UNIT-3

(19)

Quarter wave line & impedance matching: ($\lambda/4$).

The expression for the input impedance of a dissipation less line may be rearranged as.

$$Z_s = R_0 \left[\frac{Z_R + jR_0 \tan \frac{2\pi s}{\lambda}}{R_0 + jZ_R \tan \frac{2\pi s}{\lambda}} \right]$$

$$= R_0 \frac{\tan \frac{2\pi s}{\lambda} \left[\frac{Z_R}{\tan \frac{2\pi s}{\lambda}} + jR_0 \right]}{\tan \frac{2\pi s}{\lambda} \left[\frac{R_0}{\tan \frac{2\pi s}{\lambda}} + jZ_R \right]}$$

In Quarter wave line $S = \lambda/4$

$$Z_s = R_0 \frac{\left[\frac{Z_R}{\tan \frac{2\pi \times \lambda}{\lambda \times 4}} + jR_0 \right]}{\frac{R_0}{\tan \pi/2} + jZ_R}$$

$$= R_0 \times \frac{\cancel{j} R_0}{\cancel{j} Z_R}$$

$$\boxed{Z_s = R_0^2 / Z_R}$$

The i/p impedance of the line is equal to the square of R_0 of the line divided by the load impedance.

Application of Quarter wave Section:

(i) A quarter wave section of line may be consider as a transformer, to match a load of Z_R to a source of Z_s.

(ii) This match can be obtained, if the characteristic impedance R_0' of the matching quarter wave section

ii) on line is

$$R_o' = \left| \sqrt{Z_s Z_R} \right| .$$

Impedance inverter:

A quarter wave transformer may be considered as an impedance inverter, it can transform a low impedance into a high impedance and vice versa.

An important application of the quarter wave matching section is to couple a transmission line to a resistive load such as an antenna.

Half wave line:

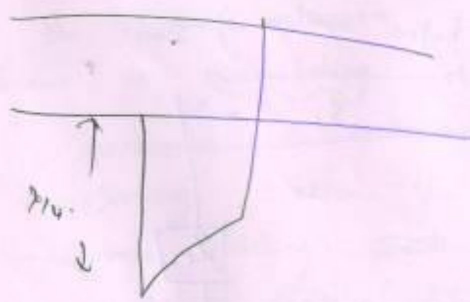
The

$$\boxed{Z_s = Z_R}$$

21

A quarter wave section of line may be considered as a transformer to match a load of Z_R to source of Z_S.

$$\boxed{R_0' = \sqrt{Z_S Z_R}}$$



Half wave line:

The input impedance of a dissipationless transmission line is.

$$Z_S = \left[\frac{Z_R + j R_0 \tan \beta s}{R_0 + j Z_R \tan \beta s} \right].$$

For half wave line $s = \lambda/2$.

$$\beta s = \frac{2\pi}{\lambda} \cdot \lambda/2 = \pi$$

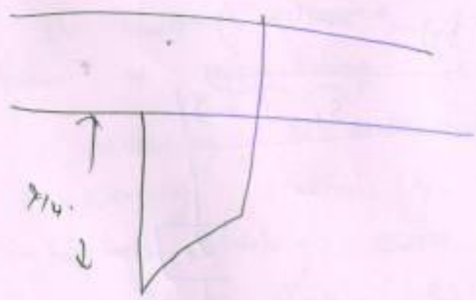
$$Z_S = R_0 \left(\frac{Z_R + j R_0 \tan \pi}{R_0 + j Z_R \tan \pi} \right)$$

$$= R_0 \left(Z_R / R_0 \right)$$

$$\boxed{Z_S = Z_R}$$

A quarter wave section of line may be considered as a transformer to match a load of Z_R to source of Z_S

$$\boxed{R_0' = \sqrt{Z_S Z_R}}$$



Half wave line:

The input impedance of a dissipationless transmission line is

$$Z_S = \left[\frac{Z_R + j R_0 \tan \beta s}{R_0 + j Z_R \tan \beta s} \right].$$

For half wave line $s = \lambda/2$.

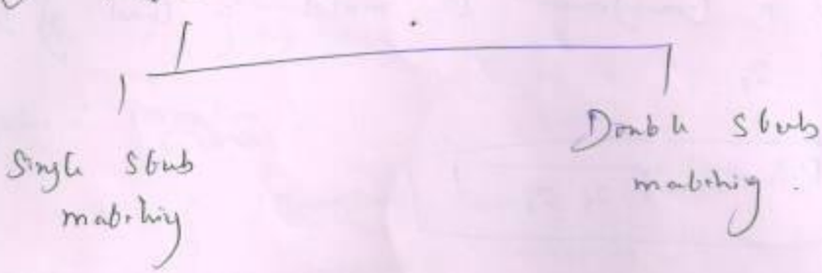
$$\beta s = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$Z_S = R_0 \left(\frac{Z_R + j R_0 \tan \pi}{R_0 + j Z_R \tan \pi} \right)$$

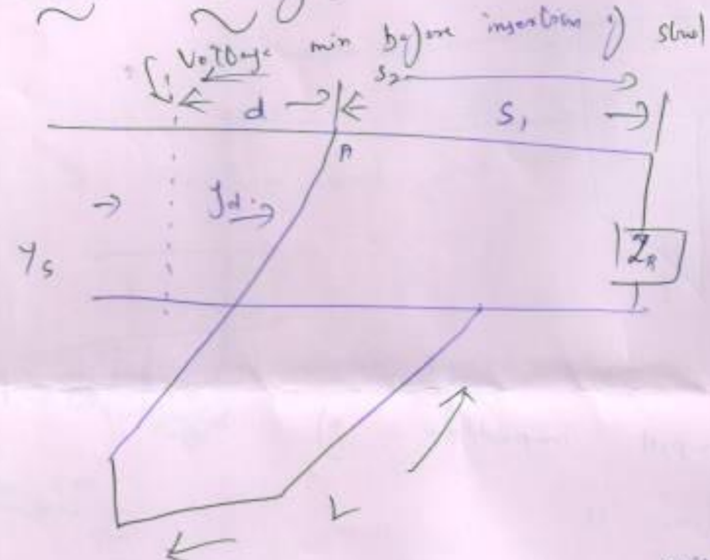
$$= R_0 \left(\frac{Z_R}{R_0} \right)$$

$$Z_S = Z_R$$

Stub matching:



Single Stub matching:



The input impedance at any point of a transmission line is given by

$$Z_s = Z_0 \cdot \frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}$$

$$Y_s = Y_0 \frac{Y_R + Y_0 \tanh \gamma l}{Y_0 + Y_R \tanh \gamma l}$$

For propagation $\gamma = j\beta$ ($\alpha = 0$).

$$Y_s = Y_0 \frac{Y_R + j Y_0 \tan \beta l}{Y_0 + j Y_R \tan \beta l}$$

Stub Matching

(21)

In general, the source (or) i/p impedance is a fixed one. By selecting the value of load impedance to be equal to the input impedance, impedance matching is achieved.

→ In some cases (Load is an antenna), the load impedance is also fixed.

→ If the load impedance is not equal to the input impedance, the maximum power transfer will not take place. This is known as mismatching.

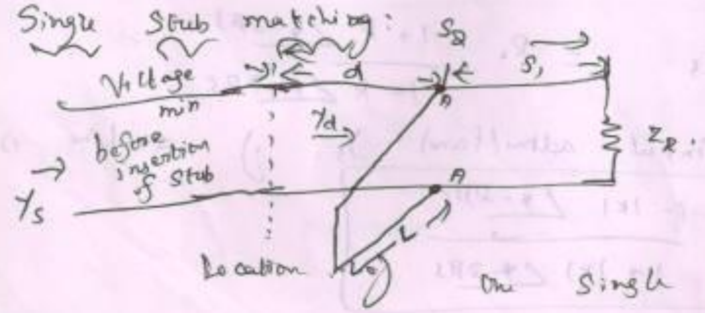
→ So, it is necessary to introduce some form of an impedance transforming section b/w the source & load to achieve impedance matching. This section is called an impedance matching device. (Eg: quarter wave transformer).

→ Another means of achieve impedance matching is the use of an open (or) short circuited line of suitable length called stub. This is called stub matching.

There are two types of stub matching.

1. Single stub matching
2. Double " "

Single Stub matching:



The single stub for impedance matching

The input impedance of the transmission line at point A,

$$\boxed{Z_S = R_0}$$

(18)

→ The line from the Source to A is then terminated in R_0 and is a Smooth line.

→ From A to the load, the reflection and Standing waves are occur.

→ To avoid reflection,

→ to select location & length of the stub.

[Voltage minimum] E.

The ~~input impedance~~, Voltage of a transmission line is

$$E_s = \frac{E_R (Z_R + R_0)}{2 Z_R} \left[e^{j\beta s} + k e^{-j\beta s} \right]$$

$$I_s = \frac{I_R (Z_R + R_0)}{2 R_0} \left(e^{j\beta s} - k e^{-j\beta s} \right)$$

$$Z_s = \frac{E_s}{I_s} = \frac{E_R \cancel{(Z_R + R_0)} / \cancel{2} Z_R}{I_R \cancel{(Z_R + R_0)} / \cancel{2} R_0} \left(\frac{e^{j\beta s} + k e^{-j\beta s}}{e^{j\beta s} - k e^{-j\beta s}} \right)$$

$$= \frac{E_R}{I_R \cdot Z_R} \times R_0$$

$$= R_0 \frac{\angle \beta s + k \angle \phi - \beta s}{\angle \beta s - k \angle \phi - \beta s}$$

$$= R_0 \frac{\cancel{\angle \beta s} \left[1 + k \angle \phi - 2\beta s \right]}{\cancel{\angle \beta s} \left[1 - k \angle \phi - 2\beta s \right]}$$

$$Z_s = R_0 \frac{1 + k \angle \phi - 2\beta s}{1 - k \angle \phi - 2\beta s}$$

The input admittance Y_s of a line is

$$\boxed{Y_s = \frac{1}{R_0} \frac{1 - |k| \angle \phi - 2\beta s}{1 + |k| \angle \phi - 2\beta s}}$$

$$G_0 = 1/R_0$$

$$Y_s = G_0\left[\frac{1-|K|\cos(\phi-2\beta s) - j|K|\sin(\phi-2\beta s)}{1+|K|\cos(\phi-2\beta s) + j|K|\sin(\phi-2\beta s)}\right]$$

$$= G_0\left[\frac{1-|K|\cos(\phi-2\beta s) - j|K|\sin(\phi-2\beta s)}{}\right] \cancel{1+|K|\cos(\phi-2\beta s)}$$

$$\cancel{= G_0\left[1-|K|^2\right.}$$ – multiply by num & denom by.

$$Y_s = G_0\left[1 + |K|\cancel{\cos(\phi-2\beta s)} - j|K|\sin(\phi-2\beta s) - |K|\cancel{\cos(\phi-2\beta s)} - |K|^2\cos^2(\phi-2\beta s) - j|K|\right.$$

$$Y_s = G_0\left[\frac{1-|K|^2 - 2j|K|\sin(\phi-2\beta s)}{1+|K|^2 + 2|K|\cos(\phi-2\beta s)}\right].$$

$$Y_s = \underset{\text{Conductance}}{G_s} + j\underset{\text{Susceptance}}{B_s}$$

$$G_s + jB_s = G_0\left[\quad\text{''}\quad\right]$$

$$\frac{G_s}{G_0} = \frac{1-|K|^2}{1+|K|^2 + 2|K|\cos(\phi-2\beta s)}.$$

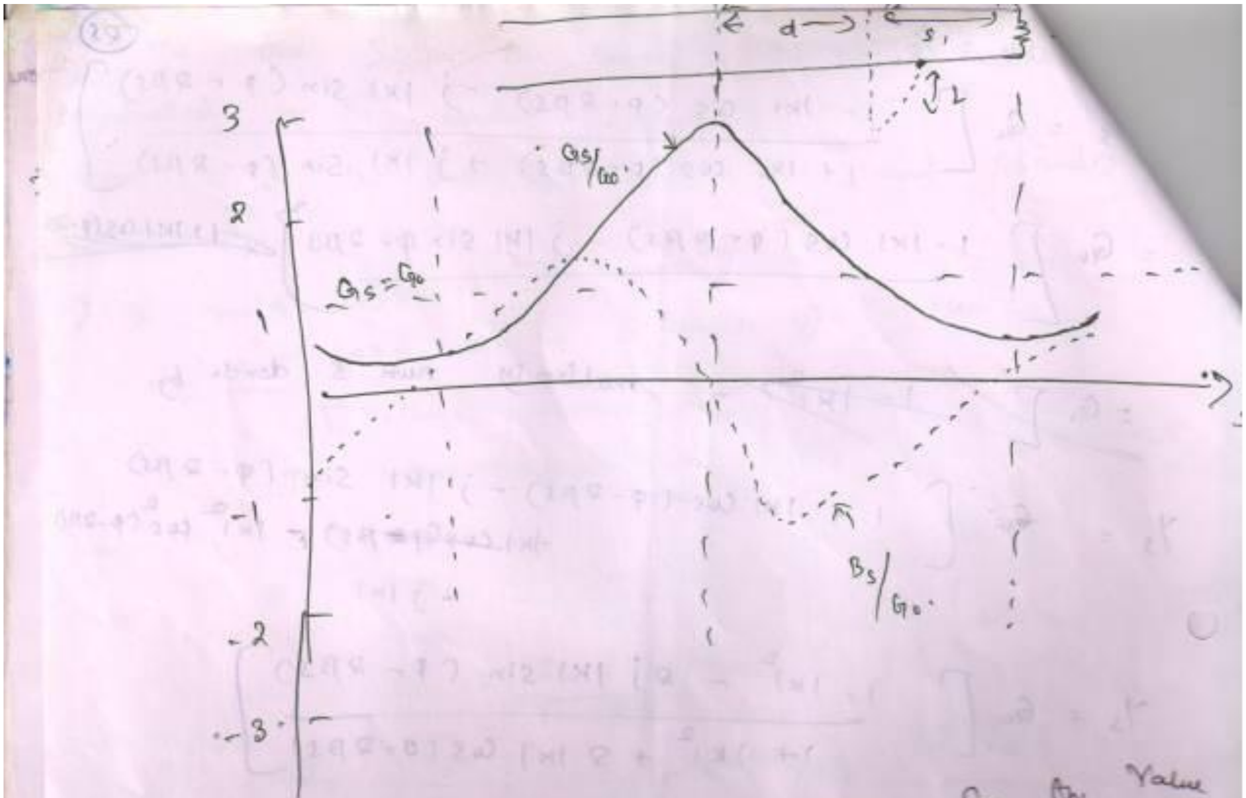
$$\frac{B_s}{G_0} = \frac{-2|K|\sin(\phi-2\beta s)}{1+|K|^2 + 2|K|\cos(\phi-2\beta s)}$$

G_s/G_0 maximum occurs for the value s at which the cosine term is -1.

$$\phi - 2\beta s_2 = -\pi$$

$$2\beta s_2 = \phi + \pi$$

$$\boxed{s_2 = \frac{\phi+\pi}{2\beta}}$$



$|K| = 0.5$

The maximum occurs for the value S_2 at which the cosine term is -1.

$$\phi - 2\beta S_2 = -\pi$$

$$\boxed{S_2 = \frac{\phi + \pi}{2\beta}}$$

At this distance from the load,

$$\frac{G_s}{G_0} = \frac{1 - |K|^2}{1 + |K|^2 - 2|K|}$$

$$= \frac{1+|K|}{1-|K|} = S.$$

$$\frac{1/R_s}{1/R_0} = \frac{R_0}{R_s} = S \qquad \boxed{R_s = R_0/S},$$

In this point of maximum G_s/G_0, is recognized as a point of minimum voltage at a distance S_2 from the load

II – 21

In other words, it is used for impedance matching.

The design Parameters of stubs are 1) Stub length 2) Stub distance from the Load.

DESIGN OF SINGLE STUB MATCHING

1) Given Z_L and Z_0 of transmission line, normalize Z_L or find $Z_n = Z_L/Z_0 = r + jx$

2) Mark the Point on the smith chart where r-circle and x-arc interest

3) Draw a circle with the radius equal to the distance from the centre of the chart to the point.

4) Draw a straight line from this Point through the centre of the chart to the other half of the chart that is travel around it through a distance of $\lambda/4$ to find the Load admittance.

5) Mark the point where the straight line intersects the circle drawn.

6) Note the distance travelled round the circumference of the chart. This is the distance of the stub from the Load.

7) Move clockwise round the Perimeter of the

II - 22

chart and find the point at which the susceptance tunes out $\pm jb$ susceptance of the line. For example if the line admittance is $1 + jb$, the required susceptance is $-jb$.

DOUBLE STUBS

In single stub matching, the stub is placed on the line at a specified point. Its location varies with Z_L and frequency. This creates some difficulties as the specified point may occur at an undesirable location. In such cases, double stubs are used. In such cases, double stubs are used such that the distance between them is fixed such that

$\lambda/16$, $\lambda/8$, $3\lambda/16$, $3\lambda/8$

DESIGN OF DOUBLE STUB MATCHING

1) Fix the distance between the two stubs and keep stub 1 at the location of the load.

2) Draw the circle corresponding to the normalised conductance, $g = 1$

3) Obtain the normalised distance of d/λ

4) Rotate the circle in anticlockwise direction by d/λ wavelength towards the load

II - (23)

. y_i is located here

5) Locate $y_L = g_L + jb_L$

6) Draw $g = g_L$ circle. This intersects the rotated $g=1$ circle at one or two points where $y_L = g_L + jb_L$.

7) Locate the corresponding y_2 points on the $g=1$ circle $y_2 = 1 + jb_2$

8) Find the stub length l_1 between the points representing y_l and y_L

9) Find the stub length l_2 & from the angle between the point representing $-jb_2$ & P_{sc}

46

Input Impedance of a dissipationless line at any point distance S away from Load is $z_S = z_0 \left[\frac{z_R + j z_0 \tan\beta s}{z_0 + j z_R \tan\beta s} \right]$

$$Y_S = \frac{Y_R + j\tan\beta s}{1 + jY_R \tan\beta s}$$

$$Y_S = \frac{Y_R(1+\tan^2\beta s)}{1+Y_R^2\tan^2\beta s} + j\frac{(1-Y_R^2)\tan\beta s}{1+Y_R^2\tan^2\beta s}$$

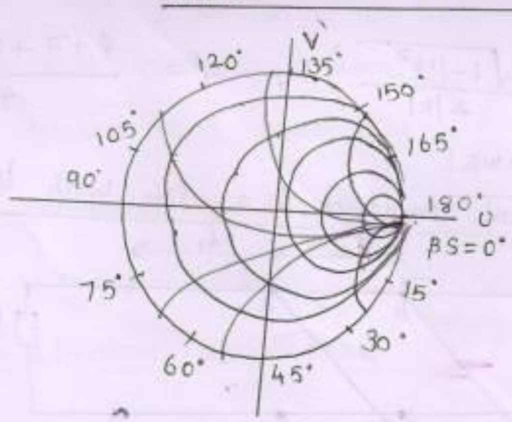
Two Stubs are Seperated by Fixed distances $\frac{\lambda}{16}$, $\frac{\lambda}{8}$, $\frac{3\lambda}{16}$, $\frac{3\lambda}{8}$ etc.....

CIRCLE DIAGRAM, SMITH CHART AND ITS APPLICATIONS:

SMITH CHART - THE SMITH CIRCLE DIAGRAM:

The Smith chart is a Valuable Graphical tool for Solving radio frequency transmission line problems.

SMITH CHART CIRCLE DIAGRAMS:



APPLICATIONS OF THE SMITH CHART:

1. Plotting an Impedance
2. Measurement of VSWR
3. Measurement of Reflection Coefficient K [magnitude and phase]
4. Measurement of Input impedance of the line.
5. Measurement of Impedance to Admittance Conversion.

1. Plotting an Impedance:

Normalized impedance $z_R = \frac{Z_R}{R_0} = \frac{60+j40}{50} = 1.2+j0.8$

The intersection of the two circles is represented by the dotted lines and the point P indicates the normalized impedance on the chart.

2. Measurement of VSWR:

Point of intersection of S-circle with the real axis at the right side of the centre indicates a VSWR for given line. - Select a centre of the circle as point O(1,0), Take a distance from O to the point P indicating normalized impedance and then draw a circle. The circle cuts the horizontal real axis. This indicates the value of VSWR.

3. Measurement of Reflection coefficient K [magnitude and phase]:

The angle of the reflection coefficient K can be obtained by extending a line from center to the outer rim of the chart through the point which indicates the normalized impedance. The point at which the extended line cuts the outer rim gives directly the value of angle of the reflection coefficient K.

4. Measurement of input impedance of the line:

Calculate Length interms of λ or degrees.

$$\lambda = \frac{v}{f} = \frac{0.6c}{f} = \frac{0.6 \times 3\times10^8}{2\times10^6} = 90\,m$$

$$S = 30\,m = \frac{30}{90}\lambda = \frac{\lambda}{3}\,m \text{ or } \frac{720}{3} = 240^\circ$$

Normalized input Impedance represented by point T is.

$$Z_{in} = 0.48 + j0.035$$

The actual value of the input impedance is

$$Z_{in} = R_0 (z_{in}) = 50\,(0.48 + j\,0.035) = 24 + j1.75\,\Omega.$$

5. Impedance to Admittance Conversion:

The terminating impedance is.

$$Z_R = 60 + j40$$

Terminating admittance is

$$Y_R = \frac{1}{Z_R} = \frac{1}{60+j40} = \frac{1}{72.111\angle 33.69} = 0.01386\angle -$$

$$= 0.0115 - j7.688\times10^{-3}\,\mho$$

$$y_R = 0.58 - j\,0.4.$$

The actual value of the admittance is

$$Y_R = G_0 [y_R] = \frac{1}{R_0}[y_R] = \frac{0.58 - j0.4}{50} = 0.0$$

(49)

Problem Solving using Smith chart:

① An R.F. Transmission line with a characteristic impedance of $300\angle 0^\circ\,\Omega$ is terminated in an impedance of $100\angle -45^\circ\,\Omega$. The Load is to be matched to the transmission line by using a short circuited stub. With the help of Smith chart. Determine the length of the stub and the distance from the load.

Solution:

$$Z_R = 100\angle -45^\circ\,\Omega = (70.71 - j70.71)\,\Omega$$

$$z_0 = R_0 = 300\angle 0^\circ = 300\,\Omega$$

1. The normalized impedance is

$$z_R = \frac{Z_R}{Z_0} = \frac{Z_R}{Z_0} = \frac{70.71 - j70.71}{300} = 0.2357 - j0.2357$$

2. Locate point A, the intersection of $r = 0.2357$ circle and $x = -0.2357$ circle. As the imaginary component (ie) reactive component of the impedance is negative, the point A is located below the horizontal axis.

3. Draw a circle with O as origin and OA as radius. This is constant S-circle. It cuts real axis. This is the value of S before stub connection.

4. Draw line from point A to O and extend to reach other end on constant S circle. In the chart A, this point is represented as B at which the normalized admittance is $y_R = 2.1 + j2.1$. Extend line AOB to the outer rim upto point B.

5. Travel along the constant S-circle in the clockwise direction from load to generator to reach a point