} = \frac{\text{sep pr}}{\rho \times g}}$

$$= \frac{8.829 \times 10^4}{1200 \times 9.81}$$

$= 7.5$ m below atmosphere.

(ii) max speed, with which the pump can run without separation taking place.

Ld N = max. speed with which pump can run without separation taking place. The separation can take place only at the beginning of suction stroke. As air vessel is not fitted on the suction pipe, there will be accelerating head acting on suction side.

Pr head at beginning of suction stroke.

$= h_s + h_{as}$ below atmosphere.

h_{as} = pr head due to acc in suction pipe

This pr should be equal to h_{sep} in the limiting case.

$7.5 = h_s + h_{as} = 2.5 + h_{as}$

$h_{as} = 7.5 - 2.5 = 5.0$ m.

But 'h_{as}' at the beginning of suction stroke.

.1125m

m^2

$$\boxed{h_{as} = \frac{l_s}{g} \times \frac{A}{a} \omega^2 r}$$

$$5.0 = \frac{3.5}{9.81} \times \frac{0.0123}{4.4178 \times 10^{-?}} \times \omega^2 \times .1125$$

$$\omega = \sqrt{\frac{5.0 \times 9.81 \times 0.00442}{3.5 \times 0.0123 \times 0.1125}}$$

$= 6.69$ rad/s.

$$\omega = \frac{2\pi N}{60}$$

$$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 6.69}{2\pi} = 63.88 \text{ r.p.m.}$$

∴ max speed with which the pump can run without separation taking place is 63.88 r.p.m.

(ii) Power req to drive the pump:

Now discharge (Q) of the single-acting pump is given by

$$Q = \frac{ALN}{60} = \frac{0.0123 \times 0.225 \times 63.88}{60}$$

$= 0.00294 \ m^3/s.$

Velocity of liquid in delivery pipe will be uniform.

Q = Area of delivery pipe × Velocity

= a × V

$$V = \frac{Q}{a} = \frac{0.00294}{0.00442} = 0.665\ m/s.$$

∴ Head loss due to friction in delivery pipe.

$$h_{fd} = \frac{4f \times l_d \times V^2}{d \times 2g}$$

$$= \frac{4 \times 0.02 \times 13.5 \times (.665)^2}{0.075 \times 2 \times 9.81}$$

$$= 0.324\ m.$$

During suction stroke, the value of max h_{fs} is given by

$$h_{fs} = \frac{4f\, l_s}{d \times 2g} \times \left[\frac{A}{a}\ \omega r \right]^2$$

$$= \frac{4 \times 0.02 \times 3.5}{0.075 \times 2 \times 9.81} \left[\frac{0.0123}{0.00442} \times 6.69 \times 0.125 \right]^2$$

$$= 0.834\ m.$$

Now power req to drive the pump in KW

$$= \frac{W \cdot Q \cdot H_m}{1000} = \frac{\rho g Q}{1000} \times \left[h_s + h_d + \frac{2}{3} h_{fs} + h_{fd} \right]$$

$$= \frac{1200 \times 9.81 \times 0.00294}{1000} \times \left[2.5 + 9.0 + \frac{2}{3} \times 0.834 + 0.324 \right].$$

$$= 0.428\ KW$$

Problem 20.16 *A double-acting reciprocating piston pump is pumping water (diameter of the piston 250 mm, diameter of piston rod, which is on one side of the piston 50 mm, piston stroke 380 mm). The suction and discharge heads are 4.5 m and 18.6 m respectively. Find the work done by the piston during outward stroke. Would the work done change for the inward stroke?*

G.D:

$$D = 250\,mm = 0.25\,m$$

$$A = \frac{\pi}{4} D^2$$

$$= 0.0491\,m^2$$

diameter of piston rod $d = 50\,mm$

$$= \frac{50}{1000} = 0.05\,m$$

$$a = \pi/_4\, d^2 = 1.963 \times 10^{-3}$$

$$= .001963\,m^2$$

Piston stroke = 380 mm

$$L = \frac{380}{1000} = 0.380\,m$$

$$h_s = 4.5\,m$$

$$h_d = 18.6\,m$$

Hence total work done during outward stroke

$$\boxed{= \rho \times g \times Q_1 \times h_s + \rho \times g \times Q_2 \times h_d}$$

find

$$Q_1 = A \times L = 0.0491 \times 0.380$$

$$= 0.01865\,m^3$$

$\frac{ALN}{60}$

NE & answer

$$Q_2 = (A-a) \times L$$

$$= (0.0491 - 0.001963) \times 0.380$$

$$= 0.01791\,m^3.$$

$$\rho \times g = 1000 \times 9.81\,N/m^3$$

Total W.D during outward stroke:

$= [1000 \times 9.81 \times 0.01865 \times 4.5 + 1000 \times 9.81 \times 0.01791 \times 18.6]$ Nm

$= \cancel{4.0} \; 4091.27 \; \cancel{kg}.$ Nm.

$= 4.0913$ KJ.

Nm convert to KJ.

$\frac{Nm}{1000} =$ KJ.

∴ Total work done during inward stroke:

$\boxed{= \rho \times g \times Q_2 \times h_s + \rho \times g \times Q_2 \times h_d}$

$= 1000 \times 9.81 \times 0.01791 \times 4.5 + 1000 \times 9.81 \times 0.01865 \times 18.6$

$= 4193.637$ Nm.

$= 4.193$ KJ.

RESULT: Total dw
outward stroke $= 4.0913$ KJ
inward stroke $= 4.193$ KJ.

inward stroke will be different.

Problem 20.17 *A single-acting reciprocating pump has a plunger diameter of 250 mm and stroke of 450 mm and it is driven with S.H.M. at 60 r.p.m. The length and diameter of delivery pipe are 60 m and 100 mm respectively. Determine the power saved in overcoming friction in the delivery pipe by fitting an air vessel on the delivery side of the pump. Assume friction factor = 0.01.*

G.D.:

Plunger Diameter = $D = 250mm = 0.250m$

$L = 450mm$

$= \frac{450}{1000} = 0.45\ m$

$r = \frac{L}{2} = 0.225$

$A = \pi/_4 D^2 = 0.049087$

$N = 60\ r.p.m.$

Angular speed $\omega = \frac{2\pi N}{60} = 2\pi\ rad/s$

Length of delivery pipe $l = 60m$

$d = 100mm$

$= \frac{100}{1000} = 0.1m^2$

$a = \pi/_4 d^2 = 7.853 \times 10^{-3}$

$= .007853\ m^2$

Friction factor $f = 0.01$

Power saved is given by:

Power saved $= \rho \times g \times Q \times \left[\frac{2}{3} (h_f)_{\text{without air vessel}} - (h_f)_{\text{with air vessel}} \right]$

$\rho \times g = 1000 \times 9.81\ N/m^3$

$Q = \frac{ALN}{60} = \frac{0.049 \times 0.45 \times 60}{60}$

$= 0.02205\ m^3/s.$

without air vessel.

length of delivery pipe

$$h_f = \frac{f \times l \times v^2}{d \times 2g}$$

$$\left[\because v = \left(\frac{A}{a} \times \omega \times r \right) \right]$$

$$= \frac{0.01 \times 60}{0.1 \times 2 \times 9.81} \times \left[\frac{0.049087}{0.007853} \times 2\pi \times 0.225 \right]^2$$

$$= 23.87 m.$$

With air vessel:

$$h_f = \frac{f \times l \times \bar{v}^2}{d \times 2g}$$

$$\bar{v} = \frac{A}{a} \times \frac{\omega \times r}{\pi}$$

$$= \frac{f \times L}{d \times 2g} \times \left[\frac{A}{a} \times \frac{\omega r}{\pi} \right)^2$$

$$= \frac{0.01 \times 60}{0.1 \times 2 \times 9.81} \times \left[\frac{0.049087}{0.007853} \times \frac{2\pi \times 0.225}{\pi} \right]^2$$

$$= 2.419 m.$$

$\therefore$ Power saved $= \rho \times g \times Q \times \left[\frac{2}{3} (h_f)_{\text{without air vessel}} - (h_f)_{\text{with air vessel}} \right] W$

$$= 1000 \times 9.81 \times 0.02209 \left[\tfrac{2}{3} \times 23.87 - 2.419 \right] W.$$

$$= 2924.26 \; W$$

$$\frac{W}{1000} = kW.$$

$$= 2.924 \; kW.$$

Problem 20.18 *A double-acting reciprocating pump runs at 120 r.p.m. When its suction pipe of 100 mm diameter is fitted with an air vessel on its suction side. The diameter of cylinder and stroke are 150 mm and 450 mm respectively. If piston is to be driven with S.H.M., find the rate of flow from or into the air vessel when the crank makes angles of 30°, 90° and 120° with the inner dead centre. Find also the crank angles at which there is no flow into or from the air vessel.*

G.D:

$$N = 120 \text{ r.p.m}$$

$$\omega = \frac{2\pi N}{60}$$

$$= 12.566 \text{ rad/s.}$$

Dia of suction pipe $d = 100 \text{ mm} = \frac{100}{1000} = 0.1 \text{ m}$

$$= 0.1 \text{ m.}$$

∴ Area of suction pipe $a = \frac{\pi}{4} d^2$

$$= \frac{\pi}{4}(0.1)^2 = 7.8539 \times 10^{-3}$$

$$= 0.007854 \text{ m}^2$$

Dia of cylinder $D = 150 \text{ mm}$

$$= \frac{150}{1000} = 0.15 \text{ m}$$

$$A = \frac{\pi}{4} D^2$$

$$= 0.01767 \text{ m}^2$$

Stroke length $L = 450 \text{ m}$

$$= \frac{450}{1000} = 0.45 \text{ m}$$

crank radius $r = \frac{L}{2} = \frac{0.45}{2} = 0.225 \text{ m.}$

(i) Rate of flow of liquid into air vessel:

$$= A\omega r\left(\sin\theta - \frac{2}{\pi}\right)$$

$$= 0.01767\,(12.566)\,(0.225)\left(\sin\theta - \frac{2}{\pi}\right)$$

$$= 0.04996\left(\sin\theta - \frac{2}{\pi}\right)$$

$\theta = 30°$

The also rate of flow $= 0.04996(\sin 30 - \frac{2}{\pi})$

$= -6.825 \times 10^{-3}$

$= -0.00682\ m^3/s.$

$\theta = 90°$

The rate of flow becomes

$= 0.04996\left(\sin 90 - \frac{2}{\pi}\right)$

$= 0.0181\ m^3/s.$

$\theta = 120°$

The rate of flow becomes

$= 0.04996\left(\sin 120 - \frac{2}{\pi}\right)$

$= 0.01146\ m^3/s.$

(ii) Crank angle at which there is no flow.

But rate of flow.

$= 0.04996\left(\sin\theta - \frac{2}{\pi}\right)$

For no flow from or into air vessel,

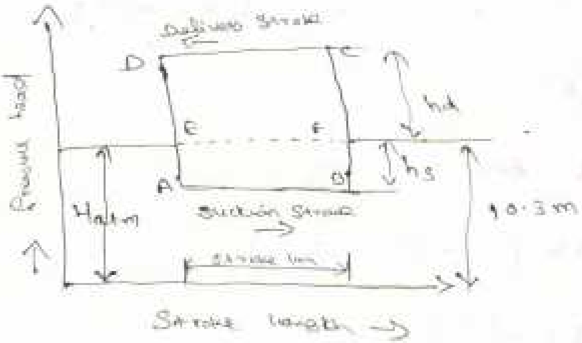
$0.04996\left(\sin\theta - \frac{2}{\pi}\right) = 0$

$\theta = \sin^{-1}(0.6366)$

$= 39° 39'$

INDICATOR DIAGRAM:

Ideal Indicator Diagram:



H_{atm} = atmospheric pressure head

= 10.3 m of water

L = length of stroke.

h_s = suction head,

h_d = Delivery head.

We know that the work done by pump per second

$$= \frac{\rho \times g \times ALN}{60} \times (h_s + h_d)$$

$$= K \times L\,(h_s + h_d)$$

$$\left[\text{where } K = \frac{\rho g A N}{60} = \text{constant}\right]$$

$$\propto L \times (h_s + h_d)$$

But from Fig 20.4, area of indicator diagram

$$= AB \times BC = AB \times (BF + FC) = L \times (h_s + h_d)$$

Effect of Acceleration in suction and delivery pipes on Indicator Diagram:

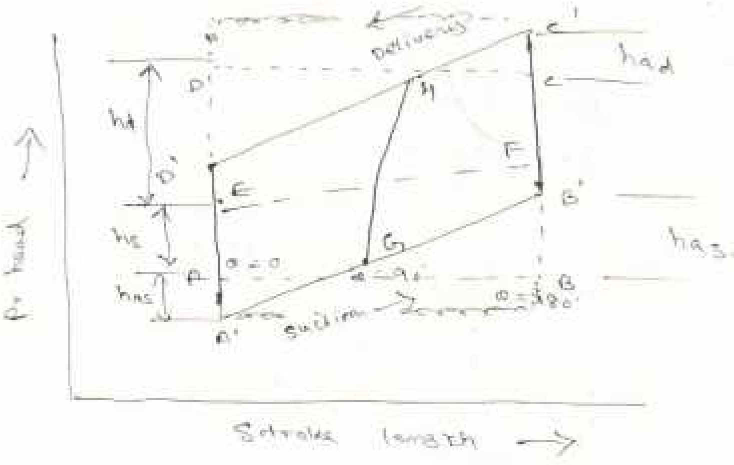
The Pr head due to acceleration in the suction pipe is given by eqn.

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r\cos\theta$$

when $\theta = 0°$, $\cos\theta = 1$ and $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

when $\theta = 90°$, $\cos\theta = 0$ and $h_{as} = 0$

when $\theta = 180°$, $\cos\theta = -1$ and $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$.



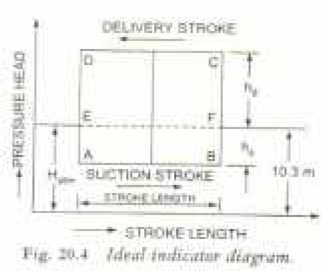


Fig. 20.4 *Ideal indicator diagram.*

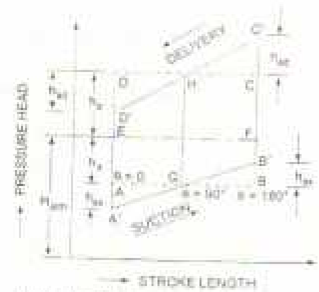


Fig. 20.5 *Effect of acceleration on indicator diagram.*

Problem 20.4:

The length and diameter of a suction pipe of a single-acting reciprocating pump are 5m and 10cm respectively. The pump has a plunger of diameter 15cm and a stroke length of 35cm. The centre of the pump is 3m above the water surface in the pump. The atm pr head is 10.3 m of water and pump is running at 35 r.p.m. Determine.

(i) pr head due to acceleration at the beginning of the suction stroke.

(ii) max pr head due to acceleration and

(iii) pr head in the cylinder at the beginning and at the end of the stroke.

G.D.

$l_s = 5m$

$d = 10cm$

$= \frac{10}{100} = 0.1 m$

$a_s = \pi/4\, d^2 = 7.854 \times 10^{-3}$

$= 0.007854\ m^2$

$D = 15cm = \frac{15}{100} = 0.15 m$

$A = \pi/4\, D^2$

$= 0.01767\ m^2$

$L = 35cm$

$= \frac{35}{100} = 0.35\ m^2$

$r = L/2 = \frac{0.35}{2} = 0.175\ m.$

The centre of pump is 3m above the water surface in the pump } suction head $h_s = 3m$

$H_{atm} = 10.3m$ of water.

$N = 35$ r.p.m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 35}{60}$$

$= 3.665$ rad/sec.

(i) Pr head due to acceleration in the suction pipe:

$$\boxed{h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos\theta}$$

At the beginning stroke $\theta = 0°$

hence ~~cos 0 = 0~~ $\cos(0) = 1$.

$$\therefore h_{as} = \frac{5}{9.81} \times \frac{0.01767}{0.007854} \times 3.665^2 (0.175) \cos 1$$

$= 2.695$ m.

(ii) Max Pr head due to acceleration in suction pipe:

$$\boxed{(h_{as})_{max} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r}$$

$= 2.695$ m.

(iii) Pr head in the cylinder at the beginning of suction stroke:

$= h_s + h_{as} = 3.0 + 2.695 = 5.695$.

Pr head in the cylinder is below the atm pr head.

$\therefore$ Absolute pr head in the cylinder at the beginning of suction stroke.

$=$ Atm pr head $- 5.695$

$= 10.3 - 5.695 = 4.605$ m of water (abs)

iii) Pr head in the cylinder at the end of suction stroke:

$= h_s - h_{as} = 3.0 - 2.695 = 0.305$ m below atm pr head

$= 10.3 - 0.305$

$= 9.995$ m of water (abs)

Problem 20.6:

A single acting reciprocating pump has piston diameter 12.5 cm and stroke length 30 cm. The centre of the pump is 4 m above the water level in the sump. The diameter and length of suction pipe are 7.5 cm and 7 m respectively. The separation occurs if the absolute pr head in the cylinder during suction stroke falls below 2.5 m of water. Calculate the max speed at which the pump can run without separation. Take atm pr head = 10.3 m of water.

G.D:

$D = 12.5\text{ cm} = \frac{12.5}{100} = 0.125$ m

$A = \pi/4\, D^2$

$= 0.01227\ m^2$

$L = 30$ cm

$= \frac{30}{2} = 0.3$ m

$r = \frac{L}{2} = \frac{0.3}{2} = 0.15$ m

~~l_{ds}~~

~~$h_s = $~~

$d_s = 7.5 cm$

$= \frac{7.5}{100} = 0.075 m$

$a_s = \pi/4 \, d^2$

$= 0.004418 \, m^2$

$l_s = 7 m.$

Separation pr head $h_{sep} = 2.5 m (abs)$

Atm pr head $H_{atm} = 10.3 m$.

(i) Thus the sep can take place at the beginning of stroke only.

(ii) Pr head in cylinder at beginning of suction stroke.

$$\boxed{= H_{atm} - (h_s + h_{as}) \, m \, (abs)}$$

$= 10.3 - (4.0 + h_{as})$

(ii) In what case pr head in the cylinder at the beginning of stroke becomes $= h_{sep}$

$\therefore h_{sep} = 10.3 - (4.0 + h_{as})$.

$2.5 = 10.3 - 4.0 - h_{as}$

$h_{as} = 10.3 - 4.0 - 2.5 = 3.80 m.$ — (1).

But from eqn, h_{as} at beginning of suction stroke is given by relat

$$\boxed{h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r}$$

$\because \theta = 0° \therefore \cos\theta = 1$ — (2).

eqn (1) and (2), we get

$3.80 = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

$3.80 = \frac{7.0}{9.81} \times \frac{0.01227}{0.004418} \times \omega^2 \times 0.15$

$$3.80 = \frac{7.0}{9.81} \times \frac{0.01227}{0.004418} \times \omega^2 \times 0.15$$

$$3.80 \times 9.81 \times 0.004418 = 7.0 \times 0.01227 \times 0.15 \times \omega^2$$

$$0.16469 = 0.012883\,\omega^2$$

$$\frac{0.16469}{0.012883} = \omega^2$$

$$12.783 = \omega^2$$

$$\omega = \sqrt{12.783} = 3.575 \text{ rad/s}$$

$$\omega = \frac{2\pi N}{60}$$

$$3.575 = \frac{2 \times \pi \times N}{60}$$

$$60 \times 3.575 = 62.83 \times N$$

$$214.5 = 62.83 \times N$$

$$34.13 = N$$

$$N = 34.14 \text{ r.p.m.}$$

Problem 20:

The diameter and stroke length of a single-acting reciprocating pump are 100mm and 300mm respectively. The water is lifted to a height of 20m above the centre of the pump. Find the max speed at which the pump may be run so that no separation occurs during the delivery stroke if the diameter and length of delivery pipe are 50mm and 25m respectively. Separation occurs if the absolute pr head in the cylinder during delivery stroke falls below 2.50 m of water.

Take atm pr head = 10.3 m of water

G.D:

Diameter of pump $D = 100\text{mm}$ ~~[illegible]~~

$$= \frac{100}{1000} = 0.1\text{m}$$

$L = 300\text{mm}$
$= 300/1000 = 0.3\text{m}$

Crank radius $r = \frac{L}{2} = \frac{0.30}{2} = 0.15\text{m}$

Delivery head $h_d = 20\text{m}$

Diamet of delivery pipe $d_d = 50\text{mm}$

$$= \frac{50}{1000} = 0.5\text{m}.$$

$$a_d = \pi/4 \, d^2$$

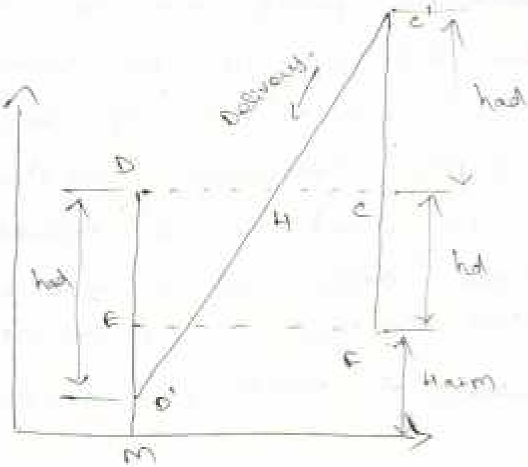
$$= 0.1963\text{ m}^2$$

length of delivery pipe $l_d = 25\text{m}$

Separation pr head $h_{sep} = 2.5\text{m (ab)}$

Atm pr head $H_{atm} = 10.3\text{ m of water}$.

Indicator diagram for delivery stroke only.



$$D'M = DM - DD'$$

$$= (DE + EM) - DD'$$

$$= (h_d + H_{atm}) - h_{ad}$$

$$\therefore h_{sep} = (h_d + H_{atm}) - h_{ad}$$

$$2.5 = (20 + 10.3) - h_{ad}$$

$$\therefore h_{ad} = (20 + 10.3) - 2.5 = 27.8\,m$$

But acceleration head h_{ad} at the end of delivery stroke is given by.

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d}\,\omega^2 r$$

$$27.8 = \frac{25}{9.81} \times \frac{\pi/4\, D^2}{\pi/4\, d_d^2} \times w^2 \times 0.15$$

$$= \frac{25}{9.81} \times \frac{D^2}{d_d^2} \times w^2 \times 0.15$$

$$= \frac{25}{9.81} \times \left(\frac{0.1}{.05}\right)^2 \times w^2 \times .15$$

$$= 1.529\, w^2$$

$$w = \sqrt{27.8/1.529} = 4.264 \text{ rad/s.}$$

We know that

$$w = \frac{2\pi N}{60}$$

$$N = \frac{60 \times w}{2\pi} = \frac{60 \times 4.264}{2\pi} = 40.72 \text{ r.p.m}$$