

Introduction

Def 1: The creation, storage and manipulation of images and drawings using a digital computer is called **computer graphics**.

Def 2: Computer graphics are graphics created using computers and more generally the representation and manipulation of image data by a computer with help from specialized software and hardware.

Computer contains two components.

- **Computer hardware**

Computer hardware contains the graphics workstations, graphic **input devices** and graphic **output devices**.

- **Computer Software**

Computer software contains the **Operating systems** which controls the basic operations, **software packages** used for geometric modeling like solid modeling, wire frame modeling and drafting, **Application software** which includes the program for design analysis, several application-specific software packages.

The major use of computer graphics is in design processes like engineering and architectural systems. Now almost all the products are computer designed. It is generally referred as CAD (Computer Aided Design) . It is mainly used in the design of buildings, automobiles, aircraft, computers, textiles, etc.

LP 1: Line Drawing algorithm

Slope-intercept equation for a straight line is

$$Y=mx+b$$

m- slope of the line.

b-constant

Two end points of the line segment are (x_1,y_1) , (x_2,y_2)

Slope $m = \frac{y_2-y_1}{x_2-x_1} \rightarrow \Delta y/\Delta x$

Δx - x interval – x_2-x_1

Δy – y interval – y_2-y_1

$$\Delta y = m. \Delta x$$

$$\Delta x = \Delta y/m$$

If the interval is known, we can find the next point.

$$x_{i+1} = x_i + \Delta x \rightarrow x_i + \Delta y/m$$

$$y_{i+1} = y_i + \Delta y \rightarrow y_i + m \cdot \Delta x$$

we sample at unit x interval and y interval then this equation becomes

$$x_{i+1} = x_i + \Delta x \rightarrow x_i + (1/m)$$

$$y_{i+1} = y_i + \Delta y \rightarrow y_i + m \cdot 1$$

The above equations are for the lines which are processed from left to right.

The equations are for the lines which are processed from right to left is

$$x_{i+1} = x_i + \Delta x \rightarrow x_i - (1/m)$$

$$y_{i+1} = y_i + \Delta y \rightarrow y_i - m \cdot 1$$

Since m can be any real number between 0 and 1 the y value must be rounded to the nearest integer.

DDA Algorithm (Digital differential analyzer)

1. Get the two end points
2. Calculate the Horizontal and vertical difference between two end points (dx,dy)
3. The greatest difference value is taken as the length value.
4. Calculate the step value $dx = dx/\text{length}$, $dy = dy / \text{length}$
5. Start plotting the line from the first point.
6. Repeat through 9 step up to length value times
7. if $dx > dy$ and $x_a < x_b$ then increment x by 1 and increment y by m
8. if $dx > dy$ and $x_a > x_b$ then decrement x by 1 and decrement y by m
9. if $dx < dy$ and $y_a < y_b$ then increment y by 1 and increment x by 1/m
10. if $dx < dy$ and $y_a > y_b$ then decrement y by 1 and decrement x by 1/m

DDA Line Algorithm

It generates lines from their differential equations.

Advantages

1. It is the faster method for calculating pixel positions.
2. It is simplest algorithm. It does not require special skills for implementation.

Disadvantages

Floating point arithmetic in DDA algorithm is still time consuming

The algorithm is orientation dependent. Hence end point accuracy is poor.

Bresenham's Line Algorithm

This algorithm uses only integer addition, subtraction, and multiplication by 2. So it is efficient for scan converting algorithms. This algorithm increments either x or y by one unit depending on the slope of the line. The increment in the other variable is determined by examine the distance between the actual line location and the nearest pixel. This distance is called **decision variable** or the **error**.

In mathematical terms the error or decision variable is defined as

$$e = D_b - D_a$$

If $e > 0$ then the pixel above the line is closer to the true line.

Else the pixel below the line is closer to the true line.

Assume the current pixel is (x_k, y_k)

We have to find the next pixel position either $((x_{k+1}, y_k)$ and (x_{k+1}, y_{k+1})

The y coordinate on the line at the pixel position x_{k+1} is $y = m(x_{k+1}) + c$

Then the distance $d_1 = y - y_k = m(x_{k+1}) + c - y_k$

$$d_2 = (y_{k+1} - y) = y_{k+1} - m(x_{k+1}) + c$$

$$d_1 - d_2 = m(x_{k+1}) + c - y_k - [y_{k+1} - m(x_{k+1}) + c]$$

$$= 2m(x_{k+1}) - 2y_k + 2c - 1$$

The error term is initially set as $e = 2\Delta y - \Delta x$ where $\Delta y = y_2 - y_1$, $\Delta x = x_2 - x_1$

Bresenham's algorithm

1. Get the two end points
2. Calculate the values $dx, dy, 2dy$ and $2dy - 2dx$ where $dx = X_2 - X_1$ and $dy = Y_2 - Y_1$
3. Calculate the starting decision parameter $d = 2dy - dx$
4. plot the first point
5. At each X_k along the line, starting at $k=0$, perform the following test

If $p_k < 0$, the next point to plot is (X_{k+1}, Y_k) and $P_{k+1} = p_k + 2dy$

Otherwise the next point to plot is (X_{k+1}, Y_{k+1}) $P_{k+1} = p_k + 2dy - 2dx$

6. Repeat step 5 for dx times.

The following program is used to for generalized bresenham algorithm, which will work for all the four coordinates.

Parallel Line algorithm

We can calculate the pixel position along the path simultaneously by partitioning the computations among the various processors available. One approach to the partitioning problem is to adapt an existing sequential algorithm to take advantage of multiple processors. Alternatively, we can look for other ways to set up the processing so that pixel positions can be calculated efficiently in parallel.

LP 2: Circle Generating algorithm

Properties of the Circle

Circle is defined as a set of points that are all at a given distance r from a center position (X_c, Y_c) . This distance relationship is expressed by the Pythagorean theorem in Cartesian coordinates as

$$(X-X_c)^2 + (Y-Y_c)^2 = r^2$$

Bresenham's line algorithm for raster display is adapted to circle generation by setting up the decision parameters for finding the closest pixel for each sampling step.

A method for direct distances comparison is to test the halfway position between two pixels, to determine if this midpoint is inside or outside the circle boundary. This method is more easy. For an integer circle radius, the midpoint approach generates the same pixel position.

Midpoint Circle Algorithm

1. Input radius r and circle center (x_c, y_c) and obtain the first point on the circumference of a circle centered on the origin as

$$(x_0, y_0) = (0, r)$$

2. Calculate the initial value of the decision parameter as

$$P_0 = 5/4 - r$$

3 At each x_k position, starting at $k=0$, perform the following test:

if $P_k < 0$, then next point along the circle centered on $(0,0)$ is (x_{k+1}, y_k)

and

$$P_{k+1} = P_k + 2x_{k+1} + 1$$

otherwise the next point along the circle is (x_{k+1}, y_{k-1}) and

$$P_{k+1} = P_k + 2x_{k+1} + 1 - 2y_{k+1}$$

$$\text{where } 2x_{k+1} = 2x_k + 2, 2y_{k+1} = 2y_k - 2$$

4. Determine the symmetry points in the other seven octants

5. Move each calculated position (x, y) onto the circular path centered on (x_c, y_c) and plot the coordinate values $x = x + x_c, y = y + y_c$

6.Repeat steps 3 to 5 until $x \geq y$

LP 3: Ellipse generating Algorithm

Properties of the Ellipse

An ellipse is a set of points such that the sum of the distances from two fixed positions (foci) is the same for all points. If the distances to any two foci from any point $P=(x,y)$ on the ellipse are labeled d_1 and d_2 then the general equation of an ellipse can be stated as $d_1 + d_2$ is constant.

An ellipse in standard position is symmetric between quadrants. But it not symmetric between the two octants of the quadrant. So, we must calculate the pixel positions along the elliptical arc throughout one quadrant, then we obtained positions in the remaining three quadrants by symmetry.

Midpoint Ellipse Algorithm

1.Input r_x, r_y and ellipse center (x_c, y_c) and obtain the first point on an ellipse centered on the origin as

$$(x_0, y_0) = (0, r_y)$$

2.Calculate the initial value of the decision parameter in region 1 as

$$p_{10} = r_y^2 - r_x^2 x_0 y_0 + \frac{1}{4} r_x^2$$

3.At each x_k position in region 1, starting at $k=0$, perform the following test if $p_{1k} < 0$, the next point along the ellipse centered on $(0,0)$ is (x_{k+1}, y_k)

otherwise the next point along the circle is (x_{k+1}, y_{k-1}) and

$$p_{1k+1} = p_{1k} + 2r_x^2 y_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

with

$$2r_x^2 y_{k+1} = 2r_x^2 y_k + 2r_x^2$$

$$2r_x^2 y_k - 2r_x^2 y_{k-1} - 2r_x^2$$

4.Calculate the initial value of the decision parameter in region 2 using the last point (x_0, y_0) calculated in region as

$$p_{20} = r_y^2 (x_0 + 1/2)^2 + r_x^2 (y_0 - 1)^2 - r_x^2 r_y^2$$

5.At each y_k position in region 2 starting at $k=0$, perform the following test if $p_{2k} > 0$ the next point along the ellipse centered on $(0,0)$ is

(x_k, y_{k-1}) and

$$p_{2k+1} = p_{2k} - 2r_x^2 y_{k+1} + r_x^2$$

otherwise the next point along the circle is (x_{k+1}, y_{k-1}) and

$$p_{2k+1} = p_{2k} + 2r_x^2 y_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

using the same incremental calculations for x and y as in region 1

6.Determine symmetry points in the other three quadrants.

7.Move each calculated pixel position (x,y) onto the elliptical path centered on (x_c, y_c) and plot the coordinate values.

$$X = x + x_c, y = y + y_c$$

8.Repeat the steps for region 1 until $2r_x^2 y > 2r_x^2 x$

LP 5: Attributes

Any parameter that affects the way a primitive is to be displayed referred to as an attribute parameter

Line Attributes**1.Line Type:**

possible line type attribute include solid lines, dashed lines, dotted lines

2.Line Width

possible line width attribute include thicker line, thinner line and standard line.

3. Line Color

no of colors that can be provided to an output primitives depends upon the display device we used.

4. Pen and brush options

lines generated with pen or brush shapes can be displayed in various widths by changing the size of the mask. These shapes can be stored in a pixel mask that identifies the array of pixel positions that are to be set along the line path

Curve Attribute**Area Fill Attributes**

Options of filling a defined region include a choice between a solid color or a patterned fill and choices for the particular colors and patterns.

Basic Fill Styles:

Hollow fill, Solid fill , Patterned fill

Character Attributes:**Text Attributes**

A set of characters are affected by a particular attribute. (Font,size,style,color,alignment,height,bold,italic,)

Marker Attributes

A particular character is affected by a particular attribute. (marker type, marker precision).

Antialiasing

The distortion (or) deformation of information due to low frequency sampling is called aliasing. The aliasing effect can be reduced by adjusting intensities of pixels along the line to minimize the effect of alising is called antialiasing.

We can improve the appearance of displayed raster lines by applying antialisaing methods that compensate for the under sampling process.

Methods of antialising

1. Increasing Resolution
2. Unweighted area sampling
3. Weighted area sampling
4. Thick line segments

1. Increasing Resolution

The aliasing effect can be minimized by increasing resolution of the raster display. By increasing resolution and making it twice the original one, the line passes through twice as many column of pixels and therefore has twice as many jags, but each jag is half as large in x and in y direction.

This improvement cause increase in cost of memory, bandwidth of memory and scan-conversion time. So it is a expensive method to reduce the aliasing method.

2. Unweighted area sampling

In general the line drawing algorithm select the pixels which is closer to the true line. In antialiasing instead of picking closest pixel, both pixels are high lighted. However their intensity values may differ.

In unweighted area sampling, the intensity of pixel is proportional to the amount of line area occupied by the pixel. It produces better results than does setting pixels either to full intensity or to zero intensity.

3. Weighted area sampling

In weighted area sampling small area closer to the pixel center has greater intensity than does one at a greater distance. Thus in weighted area sampling the intensity of the pixel is dependent on the line area occupied and the distance of area from the pixel's center.

4. Thick line segment

In raster displays it is possible to draw lines with thickness greater than one pixel. To produce a thick line, we have to run two line drawing algorithms in parallel to find the pixels along the line edges, and while stepping along the line we have to turn on all the pixels which lie between the boundaries.

LP 6: TWO DIMENSIONAL GRAPHICS TRANSFORMATIONS

Geometric Transformations

Changes in size, shape are accomplished with geometric transformation. It alter the coordinate descriptions of object.

The basic transformations are Translation, Rotation, Scaling. Other transformations are Reflection and shear. Basic transformations used to reposition and resize the two dimensional objects.

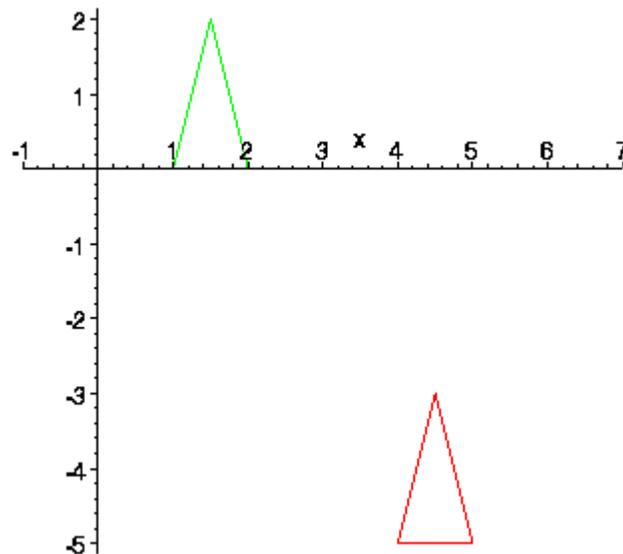
Two Dimensional Transformations

Translation

A Translation is applied to an object by repositioning it along a straight line path from one coordinate location to another. We translate a two dimensional point by adding translation distances t_x and t_y to the original position (x,y) to move the point to a new location (x',y')

$$X' = x + t_x \quad Y' = y + t_y$$

$$\text{triangle} = \{ p1=(1,0), p2=(2,0), p3=(1.5,2) \}$$



The translation distance pair (t_x, t_y) is called a translation vector or shift vector.

$$\begin{array}{ccc} P = X1 & P' = X1' & T = t_x \\ & X2' & t_y \\ & X2 & \end{array}$$

$$\begin{array}{ccc} P' = P + T & & \\ & p' = X1 + t_x & \\ & & X2 + t_y \end{array}$$

It moves objects without deformation. (ie) Every point on the object is translated by the same amount. It can be applied to lines, polygons.

Rotation

A two dimensional rotation is applied to an object by repositioning it along a circular path in the xy plane. To generate a rotation, we specify a rotation angle θ and the position (x_r, y_r) of the rotation point (or pivot point) about which the object is to be rotated.

Positive value of the rotation angle defines counter clock wise rotation.

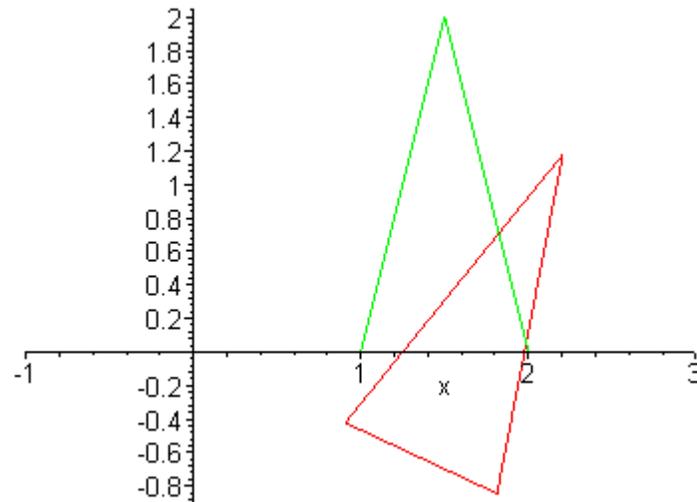
Negative value of the rotation angle defines the clock wise rotation.

$$X' = x \cos \theta - y \sin \theta$$

$$Y' = x \sin \theta + y \cos \theta$$

Using column vector $P' = P * R$

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$



Rotation of an arbitrary pivot point

Rotation of a point about any specified rotation position (x_r, y_r)

$$X' = X_r + (X - X_r) \cos \theta - (Y - Y_r) \sin \theta$$

$$Y' = Y_r + (X - X_r) \sin \theta + (Y - Y_r) \cos \theta$$

It moves objects without deformations. Every point on an object is rotated through the same angle.

Scaling

A scaling transformation alters the size of an object. This operation can be carried out for polygon by multiplying the coordinate values (x, y) of each vertex by scaling factors s_x and s_y to produce the transformed coordinates (x', y') .

$$X' = x \cdot s_x$$

$$Y' = y \cdot s_y$$

$$P = \begin{pmatrix} X_1 & X_2 \\ Y_1 & Y_2 \end{pmatrix} \quad P' = \begin{pmatrix} X_1' & X_2' \\ Y_1' & Y_2' \end{pmatrix} \quad S = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix}$$

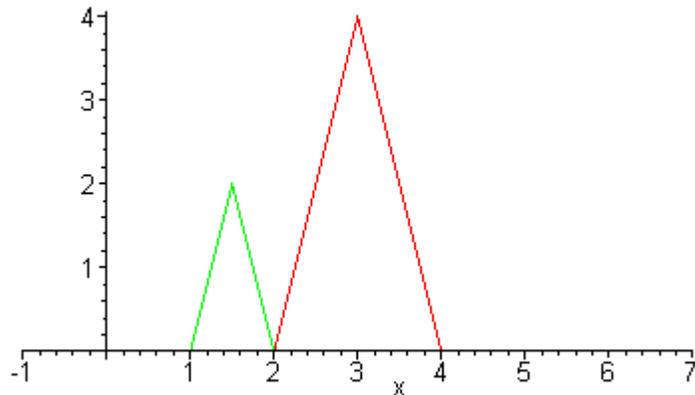
$$P' = P * S$$

If $s_x = s_y$, then it produces the uniform scaling

$s_x \neq s_y$, different scaling.

If $s_x, s_y < 0$, then it produces the reduced object size

If $s_x, s_y > 0$, then it produces the enlarged size objects.



By choosing the position called fixed point, we can control the location of the scaled object. This point is remain unchanged after the scaling transformation.

$$X' = Xf + (X - Xf)s_x \Rightarrow X' = X.s_x + (Xf(1 - s_x))$$

$$Y' = Yf + (Y - Yf)s_y \Rightarrow Y' = Y.s_y + Yf(1 - s_y)$$

Matrix representations and homogeneous coordinates

Graphics applications involves sequences of geometric transformations. The basic transformations expressed in terms of

$$P' = M1 * P + M2$$

$P, P' \rightarrow$ Column vectors.

$M1 \rightarrow 2 \times 2$ array containing multiplicative factors

$M2 \rightarrow 2$ Element column matrix containing translation terms

For translation $\rightarrow M1$ is the identity matrix

For rotation or scaling $\rightarrow M2$ contains transnational terms associated with the pivot point or scaling fixed point.

For coordinate positions are scaled, then rotated then translated, these steps are combined together into one step, final coordinate positions are obtained directly from the initial coordinate values.

To do this expand the 2×2 matrix into 3×3 matrix.

To express 2 dimensional transformation as a multiplication, we represent each cartesian coordinate position (x, y) with the homogeneous co ordinate triple (X_h, Y_h, h) where

$$X = xh/h, Y = Yh/h$$

So we can write $(h.x, h.y, h)$, set $h=1$. Each two dimensional position is represented with homogeneous coordinates $(x,y,1)$. Coordinates are represented with three element column vector. Transformation operations are written as 3 by 3 matrices.

For translation

$$\begin{array}{ccccc} X' & 1 & 0 & tx & X \\ Y' & 0 & 1 & ty & Y \\ 1 & 0 & 0 & 1 & 1 \end{array}$$

$$P' = T(tx, ty) * P$$

Inverse of the translation matrix is obtained by replacing tx, ty by $-tx, -ty$

Similarly rotation about the origin

$$\begin{array}{ccccc} X' & \cos\theta & -\sin\theta & 0 & X \\ Y' & \sin\theta & \cos\theta & 0 & Y \\ 1 & 0 & 0 & 1 & 1 \end{array}$$

$$P' = R(\theta) * P$$

We get the inverse rotation matrix when θ is replaced with $(-\theta)$

Similarly scaling about the origin

$$\begin{array}{ccccc} X' & Sx & 0 & 0 & X \\ Y' & 0 & Sy & 0 & Y \\ 1 & 0 & 0 & 1 & 1 \end{array}$$

$$P' = S(sx, sy) * P$$

Composite transformations

Sequence of transformations is called as composite transformation. It is obtained by forming products of transformation matrices is referred as a concatenation (or) composition of matrices.

Translation: -

Two successive translations

$$\begin{array}{ccc} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{array} \quad \begin{array}{ccc} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{array} \quad \begin{array}{ccc} 1 & 0 & tx_1+tx_2 \\ 0 & 1 & ty_1+ty_2 \\ 0 & 0 & 1 \end{array}$$

$$T(tx_1, ty_1) + T(tx_2, ty_2) = T(tx_1+tx_2, ty_1+ty_2)$$

Two successive translations are additive.

Rotation

Two successive rotations are additive.

$$R(\theta_1) * R(\theta_2) = R(\theta_1 + \theta_2)$$

$$P' = P \cdot R(\theta_1 + \theta_2)$$

Scaling

$$\begin{array}{ccc} Sx1 & 0 & 0 \\ 0 & Sy1 & 0 \\ 0 & 0 & 1 \end{array} \quad \begin{array}{ccc} Sx2 & 0 & 0 \\ 0 & Sy2 & 0 \\ 0 & 0 & 1 \end{array} \quad \begin{array}{ccc} Sx1 \cdot x2 & 0 & 0 \\ 0 & Sy1 \cdot y2 & 0 \\ 0 & 0 & 1 \end{array}$$

$$S(x1, y1) \cdot S(x2, y2) = S(sx1 \cdot sx2, sy1 \cdot sy2)$$

- the order we perform multiple transforms can matter
 - eg. translate + scale can differ from scale + translate
 - eg. rotate + translate can differ from translate + rotate
 - eg. rotate + scale can differ from scale + rotate (when scale_x differs from scale_y)
- When does $M1 + M2 = M2 + M1$?

M1	M2
translate	translate
scale	scale
rotate	rotate
scale (sx = sy)	rotate

General pivot point rotation

Rotation about any selected pivot point (xr, yr) by performing the following sequence of translate – rotate – translate operations.

- Translate the object so that the pivot point is at the co-ordinate origin.
- Rotate the object about the coordinate origin
- Translate the object so that the pivot point is returned to its original position

$$\begin{array}{ccc} 1 & 0 & -xr \\ 0 & 1 & -yr \\ 0 & 0 & 1 \end{array} \quad \begin{array}{ccc} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{array} \quad \begin{array}{ccc} 1 & 0 & xr \\ 0 & 1 & yr \\ 0 & 0 & 1 \end{array}$$

Concatenation properties

$$T(xr, yr) \cdot R(\theta) \cdot T(-xr, -yr) = R(xr, yr, \theta)$$

Matrix multiplication is associative. Transformation products may not be commutative.

Combination of translations, rotations, and scaling can be expressed as

$$X' = rSxx \quad rSxy \quad trSx \quad X$$

$$Y' = \begin{bmatrix} r_{Sxx} & r_{Sxy} \\ r_{Syy} & r_{Sxy} \end{bmatrix} Y$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Other transformations

Besides basic transformations other transformations are reflection and shearing

Reflection :

Reflection is a transformation that produces the mirror image of an object relative to an axis of reflection. The mirror image is generated relative to an axis of reflection by rotating the object by 180 degree about the axis.

Reflection about the line $y=0$ (ie about the x axis), the x-axis is accomplished with the transformation matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

It keeps the x values same and flips the y values of the coordinate positions.

Reflection about the y-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

It keeps the y values same and flips the x values of the coordinate positions.

Reflection relative to the coordinate origin.

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Reflection relative to the diagonal line $y=x$, the matrix is

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Reflection relative to the diagonal line $y=x$, the matrix is

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Shear

A transformation that alter the shape of an object is called the shear transformation.

Two shearing transformations

1. Shift x coordinate values (X- shear)

2. Shifts y coordinate values. (Y-shear)

In both cases only one coordinate (x or y) changes its coordinates and other preserves its values.

X –Shear

It preserves the y value and changes the x value which causes vertical lines to tilt right or left

$$\text{X-sh} = \begin{pmatrix} 1 & 0 & 0 \\ \text{shx} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X' = X + \text{shx} * y$$

$$Y' = Y$$

Y –Shear

It preserves the x value and changes the y value which causes vertical lines to tilt right or left

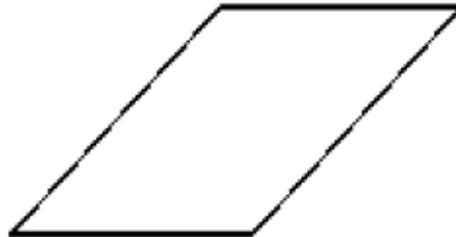
$$\text{Y-sh} = \begin{pmatrix} 1 & \text{shy} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Y' = Y + \text{shy} * X$$

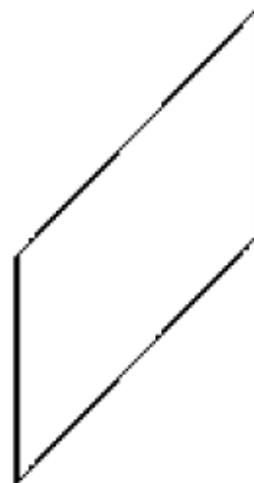
$$X' = X$$



Unit cube



Sheared in X direction



Sheared in Y direction

Shearing Relative to other reference line

We can apply x and y shear transformations relative to other reference lines. In x shear transformation we can use y reference line and in y shear we can use x reference line.

The transformation matrices for both are given below.

$$\text{X shear with y reference line} \quad \begin{pmatrix} 1 & \text{shx} & -\text{shx} * y_{\text{ref}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$x' = x + shx(y - y_{ref}), y' = y$$

$$\begin{matrix} Y \text{ shear with } x \text{ reference line} & \begin{matrix} 1 & 0 & 0 \\ shy & 1 & -shy \cdot x_{ref} \\ 0 & 0 & 1 \end{matrix} \end{matrix}$$

which generates transformed coordinate positions.

$$x' = x, y' = shy(x - x_{ref}) + y$$

This transformation shifts a coordinate position vertically by an amount proportional to its distance from the reference line $x = x_{ref}$.

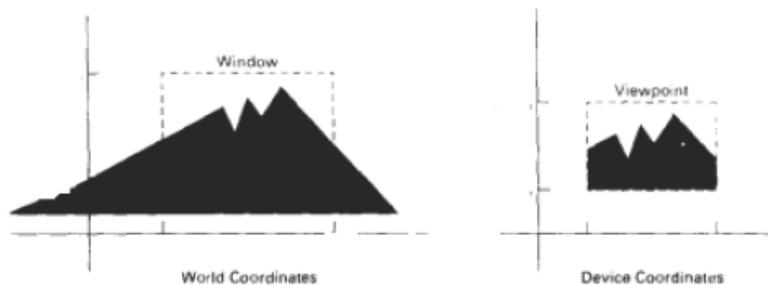
Transformations between coordinate systems

Transformations between Cartesian coordinate systems are achieved with a sequence of translate-rotate transformations. One way to specify a new coordinate reference frame is to give the position of the new coordinate origin and the direction of the new y-axis. The direction of the new x-axis is then obtained by rotating the y direction vector 90 degree clockwise. The transformation matrix can be calculated as the concatenation of the translation that moves the new origin to the old co-ordinate origin and a rotation to align the two sets of axes. The rotation matrix is obtained from unit vectors in the x and y directions for the new system

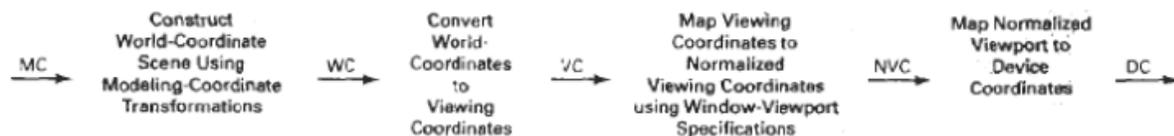
LP 7: Two dimensional viewing

Two dimensional viewing The viewing pipeline A world coordinate area selected for display is called a window. An area on a display device to which a window is mapped is called a view port. The window defines what is to be viewed the view port defines where it is to be displayed. The mapping of a part of a world coordinate scene to device coordinate is referred to as viewing transformation. The two dimensional viewing transformation is referred to as window to view port transformation of windowing transformation.

A viewing transformation using standard rectangles for the window and viewport



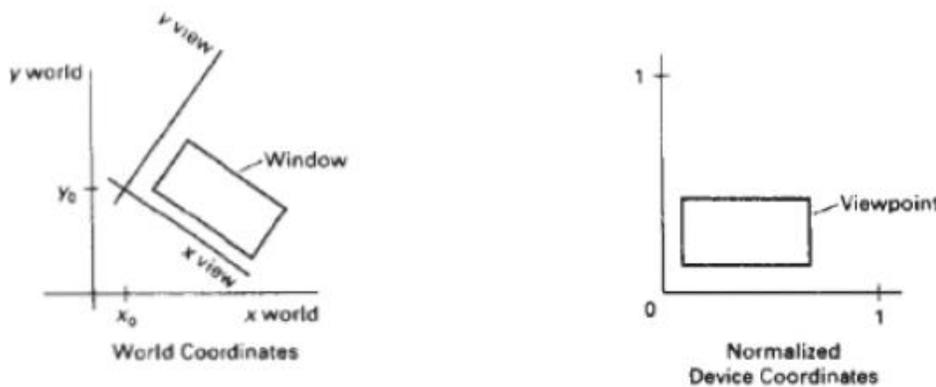
The two dimensional viewing transformation pipeline



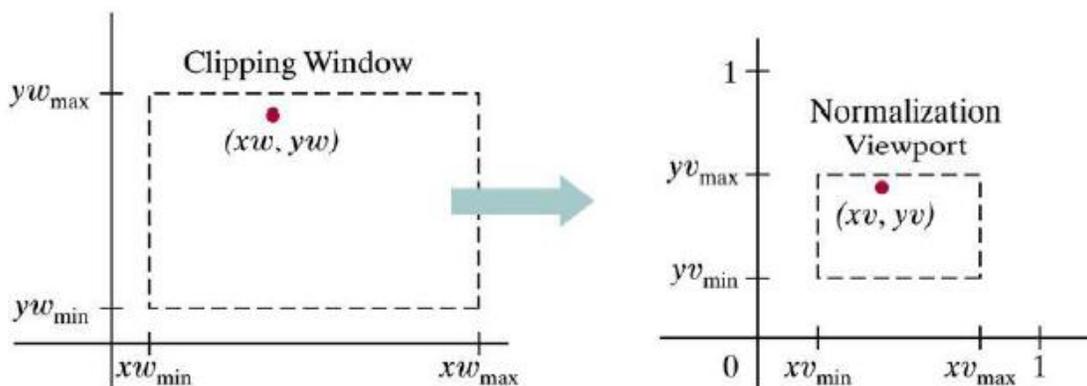
The viewing transformation in several steps, as indicated in Fig. First, we construct the scene in world coordinates using the output primitives. Next to obtain a particular orientation for the window, we can set up a two-dimensional viewing-coordinate system in the world coordinate plane, and define a window in the viewing-coordinate system. The viewing-coordinate reference frame is used to provide a method for setting up arbitrary orientations for rectangular windows.

Once the viewing reference frame is established, we can transform descriptions in world coordinates to viewing coordinates. We then define a viewport in normalized coordinates (in the range from 0 to 1) and map the viewing-coordinate description of the scene to normalized coordinates.

At the final step all parts of the picture that lie outside the viewport are clipped, and the contents of the viewport are transferred to device coordinates. By changing the position of the viewport, we can view objects at different positions on the display area of an output device.



Window to view port coordinate transformation:



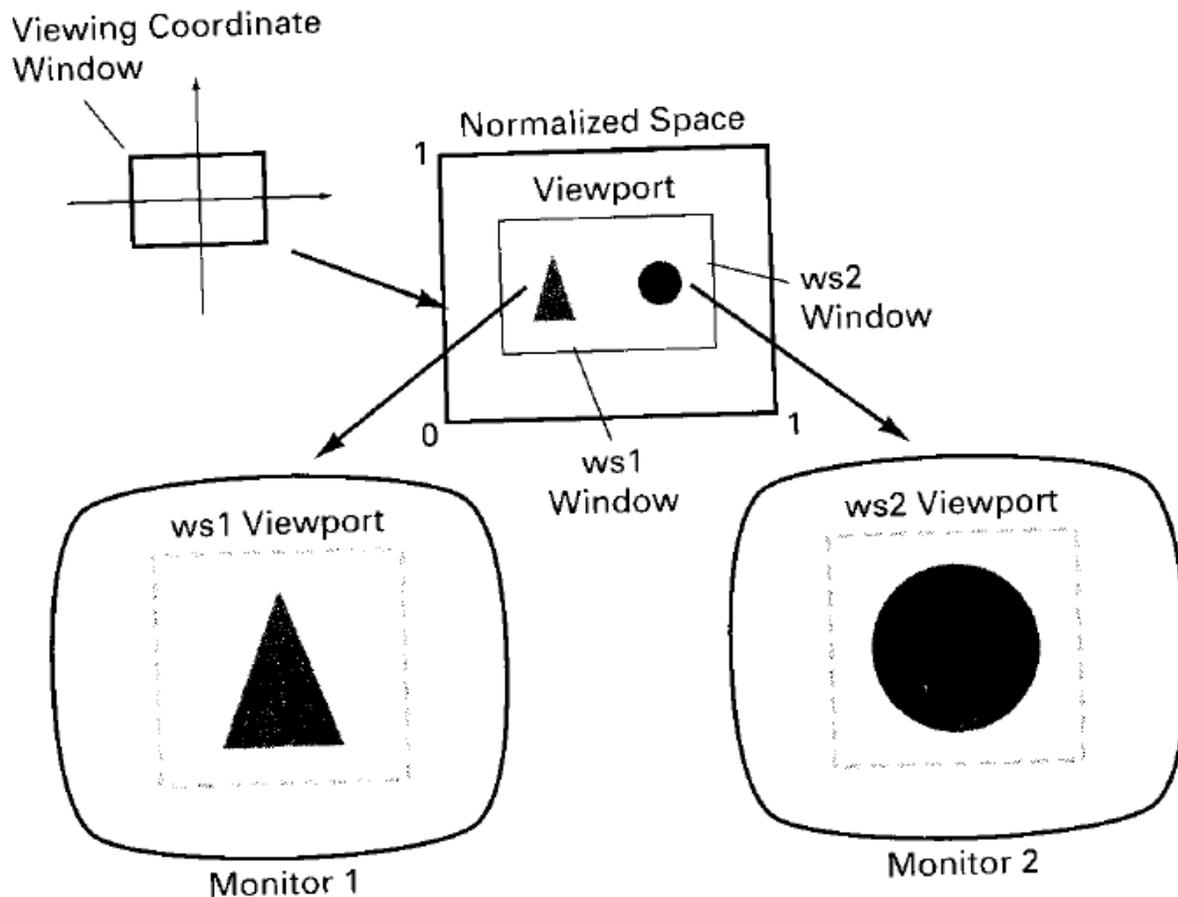
A point at position (x_w, y_w) in a designated window is mapped to viewport coordinates (x_v, y_v) so that relative positions in the two areas are the same. The figure illustrates the window

to view port mapping. A point at position (x_w, y_w) in the window is mapped into position (x_v, y_v) in the associated view port. To maintain the same relative placement in view port as in window. The conversion is performed with the following sequence of transformations.

1. Perform a scaling transformation using point position of $(x_w \text{ min}, y_w \text{ min})$ that scales the window area to the size of view port.
2. Translate the scaled window area to the position of view port. Relative proportions of objects are maintained if scaling factor are the same ($S_x = S_y$).

Otherwise world objects will be stretched or contracted in either the x or y direction when displayed on output device. For normalized coordinates, object descriptions are mapped to various display devices. Any number of output devices can be open in particular application and another window view port transformation can be performed for each open output device. This mapping called the work station transformation is accomplished by selecting a window area in normalized space and a view port are in coordinates of display device.

Mapping selected parts of a scene in normalized coordinate to different video monitors with work station transformation.



Window to Viewport transformation

The window defined in world coordinates is first transformed into the normalized device coordinates. The normalized window is then transformed into the viewport coordinate. The window to viewport coordinate transformation is known as workstation transformation. It is achieved by the following steps

1. The object together with its window is translated until the lower left corner of the window is at the origin.
2. Object and window are scaled until the window has the dimensions of the viewport
3. Translate the viewport to its correct position on the screen.

The relation of the window and viewport display is expressed as

$$\frac{XV - XV_{\min}}{XV_{\max} - XV_{\min}} = \frac{XW - XW_{\min}}{XW_{\max} - XW_{\min}}$$

$$\frac{YV - YV_{\min}}{YV_{\max} - YV_{\min}} = \frac{YW - YW_{\min}}{YW_{\max} - YW_{\min}}$$

$$XV = XV_{\min} + (XW - XW_{\min})S_x$$

$$YV = YV_{\min} + (YW - YW_{\min})S_y$$

$$S_x = \frac{XV_{\max} - XV_{\min}}{XW_{\max} - XW_{\min}}$$

$$S_y = \frac{YV_{\max} - YV_{\min}}{YW_{\max} - YW_{\min}}$$

2D Clipping

The procedure that identifies the portion of a picture that are either inside or outside of a specified region of space is referred to as clipping. The region against which an object is to be clipped is called a clip window or clipping window.

The clipping algorithm determines which points, lines or portions of lines lie within the clipping window. These points, lines or portions of lines are retained for display. All other are discarded.

Possible clipping are

1. Point clipping
2. Line clipping
3. Area clipping
4. Curve Clipping
5. Text Clipping

Point Clipping:

The points are said to be interior to the clipping if

$$XW_{\min} \leq X \leq XW_{\max}$$

$$YW_{\min} \leq Y \leq YW_{\max}$$

The equal sign indicates that points on the window boundary are included within the window.

Line Clipping:

- The lines are said to be interior to the clipping window, if the two end points of the lines are interior to the window.

- If the lines are completely right of, completely to the left of, completely above, or completely below the window, then it is discarded.

- Both end points of the line are exterior to the window, then the line is partially inside and partially outside the window. The lines which cross one or more clipping boundaries requires calculation of multiple intersection points to decide the visible portion of them. To minimize the intersection calculation and increase the efficiency of the clipping algorithm, initially completely visible and invisible lines are identified and then intersection points are calculated for remaining lines.

There are many clipping algorithms. They are

1. Sutherland and Cohen subdivision line clipping algorithm

It is developed by Dan Cohen and Ivan Sutherland. To speed up the processing this algorithm performs initial tests that reduces the number of intersections that must be calculated.

given a line segment, repeatedly:

1. check for trivial acceptance
both
2. check for trivial rejection

both endpoints of the same side of clip rectangle

3. both endpoints outside clip rectangle

Divide segment in two where one part can be trivially rejected

Clip rectangle extended into a plane divided into 9 regions . Each region is defined by a unique 4-bit string

- left bit = 1: above top edge ($Y > Y_{max}$)
- 2nd bit = 1: below bottom edge ($Y < Y_{min}$)
- 3rd bit = 1: right of right edge ($X > X_{max}$)
- right bit = 1: left of left edge ($X < X_{min}$)
- left bit = sign bit of ($Y_{max} - Y$)
- 2nd bit = sign bit of ($Y - Y_{min}$)
- 3rd bit = sign bit of ($X_{max} - X$)
- right bit = sign bit of ($X - X_{min}$)

(The sign bit being the most significant bit in the binary representation of the value. This bit is '1' if the number is negative, and '0' if the number is positive.)

The frame buffer itself, in the center, has code 0000.

1001 | 1000 | 1010

0001 | 0000 | 0010

0101 | 0100 | 0110

For each line segment:

1. each end point is given the 4-bit code of its region
2. repeat until acceptance or rejection

1. if both codes are 0000 -> trivial acceptance
2. if logical AND of codes is not 0000 -> trivial rejection
3. divide line into 2 segments using edge of clip rectangle

1. find an endpoint with code not equal to 0000
2. lines that cannot be identified as completely inside or outside are checked for the intersection with two boundaries.
3. break the line segment into 2 line segments at the crossed edge
4. forget about the new line segment lying completely outside the clip rectangle
5. draw the line segment which lies within the boundary region.

2. Mid point subdivision algorithm

If the line partially visible then it is subdivided in two equal parts. The visibility tests are then applied to each half. This subdivision process is repeated until we get completely visible and completely invisible line segments.

Mid point sub division algorithm

1. Read two end points of the line $P1(x1,x2)$, $P2(x2,y2)$
2. Read two corners (left top and right bottom) of the window, say ($Wx1,Wy1$ and $Wx2, Wy2$)

3. Assign region codes for two end points using following steps

Initialize code with bits 0000

Set Bit 1 – if ($x < Wx1$)

Set Bit 2 – if ($x > Wx1$)

Set Bit 3 – if ($y < Wy1$)

Set Bit 4 – if ($y > Wy2$)

4. Check for visibility of line

a. If region codes for both endpoints are zero then the line is completely visible. Hence draw the line and go to step 6.

b. If the region codes for endpoints are not zero and the logical ANDing of them is also nonzero then the line is completely invisible, so reject the line and go to step 6.

c. If region codes for two end points do not satisfy the condition in 4a and 4b the line is partially visible.

5. Divide the partially visible line segments in equal parts and repeat steps 3 through 5 for both subdivided line segments until you get completely visible and completely invisible line segments.

6. Stop.

This algorithm requires repeated subdivision of line segments and hence many times it is slower than using direct calculation of the intersection of the line with the clipping window edge.

3. Liang-Barsky line clipping algorithm

The Cohen Sutherland clip algorithm requires the large no of intersection calculations. here this is reduced. The update parameter requires only one division and windows intersection lines are computed only once.

The parameter equations are given as

$$X = x_1 + u \Delta x, Y = Y_1 + u \Delta y$$

$$0 \leq u \leq 1, \text{ where } \Delta x = x_2 - x_1, \Delta y = y_2 - y_1$$

Algorithm

1. Read the two end points of the line $p_1(x_1, y_1), p_2(x_2, y_2)$

2. Read the corners of the window $(x_{wmin}, y_{wmax}), (x_{wmax}, y_{wmin})$

3. Calculate the values of the parameter p_1, p_2, p_3, p_4 and q_1, q_2, q_3, q_4 such that

$$4. p_1 = \Delta x, q_1 = x_1 - x_{wmin}$$

$$p_2 = -\Delta x, q_2 = x_{wmax} - x_1$$

$$p_3 = \Delta y, q_3 = y_1 - y_{wmin}$$

$$p_4 = -\Delta y, q_4 = y_{wmax} - y_1$$

5. If $p_i = 0$ then that line is parallel to the i th boundary. if $q_i < 0$ then the line is completely outside the boundary. So discard the line segment and goto stop.

Else

{

Check whether the line is horizontal or vertical and check the line endpoint with the corresponding boundaries. If it is within the boundary area then use them to draw a line. Otherwise use boundary coordinate to draw a line. Goto stop.

}

6. initialize values for U1 and U2 as U1=0,U2=1

7. Calculate the values for $U = q_i/p_i$ for $I=1,2,3,4$

8. Select values of q_i/p_i where $p_i < 0$ and assign maximum out of them as u1

9. If ($U1 < U2$)

{

Calculate the endpoints of the clipped line as follows

$XX1 = X1 + u1 \square x$

$XX2 = X1 + u2 \square x$

$YY1 = Y1 + u1 \square y$

$YY2 = Y1 + u2 \square y$

}

10. Stop.

4. Nicholl-lee Nicholl line clipping

It Creates more regions around the clip window. It avoids multiple clipping of an individual line segment. Compare with the previous algorithms it perform few comparisons and divisions . It is applied only 2 dimensional clipping. The previous algorithms can be extended to 3 dimensional clipping.

1. For the line with two end points $p1, p2$ determine the positions of a point for 9 regions.

Only three regions need to be considered (left, within boundary, left upper corner).

2. If $p1$ appears any other regions except this, move that point into this region using some reflection method.

3. Now determine the position of $p2$ relative to $p1$. To do this depends on $p1$ creates some new region.

a. If both points are inside the region save both points.

b. If $p1$ inside , $p2$ outside setup 4 regions. Intersection of appropriate boundary is calculated depends on the position of $p2$.

c. If $p1$ is left of the window, setup 4 regions . L, Lt, Lb, Lr

1. If $p2$ is in region L, clip the line at the left boundary and save this intersection to $p2$.

2. If $p2$ is in region Lt, save the left boundary and save the top boundary.

3. If not any of the 4 regions clip the entire line.

d. If $p1$ is left above the clip window, setup 4 regions . T, Tr, Lr, Lb

1. If $p2$ inside the region save point.

2. else determine a unique clip window edge for the intersection calculation.
- e. To determine the region of p2 compare the slope of the line to the slope of the boundaries of the clip regions.

Line clipping using non rectangular clip window

Circles and other curved boundaries clipped regions are possible, but less commonly used. Clipping algorithm for those curve are slower.

1. Lines clipped against the bounding rectangle of the curved clipping region. Lines outside the region is completely discarded.
2. End points of the line with circle center distance is calculated . If the square of the 2 points less than or equal to the radius then save the line else calculate the intersection point of the line.

Polygon clipping

Splitting the concave polygon

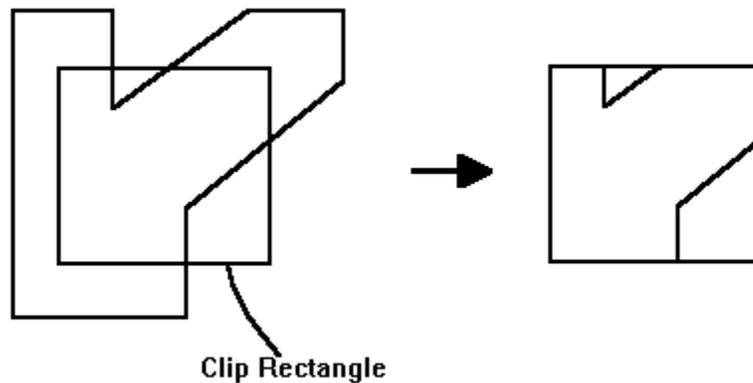
It uses the vector method , that calculate the edge vector cross products in a counter clock wise order and note the sign of the z component of the cross products. If any z component turns out to be negative, the polygon is concave and we can split it along the line of the first edge vector in the cross product pair.

Sutherland – Hodgeman polygon Clipping Algorithm

1. Read the coordinates of all vertices of the polygon.
2. Read the coordinates of the clipping window.
3. Consider the left edge of the window.
4. Compare the vertices of each edge of the polygon, Individually with the clipping plane.
5. Save the resulting intersections and vertices in the new list of vertices according to four possible relationships between the edge and the clipping boundary discussed earlier.
6. Repeats the steps 4 and 5 for remaining edges of the clipping window. Each time the resultant vertices is successively passed the next edge of the clipping window.
7. Stop.

The Sutherland –Hodgeman polygon clipping algorithm clips convex polygons correctly, But in case of concave polygons clipped polygon may be displayed with extraneous lines. It can be solved by separating concave polygon into two or more convex polygons and processing each convex polygons separately.

The following example illustrates a simple case of polygon clipping.



WEILER –Atherton Algorithm

Instead of proceeding around the polygon edges as vertices are processed, we sometime wants to follow the window boundaries. For clockwise processing of polygon vertices, we use the following rules.

- For an outside to inside pair of vertices, follow the polygon boundary.
- For an inside to outside pair of vertices, follow a window boundary in a clockwise direction.

Curve Clipping

It involves non linear equations. The boundary rectangle is used to test for overlap with a rectangular clipwindow. If the boundary rectangle for the object is completely inside the window , then save the object (or) discard the object. If it fails we can use the coordinate extends of individual quadrants and then octants for preliminary testing before calculating curve window intersection.

Text Clipping

The simplest method for processing character strings relative to a window boundary is to use the all or none string clipping strategy. If all the string is inside then accept it else omit it.

We discard only those character that are not completely inside the window. Here the boundary limits of individual characters are compared to the window.

Exterior clipping

The picture part to be saved are those that are outside the region. This is referred to as exterior clipping. An application of exterior clipping is in multiple window systems.

Objects within a window are clipped to the interior of that window. When other higher priority windows overlap these objects , the objects are also clipped to the exterior of the overlapping window.