

## LP 1: Three Dimensional Object Representations

Representation schemes for solid objects are divided into two categories as follows:

### 1. Boundary Representation ( B-reps)

It describes a three dimensional object as a set of surfaces that separate the object interior from the environment. Examples are polygon facets and spline patches.

### 2. Space Partitioning representation

It describes the interior properties, by partitioning the spatial region containing an object into a set of small, nonoverlapping, contiguous solids(usually cubes). Eg: Octree Representation.

## Polygon Surfaces

Polygon surfaces are boundary representations for a 3D graphics object is a set of polygons that enclose the object interior.

## Polygon Tables

The polygon surface is specified with a set of vertex coordinates and associated attribute parameters.

For each polygon input, the data are placed into tables that are to be used in the subsequent processing.

Polygon data tables can be organized into two groups: Geometric tables and attribute tables.

**Geometric Tables** Contain vertex coordinates and parameters to identify the spatial orientation of the polygon surfaces.

**Attribute tables** Contain attribute information for an object such as parameters specifying the degree of transparency of the object and its surface reflectivity and texture characteristics. A convenient organization for storing geometric data is to create three lists:

### 1. The Vertex Table

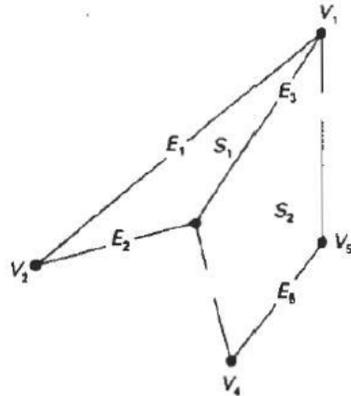
Coordinate values for each vertex in the object are stored in this table.

### 2. The Edge Table

It contains pointers back into the vertex table to identify the vertices for each polygon edge.

### 3. The Polygon Table

It contains pointers back into the edge table to identify the edges for each polygon. This is shown in fig

**Vertex table**

V1 : X1, Y1, Z1  
 V2 : X2, Y2, Z2  
 V3 : X3, Y3, Z3  
 V4 : X4, Y4, Z4  
 V5 : X5, Y5, Z5

**Edge Table**

E1 : V1, V2  
 E2 : V2, V3  
 E3 : V3, V1  
 E4 : V3, V4  
 E5 : V4, V5  
 E6 : V5, V1

**Polygon surface table**

S1 : E1, E2, E3  
 S2 : E3, E4, E5, E6

Listing the geometric data in three tables provides a convenient reference to the individual components (vertices, edges and polygons) of each object.

The object can be displayed efficiently by using data from the edge table to draw the component lines.

Extra information can be added to the data tables for faster information extraction. For instance, edge table can be expanded to include forward points into the polygon table so that common edges between polygons can be identified more rapidly.

E1 : V1, V2, S1  
 E2 : V2, V3, S1  
 E3 : V3, V1, S1, S2  
 E4 : V3, V4, S2  
 E5 : V4, V5, S2  
 E6 : V5, V1, S2

is useful for the rendering procedure that must vary surface shading smoothly across the edges from one polygon to the next. Similarly, the vertex table can be expanded so that vertices are cross-referenced to corresponding edges.

Additional geometric information that is stored in the data tables includes the slope for each edge and the coordinate extents for each polygon. As vertices are input, we can calculate edge slopes and we can scan the coordinate values to identify the minimum and maximum x, y and z values for individual polygons.

The more information included in the data tables will be easier to check for errors.

Some of the tests that could be performed by a graphics package are:

1. That every vertex is listed as an endpoint for at least two edges.
2. That every edge is part of at least one polygon.
3. That every polygon is closed.
4. That each polygon has at least one shared edge.
5. That if the edge table contains pointers to polygons, every edge referenced by a polygon pointer has a reciprocal pointer back to the polygon.

### Plane Equations:

To produce a display of a 3D object, we must process the input data representation for the object through several procedures such as,

- Transformation of the modeling and world coordinate descriptions to viewing coordinates.
- Then to device coordinates:
- Identification of visible surfaces
- The application of surface-rendering procedures.

For these processes, we need information about the spatial orientation of the individual surface components of the object. This information is obtained from the vertex coordinate value and the equations that describe the polygon planes.

The equation for a plane surface is

$$Ax + By + Cz + D = 0 \text{ ----(1)}$$

Where (x, y, z) is any point on the plane, and the coefficients A,B,C and D are constants describing the spatial properties of the plane.

We can obtain the values of A, B,C and D by solving a set of three plane equations using the coordinate values for three non collinear points in the plane.

For that, we can select three successive polygon vertices (x1, y1, z1), (x2, y2, z2) and (x3, y3, z3) and solve the following set of simultaneous linear plane equations for the ratios A/D, B/D and C/D.

$$(A/D)x_k + (B/D)y_k + (C/D)z_k = -1, k=1,2,3 \text{ -----(2)}$$

The solution for this set of equations can be obtained in determinant form, using Cramer's rule as

$$A = \frac{\begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix}}{\begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix}} \quad B = \frac{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}} \quad D = \frac{\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}} \text{ ----(3)}$$

Expanding the determinants, we can write the calculations for the plane coefficients in the form:

$$A = y_1(z_2 - z_3) + y_2(z_3 - z_1) + y_3(z_1 - z_2)$$

$$B = z_1(x_2 - x_3) + z_2(x_3 - x_1) + z_3(x_1 - x_2)$$

$$C = x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)$$

$$D = -x_1 (y_2 z_3 - y_3 z_2) - x_2 (y_3 z_1 - y_1 z_3) - x_3 (y_1 z_2 - y_2 z_1) \text{ -----(4)}$$

As vertex values and other information are entered into the polygon data structure, values for A, B, C and D are computed for each polygon and stored with the other polygon data.

Plane equations are used also to identify the position of spatial points relative to the plane surfaces of an object. For any point (x, y, z) not on a plane with parameters A,B,C,D, we have  $Ax + By + Cz + D \neq 0$

We can identify the point as either inside or outside the plane surface according to the sign (negative or positive) of  $Ax + By + Cz + D$ :

If  $Ax + By + Cz + D < 0$ , the point (x, y, z) is inside the surface. If  $Ax + By + Cz + D > 0$ , the point (x, y, z) is outside the surface.

These inequality tests are valid in a right handed Cartesian system, provided the plane parameters A,B,C and D were calculated using vertices selected in a counter clockwise order when viewing the surface in an outside-to-inside direction.

### Polygon Meshes

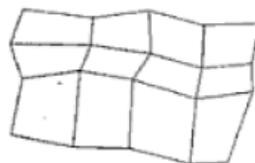
A single plane surface can be specified with a function such as **fillArea**. But when object surfaces are to be tiled, it is more convenient to specify the surface facets with a mesh function.

One type of polygon mesh is the **triangle strip**. A triangle strip formed with 11 triangles connecting 13 vertices.



This function produces n-2 connected triangles given the coordinates for n vertices.

Another similar function is the **quadrilateral mesh**, which generates a mesh of (n-1) by (m-1) quadrilaterals, given the coordinates for an n by m array of vertices. Figure shows 20 vertices forming a mesh of 12 quadrilaterals.



**Curved Lines and Surfaces** Displays of three dimensional curved lines and surface can be generated from an input set of mathematical functions defining the objects or from a set of user specified data points. When functions are specified, a package can project the defining equations for a curve to the display plane and plot pixel positions along the path of the projected function. For surfaces, a functional description is decorated to produce a polygon-mesh approximation to the surface.

**Spline Representations** A Spline is a flexible strip used to produce a smooth curve through a designated set of points. Several small weights are distributed along the length of the strip to hold it in position on the drafting table as the curve is drawn.

The **Spline curve** refers to any sections curve formed with polynomial sections satisfying specified continuity conditions at the boundary of the pieces.

A **Spline surface** can be described with two sets of orthogonal spline curves. Splines are used in graphics applications to design curve and surface shapes, to digitize drawings for computer storage, and to specify animation paths for the objects or the camera in the scene. CAD applications for splines include the design of automobiles bodies, aircraft and spacecraft surfaces, and ship hulls.

**Interpolation and Approximation Splines** Spline curve can be specified by a set of coordinate positions called **control points** which indicates the general shape of the curve. These control points are fitted with piecewise continuous parametric polynomial functions in one of the two ways. 1. When polynomial sections are fitted so that the curve passes through each control point the resulting curve is said to **interpolate** the set of control points.

**A set of six control points interpolated with piecewise continuous polynomial sections**



1. When the polynomials are fitted to the general control point path without necessarily passing through any control points, the resulting curve is said to **approximate** the set of control points.

**A set of six control points approximated with piecewise continuous polynomial sections**



Interpolation curves are used to digitize drawings or to specify animation paths. Approximation curves are used as design tools to structure object surfaces. A spline curve is designed, modified and manipulated with operations on the control points. The curve can be translated, rotated or scaled with transformation applied to the control points. The convex polygon boundary that encloses a set of control points is called the **convex hull**. The shape of the convex hull is to imagine a rubber band stretched around the position of the control points so that each control point is either on the perimeter of the hull or inside it. **Convex hull shapes (dashed lines) for two sets of control points**

### Parametric Continuity Conditions

For a smooth transition from one section of a piecewise parametric curve to the next various **continuity conditions** are needed at the connection points.

If each section of a spline is described with a set of parametric coordinate functions or the form  $x = x(u)$ ,  $y = y(u)$ ,  $z = z(u)$ ,  $u_1 \leq u \leq u_2$  -----(a)

We set **parametric continuity** by matching the parametric derivatives of adjoining curve sections at their common boundary.

**Zero order parametric continuity** referred to as C0 continuity, means that the curves meet. (i.e) the values of x,y, and z evaluated at  $u_2$  for the first curve section are equal. Respectively, to the value of x,y, and z evaluated at  $u_1$  for the next curve section.

**First order parametric continuity** referred to as C1 continuity means that the first parametric derivatives of the coordinate functions in equation (a) for two successive curve sections are equal at their joining point.

**Second order parametric continuity**, or C2 continuity means that both the first and second parametric derivatives of the two curve sections are equal at their intersection.

Higher order parametric continuity conditions are defined similarly.

**Piecewise construction of a curve by joining two curve segments using different orders of continuity**

a) Zero order continuity only



b) First order continuity only



### c) Second order continuity only



### Geometric Continuity Conditions

To specify conditions for geometric continuity is an alternate method for joining two successive curve sections.

The parametric derivatives of the two sections should be proportional to each other at their common boundary instead of equal to each other.

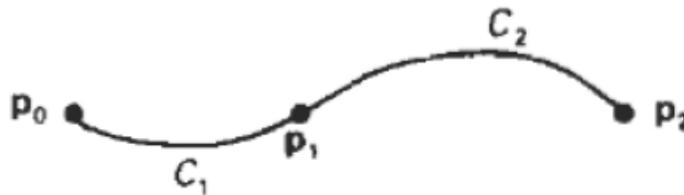
Zero order Geometric continuity referred as G0 continuity means that the two curves sections must have the same coordinate position at the boundary point.

First order Geometric Continuity referred as G1 continuity means that the parametric first derivatives are proportional at the interaction of two successive sections.

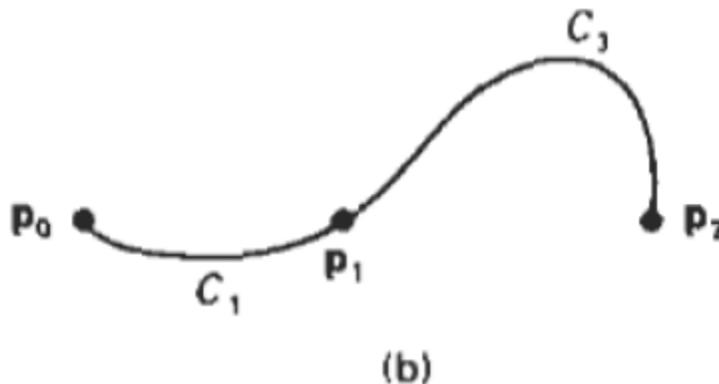
Second order Geometric continuity referred as G2 continuity means that both the first and second parametric derivatives of the two curve sections are proportional at their boundary. Here the curvatures of two sections will match at the joining position.

### Three control points fitted with two curve sections joined with

#### a) parametric continuity



b) geometric continuity where the tangent vector of curve C3 at point p1 has a greater magnitude than the tangent vector of curve C1 at p1.



**Spline specifications** There are three methods to specify a spline representation:

1. We can state the set of boundary conditions that are imposed on the spline; (or)
2. We can state the matrix that characterizes the spline; (or)
3. We can state the set of **blending functions** that determine how specified geometric constraints on the curve are combined to calculate positions along the curve path.

To illustrate these three equivalent specifications, suppose we have the following parametric cubic polynomial representation for the x coordinate along the path of a spline section.

$x(u) = au^3 + bu^2 + cu + d$   $0 \leq u \leq 1$  -----(1) Boundary conditions for this curve might be set on the endpoint coordinates  $x(0)$  and  $x(1)$  and on the parametric first derivatives at the endpoints  $x'(0)$  and  $x'(1)$ . These boundary conditions are sufficient to determine the values of the four coordinates  $a$ ,  $b$ ,  $c$  and  $d$ . From the boundary conditions we can obtain the matrix that characterizes this spline curve by first rewriting eq(1) as the matrix product

$$x(u) = [u^3 \ u^2 \ u \ 1] \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \text{ -----( 2 )}$$

= U.C

where U is the row matrix of power of parameter u and C is the coefficient column matrix.

Using equation (2) we can write the boundary conditions in matrix form and solve for the coefficient matrix C as

$C = M_{\text{spline}} \cdot M_{\text{geom}}$  -----(3) Where  $M_{\text{geom}}$  is a four element column matrix containing the geometric constraint values on the spline and  $M_{\text{spline}}$  is the  $4 \times 4$  matrix that transforms the geometric constraint values to the polynomial coefficients and provides a characterization for the spline curve.

Matrix  $M_{\text{geom}}$  contains control point coordinate values and other geometric constraints.

We can substitute the matrix representation for C into equation (2) to obtain.

$$x(u) = U \cdot M_{\text{spline}} \cdot M_{\text{geom}} \text{ -----(4)}$$

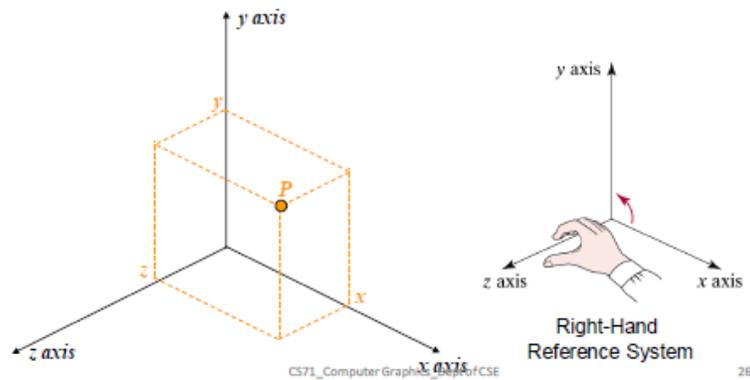
The matrix  $M_{\text{spline}}$ , characterizing a spline representation, called the **basis matrix** is useful for transforming from one spline representation to another.

Finally we can expand equation (4) to obtain a polynomial representation for coordinate x in terms of the geometric constraint parameters.

$x(u) = \sum g_k \cdot B_k(u)$  where  $g_k$  are the constraint parameters, such as the control point coordinates and slope of the curve at the control points and  $B_k(u)$  are the polynomial blending functions.

## LP 2: 3D Transformation

### 3-D Coordinate Spaces

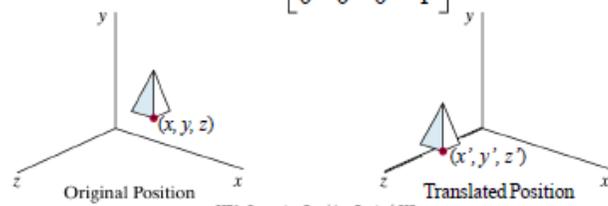


## Translations In 3-D

To translate a point in three dimensions by  $dx$ ,  $dy$  and  $dz$  .calculate the new points as follows:

$$x' = x + dx \quad y' = y + dy \quad z' = z + dz$$

$$T(d_x, d_y, d_z) = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



## Scaling In 3-D

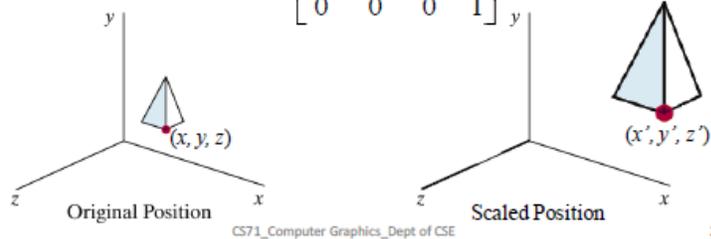
To scale a point in three dimensions by  $s_x$ ,  $s_y$  and  $s_z$  simply calculate the new points as follows:

$$x' = s_x * x$$

$$y' = s_y * y$$

$$z' = s_z * z$$

$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



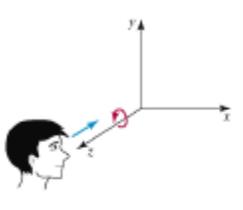
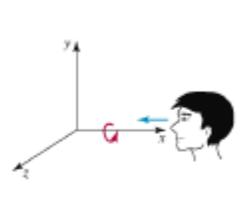
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Rotations In 3-D When we performed rotations in two dimensions we only had the choice of rotating about the z axis In the case of three dimensions we have more options – Rotate about x – Rotate about y – Rotate about z

## Rotations In 3-D (cont...)

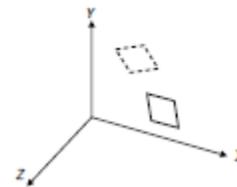
The equations for the three kinds of rotations in 3-D are as follows: (Assume – X→Y→Z→X)

		
$\begin{aligned} x' &= x \cdot \cos\theta - y \cdot \sin\theta \\ y' &= x \cdot \sin\theta + y \cdot \cos\theta \\ z' &= z \end{aligned}$	$\begin{aligned} x' &= x \\ y' &= y \cdot \cos\theta - z \cdot \sin\theta \\ z' &= y \cdot \sin\theta + z \cdot \cos\theta \end{aligned}$ <small>CS71, Computer Graphics, Dept of CSE</small>	$\begin{aligned} x' &= z \cdot \sin\theta + x \cdot \cos\theta \\ y' &= y \\ z' &= z \cdot \cos\theta - x \cdot \sin\theta \end{aligned}$ <small>3D</small>

### 3D Transformations (cont.)

The 2D rotation introduced previously is just a 3D rotation about z axis.

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



similarly we have:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### 3D Transformation Matrices

Translation by $dx, dy, dz$	$\begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Scaling by $s_x, s_y, s_z$	$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
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$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Rotate About X-Axis	$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Rotate About Y-Axis	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Rotate About Z-Axis
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General 3D Rotations • Rotation about an axis that is parallel to one of the coordinate axes : 1. Translate the object so that the rotation axis coincides with the parallel coordinate axis 2. Perform the specified rotation about the axis 3. Translate the object so that the rotation axis is moved back to its original position • Not parallel : 1. Translate the object so that the rotation axis passes through the coordinate origin 2. Rotate the object so that the axis of rotation coincides with one of the coordinate axes 3. Perform the specified rotation about the axis 4. Apply inverse rotations to bring the rotation axis back to its original orientation 5. Apply the inverse translation to bring back the rotation axis to its original position

3 D Transformation functions • Functions are – translate3(translateVector, matrixTranslate) – rotateX(thetaX, xMatrixRotate) – rotateY(thetaY, yMatrixRotate) – rotateZ(thetaZ, zMatrixRotate) – scale3(scaleVector,matrixScale) • To apply transformation matrix to the specified points , – transformPoint3(inPoint, matrix,outPoint) • We can construct composite transformations with the following functions – composeMatrix3 – buildTransformationMatrix3 – composeTransformationMatrix3 CS71\_Computer Graphics\_Dept of CSE 34 Reflections In 3-D • Three Dimensional Reflections can be performed relative to a selected reflection axis or a selected reflection plane • Consider a reflection that converts coordinate specifications from a right handed system to left handed system. • This transformation changes the sign of Z coordinate leaving x and y coordinates

$$T(d_x, d_y, d_z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$


### Shears In 3-D

Shearing transformations are used to distortions in the shape of an object. In 2D, shearing is applied to x or y axes. In 3D it can applied to z axis also

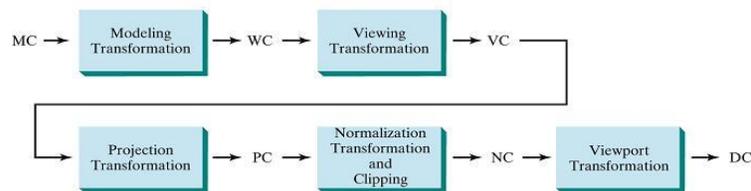
The following transformation produces an Z axis shear

$$SH_z = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Parameters a and b can be assigned any real values

### LP 3: 3D Viewing

- World coordinate system (where the objects are modeled and defined)
- Viewing coordinate system (viewing objects with respect to another user defined coordinate system)
- Projection coordinate system (coordinate positions to be on the projection plane)
- Device Coordinate System (pixel positions in the plane of the output device)



steps to establish a Viewing coordinate system or view reference coordinate system and the view plane

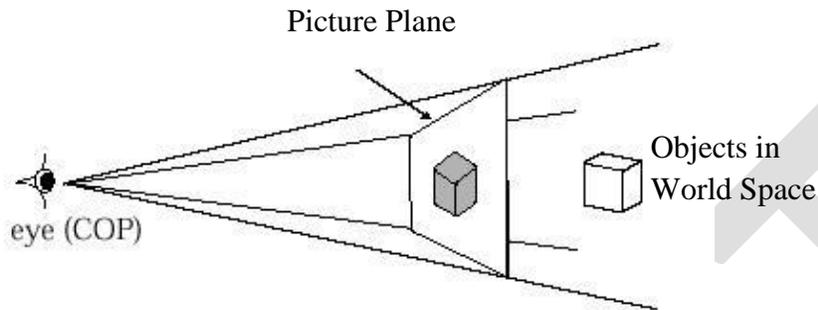
#### Transformation from world to viewing coordinates

- Translate the view reference point to the origin of the world coordinate system
- Apply rotations to align the axes
  - Three rotations of three axes
  - Composite transformation matrix (unit vectors  $u$ ,  $v$ ,  $n$ )
    - $n = N/|N|$
    - $u = (V * N) / |V * N|$

- $v = n * u$

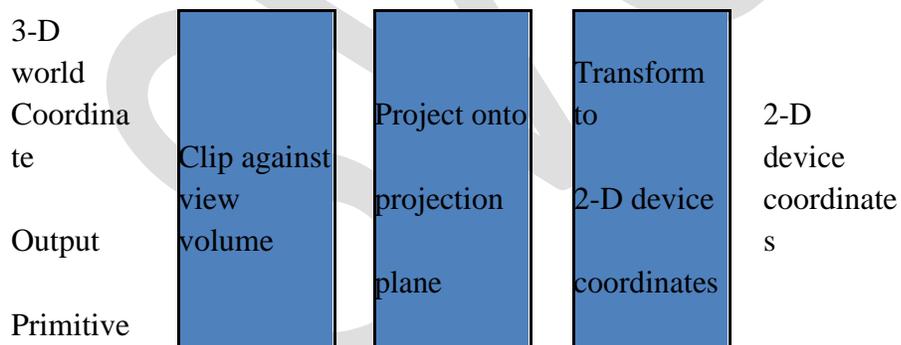
## Projections

Our 3-D scenes are all specified in 3-D world coordinates. To display these we need to generate a 2-D image - project objects onto a picture plane



## Converting From 3-D To 2-D

- Projection is just one part of the process of transforming 3D world coordinates to a 2-D projection plane



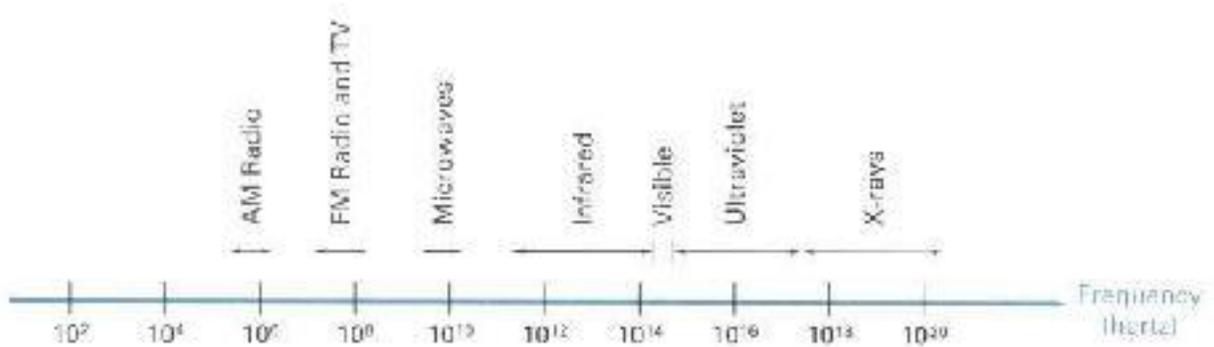
## LP 5: Color Models

Color Model is a method for explaining the properties or behavior of color within some particular context. No single color model can explain all aspects of color, so we make use of different models to help describe the different perceived characteristics of color.

### Properties of Light

Light is a narrow frequency band within the electromagnetic system.

Other frequency bands within this spectrum are called radio waves, micro waves, infrared waves and x-rays. The below fig shows the frequency ranges for some of the electromagnetic bands.



Each frequency value within the visible band corresponds to a distinct color.

At the low frequency end is a red color ( $4.3 \times 10^{14}$  Hz) and the highest frequency is a violet color ( $7.5 \times 10^{14}$  Hz)

Spectral colors range from the reds through orange and yellow at the low frequency end to greens, blues and violet at the high end.

Since light is an electro magnetic wave, the various colors are described in terms of either the frequency for the wave length  $\lambda$  of the wave.

The wave length and frequency of the monochromatic wave are inversely proportional to each other, with the proportionality constants as the speed of light

C where  $C = \lambda f$

A light source such as the sun or a light bulb emits all frequencies within the visible range to produce white light. When white light is incident upon an object, some frequencies are reflected and some are absorbed by the object. The combination of frequencies present in the reflected light determines what we perceive as the color of the object.

If low frequencies are predominant in the reflected light, the object is described as red. In this case, the perceived light has the dominant frequency at the red end of the spectrum. The dominant frequency is also called the hue, or simply the color of the light.

Brightness is another property, which in the perceived intensity of the light.

Intensity in the radiant energy emitted per unit time, per unit solid angle, and per unit projected area of the source.

Radiant energy is related to the luminance of the source.

The next property in the purity or saturation of the light.

- Purity describes how washed out or how pure the color of the light appears.
- Pastels and Pale colors are described as less pure.

The term chromaticity is used to refer collectively to the two properties, purity and dominant frequency.

Two different color light sources with suitably chosen intensities can be used to produce a range of other colors.

If the 2 color sources combine to produce white light, they are called complementary colors. E.g., Red and Cyan, green and magenta, and blue and yellow.

Color models that are used to describe combinations of light in terms of dominant frequency use 3 colors to obtain a wide range of colors, called the color gamut.

The 2 or 3 colors used to produce other colors in a color model are called primary colors.

### **Standard Primaries**

### **XYZ Color**

The set of primaries is generally referred to as the XYZ or (X,Y,Z) color model

where X,Y and Z represent vectors in a 3D, additive color space.

Any color  $C\lambda$  is expressed as

$$C\lambda = XX + YY + ZZ \text{-----} \quad (1)$$

Where X,Y and Z designates the amounts of the standard primaries needed to match  $C\lambda$ .

It is convenient to normalize the amount in equation (1) against luminance  $(X+ Y+ Z)$ . Normalized amounts are calculated as,

$$x = X/(X+Y+Z), \quad y = Y/(X+Y+Z), \quad z = Z/(X+Y+Z)$$

with  $x + y + z = 1$

Any color can be represented with just the x and y amounts. The parameters x and y are called the chromaticity values because they depend only on hue and purity.

If we specify colors only with x and y, we cannot obtain the amounts X, Y and Z. so, a complete description of a color is given with the 3 values x, y and Y.

$$X = (x/y)Y, \quad Z = (z/y)Y$$

Where  $z = 1-x-y$ .

### Intuitive Color Concepts

Color paintings can be created by mixing color pigments with white and black pigments to form the various shades, tints and tones.

Starting with the pigment for a „pure color“ the color is added to black pigment to produce different shades. The more black pigment produces darker shades.

Different tints of the color are obtained by adding a white pigment to the original color, making it lighter as more white is added.

Tones of the color are produced by adding both black and white pigments.

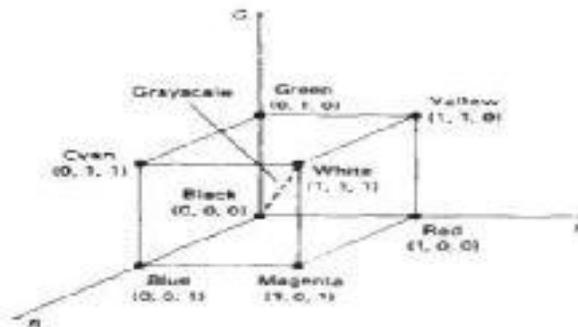
## RGB Color Model

Based on the tristimulus theory of vision, our eyes perceive color through the stimulation of three visual pigments in the cones on the retina.

These visual pigments have a peak sensitivity at wavelengths of about 630 nm (red), 530 nm (green) and 450 nm (blue).

By comparing intensities in a light source, we perceive the color of the light.

This is the basis for displaying color output on a video monitor using the 3 color primaries, red, green, and blue referred to as the RGB color model. It is represented in the below figure



Vertices of the cube on the axes represent the primary colors, the remaining vertices represents the complementary color for each of the primary colors.

The RGB color scheme is an additive model. (i.e.,) Intensities of the primary colors are added to produce other colors.

Each color point within the bounds of the cube can be represented as the triple (R,G,B) where values for R, G and B are assigned in the range from 0 to 1.

The color  $C_\lambda$  is expressed in RGB component as

$$C_\lambda = RR + GG + BB$$

The magenta vertex is obtained by adding red and blue to produce the triple (1,0,1) and white at (1,1,1) in the sum of the red, green and blue vertices.

Shades of gray are represented along the main diagonal of the cube from the origin (black) to the white vertex.

## YIQ Color Model

The National Television System Committee (NTSC) color model for forming the composite video signal in the YIQ model.

In the YIQ color model, luminance (brightness) information is contained in the Y parameter, chromaticity information (hue and purity) is contained into the I and Q parameters.

A combination of red, green and blue intensities are chosen for the Y parameter to yield the standard luminosity curve.

Since Y contains the luminance information, black and white TV monitors use only the Y signal.

Parameter I contains orange-cyan hue information that provides the flash-tone shading and occupies a bandwidth of 1.5 MHz.

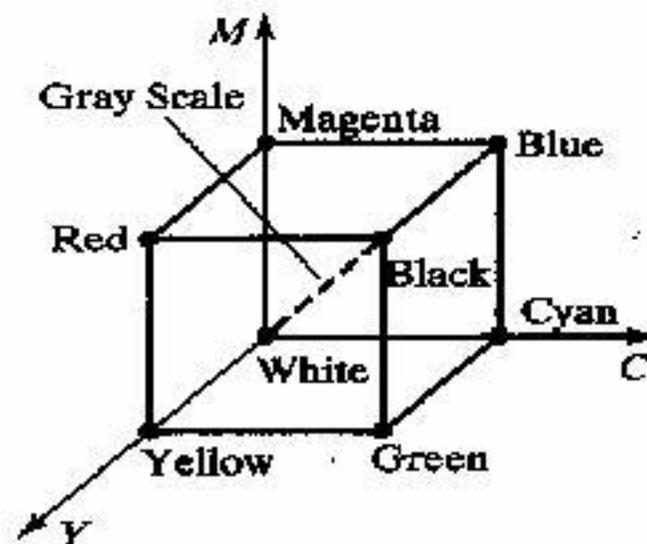
Parameter Q carries green-magenta hue information in a bandwidth of about 0.6 MHz.

## CMY Color Model

A color model defined with the primary colors cyan, magenta, and yellow (CMY) is useful for describing color output to hard copy devices.

It is a subtractive color model (i.e., cyan can be formed by adding green and blue light. When white light is reflected from cyan-colored ink, the reflected light must have no red component. i.e., red light is absorbed or subtracted by the ink.

Magenta ink subtracts the green component from incident light and yellow subtracts the blue component.



In CMY model, point (1,1,1) represents black because all components of the incident light are subtracted.

The origin represents white light.

Equal amounts of each of the primary colors produce grays along the main diagonal of the cube.

A combination of cyan and magenta ink produces blue light because the red and green components of the incident light are absorbed.

The printing process often used with the CMY model generates a color point with a collection of 4 ink dots; one dot is used for each of the primary colors (cyan, magenta and yellow) and one dot in black.

The conversion from an RGB representation to a CMY representation is expressed as

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Where the white is represented in the RGB system as the unit column vector.

Similarly the conversion of CMY to RGB representation is expressed as

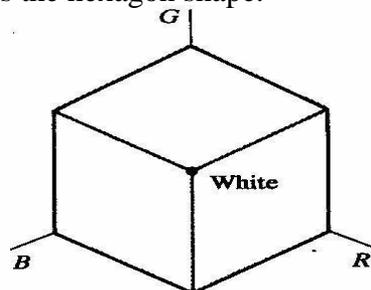
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} C \\ M \\ Y \end{bmatrix}$$

Where black is represented in the CMY system as the unit column vector.

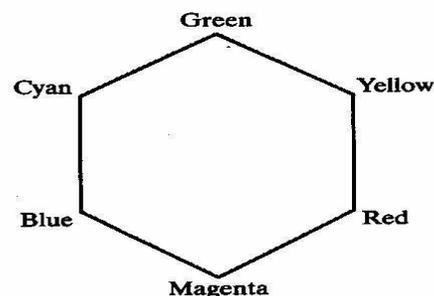
### HSV Color Model

The HSV model uses color descriptions that have a more interactive appeal to a user.

Color parameters in this model are hue (H), saturation (S), and value (V). The 3D representation of the HSV model is derived from the RGB cube. The outline of the cube has the hexagon shape.



RGB Color Cube  
(a)



Color Hexagon  
(b)

The boundary of the hexagon represents the various hues, and it is used as the top of the HSV hexcone.

In the hexcone, saturation is measured along a horizontal axis, and value is along a vertical axis through the center of the hexcone.

Hue is represented as an angle about the vertical axis, ranging from  $0^{\circ}$  at red through  $360^{\circ}$ . Vertices of the hexagon are separated by  $60^{\circ}$  intervals. Yellow is at  $60^{\circ}$ , green at  $120^{\circ}$  and cyan opposite red at  $H = 180^{\circ}$ . Complementary colors are  $180^{\circ}$  apart.

Saturation  $S$  varies from 0 to 1. the maximum purity at  $S = 1$ , at  $S = 0.25$ , the hue is said to be one quarter pure, at  $S = 0$ , we have the gray scale. Value  $V$  varies from 0 at the apex to 1 at the top.

- the apex representation black.

At the top of the hexcone, colors have their maximum intensity. When  $V = 1$  and  $S = 1$  we have the „pure“ hues.

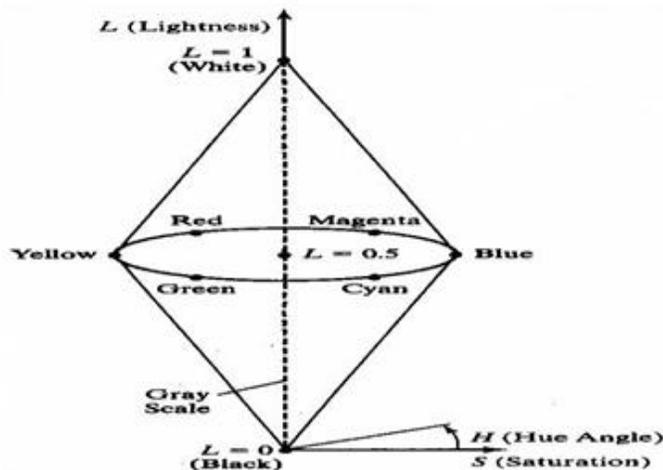
White is the point at  $V = 1$  and  $S = 0$ .

### HLS Color Model

HLS model is based on intuitive color parameters used by Tektronix.

It has the double cone representation shown in the below figure. The 3 parameters in this model are called Hue (H), lightness (L) and saturation (s).

Hue specifies an angle about the vertical axis that locates a chosen hue.



In this model  $H = 0^\circ$  corresponds to Blue.

The remaining colors are specified around the perimeter of the cone in the same order as in the HSV model.

Magenta is at  $60^\circ$ , Red in at  $120^\circ$ , and cyan in at  $H = 180^\circ$ .

The vertical axis is called lightness (L). At  $L = 0$ , we have black, and white is at  $L$

$= 1$  Gray scale in along the L axis and the “purehues” on the  $L = 0.5$  plane.

Saturation parameter S specifies relative purity of a color. S varies from 0 to 1 pure hues are those for which  $S = 1$  and  $L = 0.5$ .

- As S decreases, the hues are said to be less pure.
- At  $S = 0$ , it is said to be gray scale.

## LP 7: Animation

Computer animation refers to any time sequence of visual changes in a scene.

Computer animations can also be generated by changing camera parameters such as position, orientation and focal length.

Applications of computer-generated animation are entertainment, advertising, training and education.

**Example :** Advertising animations often transition one object shape into another.

### Frame-by-Frame animation

Each frame of the scene is separately generated and stored. Later, the frames can be recoded on film or they can be consecutively displayed in "real-time playback" mode

### Design of Animation Sequences

An animation sequence is designed with the following steps:

- Story board layout
- Object definitions
- Key-frame specifications

**Story Board:**

Generation of in-between frames.

The story board is an outline of the action.

It defines the motion sequences as a set of basic events that are to take place.

Depending on the type of animation to be produced, the story board could consist of a set of rough sketches or a list of the basic ideas for the motion.

**Object Definition**

An object definition is given for each participant in the action.

Objects can be defined in terms of basic shapes such as polygons or splines.

The associated movements of each object are specified along with the shape.

**Key frame**

A key frame is detailed drawing of the scene at a certain time in the animation sequence.

Within each key frame, each object is positioned according to the time for that frame.

Some key frames are chosen at extreme positions in the action; others are spaced so that the time interval between key frames is not too much.

**In-betweens**

In betweens are the intermediate frames between the key frames.

The number of in between needed is determined by the media to be used to display the animation.

Film requires 24 frames per second and graphics terminals are refreshed at the rate of 30 to 60 frames per seconds.

Time intervals for the motion are setup so there are from 3 to 5 in-between for each

pair of key frames.

Depending on the speed of the motion, some key frames can be duplicated.

For a 1 min film sequence with no duplication, 1440 frames are needed.

Other required tasks are

- Motion verification
- Editing
- Production and synchronization of a sound track.

### **General Computer Animation Functions**

Steps in the development of an animation sequence are,

- Object manipulation and rendering
- Camera motion
- Generation of in-betweens

Animation packages such as wave front provide special functions for designing the animation and processing individuals objects.

Animation packages facilitate to store and manage the object database.

Object shapes and associated parameter are stored and updated in the database.

Motion can be generated according to specified constraints using 2D and 3D transformations.

Standard functions can be applied to identify visible surfaces and apply the rendering algorithms.

Camera movement functions such as zooming, panning and tilting are used for motion simulation.

Given the specification for the key frames, the in-betweens can be automatically generated.

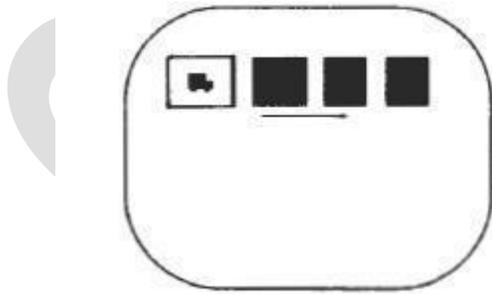
## Raster Animations

On raster systems, real-time animation in limited applications can be generated using raster operations.

Sequence of raster operations can be executed to produce real time animation of either 2D or 3D objects.

We can animate objects along 2D motion paths using the color-table transformations.

- Predefine the object as successive positions along the motion path, set the successive blocks of pixel values to color table entries.
- Set the pixels at the first position of the object to „on“ values, and set the pixels at the other object positions to the background color.
- The animation is accomplished by changing the color table values so that the object is „on“ at successive positions along the animation path as the preceding position is set to the background intensity.



## Computer Animation Languages

Animation functions include a graphics editor, a key frame generator and standard graphics routines.

The graphics editor allows designing and modifying object shapes, using spline surfaces, constructive solid geometry methods or other representation schemes.

Scene description includes the positioning of objects and light sources defining the photometric parameters and setting the camera parameters.

Action specification involves the layout of motion paths for the objects and camera.

Keyframe systems are specialized animation languages designed simply to generate the in-betweens from the user specified keyframes.

Parameterized systems allow object motion characteristics to be specified as part of the object definitions. The adjustable parameters control such object characteristics as degrees of freedom motion limitations and allowable shape changes.

Scripting systems allow object specifications and animation sequences to be defined with a user input script. From the script, a library of various objects and motions can be constructed.

## Keyframe Systems

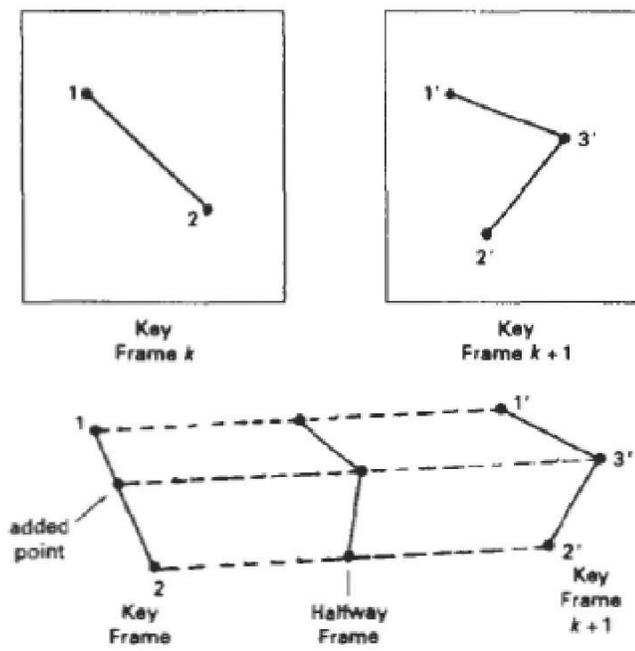
Each set of in-betweens are generated from the specification of two keyframes.

For complex scenes, we can separate the frames into individual components or objects called cells, an acronym from cartoon animation.

## Morphing

Transformation of object shapes from one form to another is called Morphing.

Morphing methods can be applied to any motion or transition involving a change in shape. The example is shown in the below figure.



Suppose we equalize the edge count and parameters  $L_k$  and  $L_{k+1}$  denote the number of line segments in two consecutive frames. We define,

$$\begin{aligned} L_{\max} &= \max(L_k, L_{k+1}) \\ L_{\min} &= \min(L_k, L_{k+1}) \\ N_e &= L_{\max} \bmod L_{\min} \\ N_s &= \text{int}(L_{\max}/L_{\min}) \end{aligned}$$

The preprocessing is accomplished by

- Dividing  $N_e$  edges of keyframe<sub>min</sub> into  $N_s+1$  section.
- Dividing the remaining lines of keyframe<sub>min</sub> into  $N_s$  sections.

For example, if  $L_k = 15$  and  $L_{k+1} = 11$ , we divide 4 lines of keyframe<sub>k+1</sub> into 2 sections each. The remaining lines of keyframe<sub>k+1</sub> are left intact.

If the vector counts in equalized parameters  $V_k$  and  $V_{k+1}$  are used to denote the number of vertices in the two consecutive frames. In this case we define

$$V_{\max} = \max(V_k, V_{k+1}), V_{\min} = \min(V_k, V_{k+1}) \quad \text{and}$$

$$N_{ls} = (V_{\max} - 1) \bmod (V_{\min} - 1)$$

$$N_p = \text{int}((V_{\max} - 1)/(V_{\min} - 1))$$

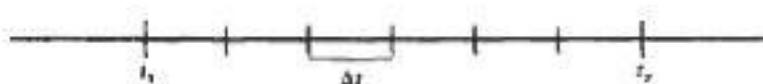
Preprocessing using vertex count is performed by

- Adding  $N_p$  points to  $N_{ls}$  line section of keyframe<sub>min</sub>.
- Adding  $N_p-1$  points to the remaining edges of keyframe<sub>min</sub>.

## Simulating Accelerations

Curve-fitting techniques are often used to specify the animation paths between key frames. Given the vertex positions at the key frames, we can fit the positions with linear or nonlinear paths. Figure illustrates a nonlinear fit of key-frame positions. This determines the trajectories for the in-betweens. To simulate accelerations, we can adjust the time spacing for the in-betweens.

For constant speed (zero acceleration), we use equal-interval time spacing for the in-betweens. Suppose we want  $n$  in-betweens for key frames at times  $t_1$  and  $t_2$ .



The time interval between key frames is then divided into  $n + 1$  subintervals, yielding an in-between spacing of

$$\Delta = (t_2 - t_1) / (n + 1)$$

we can calculate the time for any in-between as

$$t_{Bj} = t_1 + j \Delta, j = 1, 2, \dots, n$$

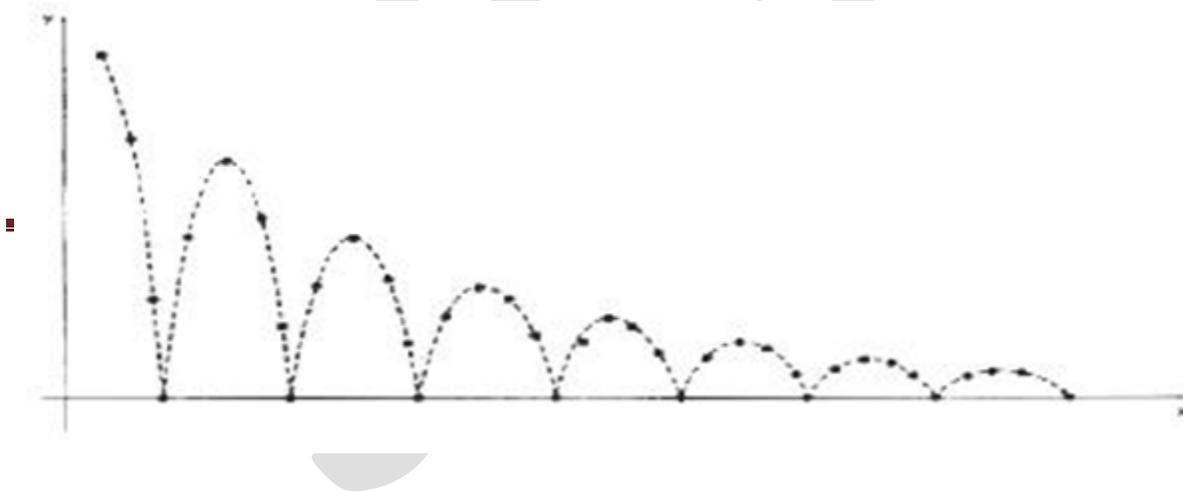
### Motion Specification

These are several ways in which the motions of objects can be specified in an animation system.

### Direct Motion Specification

Here the rotation angles and translation vectors are explicitly given.

Then the geometric transformation matrices are applied to transform coordinate positions.



We can approximate the path of a bouncing ball with a damped, rectified, sine curve

$$y(x) = A / \sin(\omega_x + \theta_0) / e^{-kx}$$

where  $A$  is the initial amplitude,  $\omega$  is the angular frequency,  $\theta_0$  is the phase angle and  $k$  is the damping constant.