Sri Vidya College of Engineering and Technology
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Department of Mechanical Engineering
Question Paper for Internal Assessment Test I

Semester & Branch: **III/ Mechanical** **Date: 27 /08/14**
Subject Name: **Strength of Materials** **Max. Marks: 50**
Subject Code: **CE6306** **Max. Time: 90 mins**

Part A

Answer all the questions **(5 x 2 = 10)**

1. Define Strain. *(Out comes – a; Learning skill - remembering)*
2. Define Poisson's ratio. *(Out comes – a; Learning skill - remembering)*
3. Define Bulk Modulus. *(Out comes – a; Learning skill- remembering, understanding)*
4. Define bending moment at a section of the beam.
   (Out comes – a; Learning skill- remembering)
5. What is meant by shear force? *(Out comes – a; Learning skill- remembering)*

PART – B **(1 x 8 + 2 x 16 = 40)**

6. **(a)** Mohr's circle when a body is subjected to two mutually Perpendicular Principal Tensile Stresses of unequal Intensities.
   (Out comes – a,b; Learning skill- understanding, applying)

Or

6. **(b)** Mohr's circle when a body is subjected to two mutually Perpendicular Principal Tensile Stresses Accompanied by a simple Shear Stress.
   (Out comes – a; Learning skill- remembering)

7. **(a)** A steel rod of 20 mm diameter passes centrally through a copper tube of 50 mm external diameter and 40 mm internal diameter. The tube is closed at each by rigid plates of negligible thickness. The nuts are tightened lightly home on the projecting parts of the rod. If the temperature of the assembly is raised by 50°C, calculate the stresses developed in copper and steel. Take E for steel and copper as 200 GN/m^2 and 100 GN/m^2 and α for steel and copper as 12 x 10^{-6} per °C and 18 x 10^{-6} per °C.
   (Out comes – a,b; Learning skill- understanding, applying)

Or

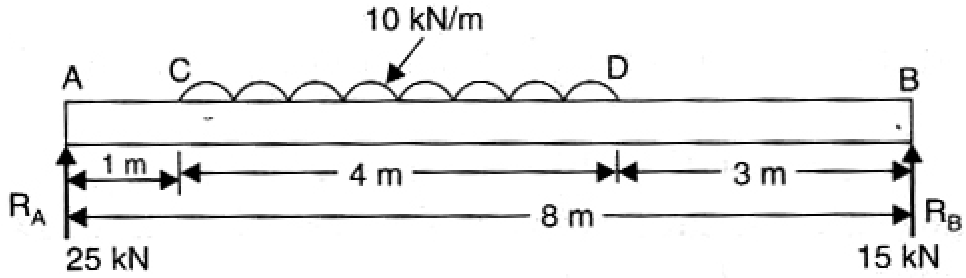
7. **(b)** A steel rod 5m long and 30 mm in diameter is subjected to an axial tensile load of 50kN. Determine the change in length, diameter and volume of the rod. Take E = 2 x 10^5 N/mm^2 and Poisson's ratio = 0.25.
   (Out comes – a,b; Learning skill- understanding, applying)

8. **(a)** Draw the shear force and B.M. diagrams for a simply supported beam of length 9 m and carrying a uniformly distributed load of 10kN/m for a distance of 6 m from the left end. Also calculate the maximum B.M. on the location.

(Out comes – a,b; Learning skill- understanding ,applying)

Or

8 **(b)** Draw the shear force and B.M. diagrams for a simply supported beam of length 8 m and carrying a uniformly distributed load of 10kN/m for a distance of 4 m as shown in fig.



(Out comes – a,b; Learning skill- understanding ,applying)

Answers

1. Strain

Strain is defined as "deformation of a solid due to stress" and can be expressed as

$\varepsilon = dl / l_o$

$= \sigma / E$ (3)

where

dl = change of length (m, in)

l_o = initial length (m, in)

ε = unit less measure of engineering strain

2. **Poisson's ratio** is the ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force. Tensile deformation is considered positive and compressive deformation is considered negative. The definition of Poisson's ratio contains a minus sign so that normal materials have a positive ratio. Poisson's ratio, also called Poisson ratio or the Poisson coefficient, or coefficient de Poisson, is usually represented as a lower case Greek nu, n.

3. **Bulk Modulus** Elasticity can be expressed as

$E = - dp / (dV / V)$ (1)

where

E = bulk modulus elasticity

dp = differential change in pressure on the object

dV = differential change in volume of the object

V = initial volume of the object

4. **Bending moments** are produced by transverse loads applied to beams. The simplest case is the cantilever **beam** , widely encountered in balconies, aircraft wings, diving boards etc.

tance from that section. It thus has units of N m. It is balanced by the **internal moment** arising from the

5. **Shearing forces** are unaligned forces pushing one part of a body in one direction, and another part of the body in the opposite direction. When the forces are aligned into each other, they are called

compression forces. An example is a deck of cards being pushed one way on the top, and the other at the bottom, causing the cards to slide.

6. a) MOHR'S CIRCLE

Mohr's Circle when a Body is Subjected to two Mutually Perpendicular Principal Tensile Stresses of Unequal Intensities. Consider a rectangular body subjected to two mutually perpendicular principal tensile stresses of unequal intensities. It is required to find the resultant stress on an oblique plane.

Let σ_1 = Major tensile stress

σ_2 = Minor tensile stress, and

θ = Angle made by the oblique plane with the axis of minor tensile stress.

Mohr's circle is drawn as : (See Fig. 3.22).

Take any point A and draw a horizontal line through A. Take $AB = \sigma_1$ and $AC = \sigma_2$ towards right from A to some suitable scale. With BC as diameter describe a circle. Let O is the centre of the circle. Now through O, draw a line OE marking an angle 2θ with OB.

From E, draw ED perpendicular on AB. Join AE. Then the normal and tangential stresses on the oblique plane are given by AD and ED respectively. The resultant stress on the oblique plane is given by AE.

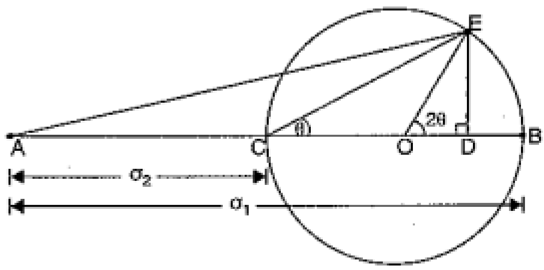


Fig. 3.22

From Fig. 3.22, we have

Length AD = Normal stress on oblique plane

Length ED = Tangential stress on oblique plane

Length AE = Resultant stress on oblique plane.

Radius of Mohr's circle $= \frac{\sigma_1 - \sigma_2}{2}$

Angle ϕ = obliquity.

Proof. (See Fig. 3.22)

$$CO = OB = OE = \text{Radius of Mohr's circle} = \frac{\sigma_1 - \sigma_2}{2}$$

$= \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$

$= \sigma_t$ or Tangential stress.

6b) Mohr's circle

Mohr's Circle when a Body is Subjected to two Mutually Perpendicular Principal Tensile Stresses Accompanied by a Simple Shear Stress. Consider a rectangular body subjected to two mutually perpendicular principal tensile stresses of unequal intensities accompanied by a simple shear stress. It is required to find the resultant stress on an oblique plane as shown in Fig. 3.26.

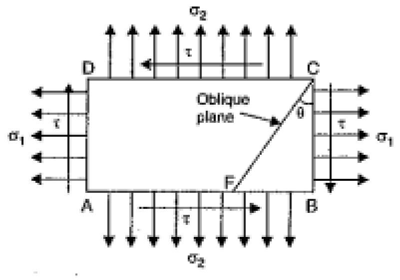


Fig. 3.26

Let σ_1 = Major tensile stress

σ_2 = Minor tensile stress

τ = Shear stress across face *BC* and *AD*

θ = Angle made by the oblique plane with the plane of major tensile stress.

According to the principal of shear stress, the faces *AB* and *CD* will also be subjected to a shear stress of τ.

Mohr's circle is drawn as given in Fig. 3.27.

Take any point *A* and draw a horizontal line through *A*.

Take $AB = \sigma_1$ and $AC = \sigma_2$ towards right of *A* to some suitable scale. Draw perpendiculars at *B* and *C* and cut off *BF* and *CG* equal to shear stress τ to the same scale. Bisect *BC* at *O*. Now with *O* as centre and radius equal to *OG* or *OF* draw a circle. Through *O*, draw a line *OE* making an angle of 2θ with *OF* as shown in Fig. 3.27. From *E*, draw *ED* perpendicular to *CB*. Join *AE*. Then length *AE* represents the resultant stress on the given oblique plane. And lengths *AD* and *ED* represents the normal stress and tangential stress respectively.

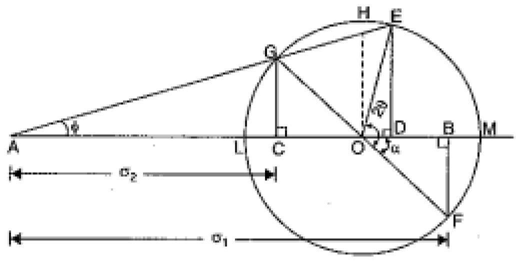


Fig. 3.27

Hence from Fig. 3.27, we have

Length *AE* = Resultant stress on the oblique plane

Length *AD* = Normal stress on the oblique plane

Length *ED* = Shear stress on the oblique plane.

Proof. (See Fig. 3.27).

$$CO = \frac{1}{2}\, CB = \frac{1}{2}\,[\sigma_1 - \sigma_2] \qquad (\because \; CB = \sigma_1 - \sigma_2)$$

$= \frac{\sigma_1 + \sigma_2}{2} + OE \cos(2\theta - \alpha)$ $[\because \ OD = OE \cos(2\theta - \alpha)]$

$= \frac{\sigma_1 + \sigma_2}{2} + OE [\cos 2\theta \cos \alpha + \sin 2\theta \sin \alpha]$

$= \frac{\sigma_1 + \sigma_2}{2} + OE \cos 2\theta \cos \alpha + OE \sin 2\theta \sin \alpha$

$= \frac{\sigma_1 + \sigma_2}{2} + OE \cos \alpha \, . \cos 2\theta + OE \sin \alpha \, . \sin 2\theta$

$= \frac{\sigma_1 + \sigma_2}{2} + OF \cos \alpha \, . \cos 2\theta + OF \sin \alpha \, . \sin 2\theta$

$(\because \ OE = OF = \text{Radius})$

$= \frac{\sigma_1 + \sigma_2}{2} + OB \cos 2\theta + BF \sin 2\theta$

$(\because \ OF \cos \alpha = OB, OF \sin \alpha = BF)$

$= \frac{\sigma_1 + \sigma_2}{2} + CO \cos 2\theta + \tau \sin 2\theta$ $(\because \ OB = CO, BF = \tau)$

$= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta$ $\left(\because \ CO = \frac{\sigma_1 - \sigma_2}{2}\right)$

$= \sigma_n$ or Normal stress

$ED = OE \sin(2\theta - \alpha) = OE (\sin 2\theta \cos \alpha - \cos 2\theta \sin \alpha)$

$= OE \sin 2\theta \cos \alpha - OE \cos 2\theta \sin \alpha$

$= OE \cos \alpha \, . \sin 2\theta - OE \sin \alpha \, . \cos 2\theta$

$= OE \cos \alpha \, . \sin 2\theta - OE \sin \alpha \, . \cos 2\theta$ $(\because \ OE = OF = \text{Radius})$

$= OB \, . \sin 2\theta - BF \cos 2\theta$ $(\because \ OF \cos \alpha = OB, OF \sin \alpha = BF)$

$= CO \, . \sin 2\theta - \tau \cos 2\theta$ $(\because \ OB = CO, BF = \tau)$

$= \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - \tau \cos 2\theta$ $\left(\because \ CO = \frac{\sigma_1 - \sigma_2}{2}\right)$

$= \sigma_t$ or Tangential stress.

7. a)

Sol. Given :

Dia. of steel rod $= 20$ mm

$\therefore$ Area of steel rod, $A_s = \frac{\pi}{4} \times 20^2 = 100\pi \text{ mm}^2$

Area of copper tube, $A_c = \frac{\pi}{4} (50^2 - 40^2) \text{ mm}^2 = 225\pi \text{ mm}^2$

Rise of temperature, $T = 50°C$

E for steel, $E_s = 200 \text{ GN/m}^2$

$= 200 \times 10^9 \text{ N/m}^2$ $\quad (\because \; G = 10^9)$

$= 200 \times 10^3 \times 10^6 \text{ N/m}^2$

$= 200 \times 10^3 \text{ N/mm}^2$ $\quad (\because \; 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2)$

E for copper, $E_c = 100 \text{ GN/m}^2 = 100 \times 10^9 \text{ N/m}^2$

$= 100 \times 10^3 \times 10^6 \text{ N/m}^2 = 100 \times 10^3 \text{ N/mm}^2$

α for steel, $\alpha_s = 12 \times 10^{-6}$ per °C

α for copper, $\alpha_c = 18 \times 10^{-6}$ per °C.

As α for copper is more than that of steel, hence the free expansion of copper will be more than that of steel when there is a rise in temperature. But the ends of the rod and the tube is fixed to the rigid plates and the nuts are tightened on the projected parts of the rod. Hence the two members are not free to expand. Hence the tube and the rod will expand by the same amount. The free expansion of the copper tube will be more than the common expansion, whereas the free expansion of the steel rod will be less than the common expansion. Hence the copper tube will be subjected to compressive stress and the steel rod will be subjected to tensile stress.

Let σ_s = Tensile stress in steel

σ_c = Compressive stress in copper.

For the equilibrium of the system,

Tensile load on steel = Compressive load on copper

or $$\sigma_s \, . \, A_s = \sigma_c \, . \, A_c$$

or $$\sigma_s = \frac{A_c}{A_s} \times \sigma_c$$

$$= \frac{225\pi}{100\pi} \times \sigma_c = 2.25\sigma_c \quad ...(i)$$

We know that the copper tube and steel rod will actually expand by the same amount.

$\therefore$ Actual expansion of steel = Actual expansion of copper ...(ii)

But actual expansion of steel

= Free expansion of steel + Expansion due to tensile stress in steel

$$= \alpha_s \, . \, T \, . \, L + \frac{\sigma_s}{E_s} \, . \, L$$

actual expansion of copper

= Free expansion of copper − Contraction due to compressive stress in copper

$$= \alpha_c \,.\, T \,.\, L - \frac{\sigma_c}{E_c} \,.\, L$$

Substituting these values in equation (*ii*), we get

$$\alpha_s \,.\, T \,.\, L + \frac{\sigma_s}{E_s} \,.\, L = \alpha_c \,.\, T \,.\, L - \frac{\sigma_c}{E_c} \,.\, L$$

$$\alpha_s \,.\, T + \frac{\sigma_s}{E_s} = \alpha_c \,.\, T - \frac{\sigma_c}{E_c}$$

$$12 \times 10^{-6} \times 50 + \frac{2.25\,\sigma_c}{200 \times 10^3} = 18 \times 10^{-6} \times 50 - \frac{\sigma_c}{100 \times 10^3} \qquad (\because \quad \sigma_s = 2.25\,\sigma_c)$$

$$\frac{2.25\,\sigma_c}{200 \times 10^3} + \frac{\sigma_c}{100 \times 10^3} = 18 \times 10^{-6} \times 50 - 12 \times 10^{-6} \times 50$$

$$1.125 \times 10^{-5}\,\sigma_c + 10^{-5}\,\sigma_c = 6 \times 10^{-6} \times 50$$

$$2.125 \times 10^{-5}\,\sigma_c = 30 \times 10^{-5}$$

$$2.125\sigma_c = 30$$

$$\therefore \qquad \sigma_c = \frac{30}{2.125} = \mathbf{14.117\ N/mm^2}.$$ **Ans.**

Substituting this value in equation (*i*), we get

$$\sigma_s = 14.117 \times 2.25$$

$$= \mathbf{31.76\ N/mm^2}.$$ **Ans.**

7 b)

Sol. Given :

Length, $L = 5 \text{ m} = 5 \times 10^3 \text{ mm}$

Diameter, $d = 30 \text{ mm}$

$\therefore$ Volume, $V = \frac{\pi}{4} d^2 \times L = \frac{\pi}{4} (30)^2 \times 5 \times 10^3 = 35.343 \times 10^5$

Tensile load, $P = 50 \text{ kN} = 50 \times 10^3$

Value of E $= 2 \times 10^5 \text{ N/mm}^2$

Poisson's ratio, $\mu = 0.25$

Let δd = Change in diameter

δL = Change in length

δV = Change in volume

Now strain of length $= \frac{\text{Stress}}{E}$

$$= \frac{\text{Load}}{\text{Area}} \times \frac{1}{E} \qquad \left(\because \quad \text{Stress} = \frac{\text{Load}}{\text{Area}}\right)$$

$$= \frac{P}{\frac{\pi}{4} \times d^2} \times \frac{1}{E} = \frac{50 \times 10^3}{\frac{\pi}{4} \times 30^2} \times \frac{1}{2 \times 10^5}$$

$$= \frac{0.4 \times 50 \times 10^3}{\pi \times 30^2 \times 2 \times 10^5} = 0.0003536$$

But strain of length $= \frac{\delta L}{L}$

$\therefore \quad \frac{\delta L}{L} = 0.0003536$

$\therefore \quad \delta L = 0.0003536 \times 5 \times 10^3$

$= \mathbf{1.768\ mm.}$ **Ans.**

Now Poisson's ratio $= \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$\therefore$ Lateral strain = Poisson's ratio × Longitudinal strain

$= 0.25 \times 0.0003536 \quad \left(\because \text{ Longitudinal strain} = \frac{\delta L}{L}\right)$

$= 0.0000884$

But Lateral strain $= \frac{\delta d}{d}$

$\therefore \quad \frac{\delta d}{d} = 0.0000884$

$\therefore \quad \delta d = 0.0000884 \times d$

$= 0.0000884 \times 30 = 0.002652$ mm

Now using equation (2.8), we get

Volumetric strain, $\frac{\delta V}{V} = \frac{\delta L}{L} - \frac{2\delta d}{d}$

$= 0.0003536 - 2 \times 0.0000884 = 0.0001768$

$\therefore \quad \delta V = V \times 0.0001768$

$= 35.343 \times 10^5 \times 0.0001768$

$= \mathbf{624.86\ mm^3.}$ **Ans.**

8 a)

Sol. First calculate reactions R_A and R_B.

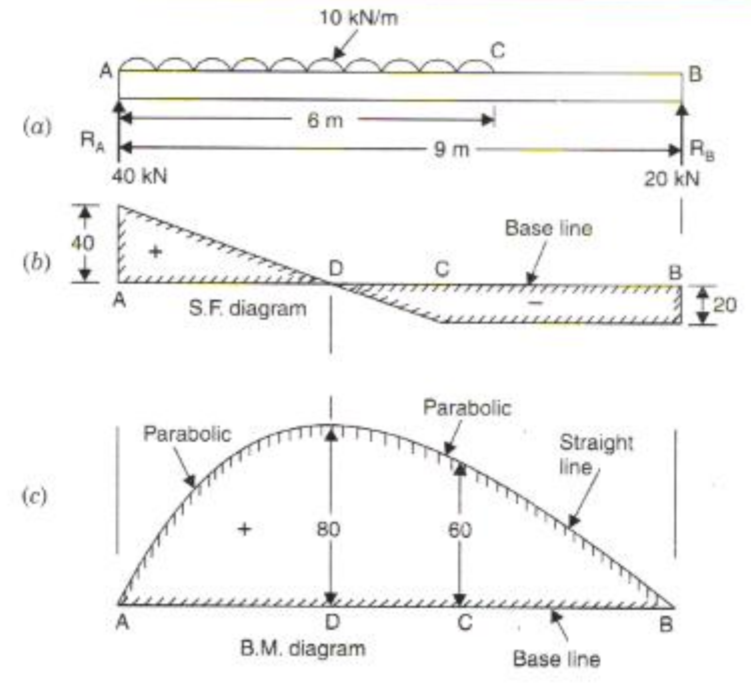


Fig. 6.28

Taking moments of the forces about A, we get

$$R_B \times 9 = 10 \times 6 \times \frac{6}{2} = 180$$

$$\therefore \quad R_B = \frac{180}{9} = 20 \text{ kN}$$

$$\therefore \quad R_A = \text{Total load on beam} - R_B = 10 \times 6 - 20 = 40 \text{ kN}.$$

Consider any section at a distance x from A between A and C. The shear force at the section is given by,

$$F_x = + R_A - 10\,x = + 40 - 10\,x \qquad ...(i)$$

Equation (i) shows that shear force varies by a straight line law between A and C.

At A, $x = 0$ hence $\qquad F_A = + 40 - 0 = 40$ kN

At C, $x = 6$ m hence $\qquad F_C = + 40 - 10 \times 6 = - 20$ kN

The shear force at A is + 40 kN and at C is − 20 kN. Also shear force between A and C varies by a straight line. This means that somewhere between A and C, the shear force is zero. Let the S.F. is zero at x metre from A. Then substituting the value of S.F. (*i.e.*, F_x) equal to zero in equation (i), we get

$$0 = 40 - 10x$$

$$\therefore \quad x = \frac{40}{10} = 4 \text{ m}$$

Hence shear force is zero at a distance 4 m from A.

The shear force is constant between C and B. This equal to − 20 kN.

Now the shear force diagram is drawn as shown in Fig. 6.28 (b). In the shear force diagram, distance AD = 4 m. The point D is at a distance 4 m from A.

B.M. Diagram

The B.M. at any section between A and C at a distance x from A is given by,

$$M_x = R_A \times x - 10\,.\,x\,.\,\frac{x}{2} = 40x - 5x^2 \qquad ...(ii)$$

Equation (ii) shows that B.M. varies according to parabolic law between A and C.

At A, $x = 0$ hence $\qquad M_A = 40 \times 0 - 5 \times 0 = 0$

At C, $x = 6$ m hence $\qquad M_C = 40 \times 6 - 5 \times 6^2 = 240 - 180 = + 60$ kNm

At D, $x = 4$ m hence $\qquad M_D = 40 \times 4 - 5 \times 4^2 = 160 - 80 = + 80$ kNm

The bending moment between C and B varies according to linear law.

B.M. at B is zero whereas at C is 60 kNm.

The bending moment diagram is drawn as shown in Fig. 6.28 (c).

Maximum Bending Moment

The B.M. is maximum at a point where shear force changes sign. This means that the point where shear force becomes zero from positive value to the negative or *vice-versa*, the B.M. at that point will be maximum. From the shear force diagram, we know that at point D, the shear force is zero after changing its sign. Hence B.M. is maximum at point D. But the B.M. at D is + 80 kNm.

$\therefore \qquad$ Max. B.M. = **+ 80 kN. Ans.**

8 b)

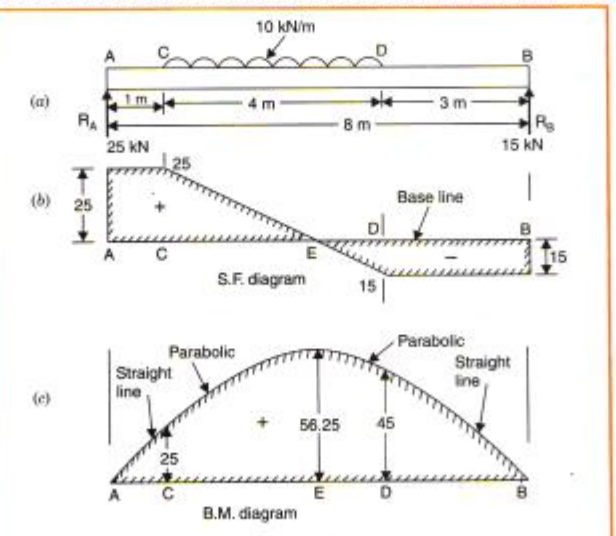


Fig. 6.29

$$\therefore \quad R_B = \frac{120}{8} = 15 \text{ kN}$$

$$\therefore \quad R_A = \text{Total load on beam} - R_B = 10 \times 4 - 15 = 25 \text{ kN}$$

Shear Force Diagram

The shear force at A is + 25 kN. The shear force remains constant between A and C and equal to + 25 kN. The shear force at B is – 15 kN. The shear force remains constant between B and D and equal to – 15 kN. The shear force at any section between C and D at a distance x from A is given by,

$$F_x = +25 - 10(x - 1)$$

At C, $x = 1$ hence $\quad F_C = +25 - 10(1 - 1) = +25$ kN

At D, $x = 5$ hence $\quad F_D = +25 - 10(5 - 1) = -15$ kN

The shear force at C is + 25 kN and at D is – 15 kN. Also shear force between C and D varies by a straight line law. This means that somewhere between C and D, the shear force is zero. Let the S.F. be zero at x metre from A. Then substituting the value of S.F. (*i.e.*, F_x) equal to zero in equation (*i*), we get

$$0 = 25 - 10(x - 1)$$

or $\quad 0 = 25 - 10x + 10 \quad$ or $\quad 10x = 35$

$$\therefore \quad x = \frac{35}{10} = 3.5 \text{ m}$$

Hence the shear force is zero at a distance 3.5 m from A.

Hence the distance AE = 3.5 m in the shear force diagram shown in Fig. 6.29 (*b*).

B.M. Diagram

B.M. at A is zero

B.M. at B is also zero

B.M. at $C = R_A \times 1 = 25 \times 1 = 25$ kNm

The B.M. at any section between C and D at a distance x from A is given by,

$$M_x = R_A \,.\, x - 10(x - 1)\,.\,\frac{(x-1)}{2} = 25 \times x - 5(x-1)^2 \qquad \ldots(ii)$$

At C, $x = 1$ hence $\quad M_C = 25 \times 1 - 5(1 - 1)^2 = 25$ kNm

At D, $x = 5$ hence $\quad M_D = 25 \times 5 - 5(5 - 1)^2 = 125 - 80 = 45$ kNm

At E, $x = 3.5$ hence $\quad M_E = 25 \times 3.5 - 5(3.5 - 1)^2 = 87.5 - 31.25 = 56.25$ kNm

B.M. will increase from 0 at A to 25 kNm at C by a straight line law. Between C and D the B.M. varies according to parabolic law as is clear from equation (ii). Between C and D, the B.M. is the maximum at E. From D to B the B.M. will decrease from 45 kNm at D to zero at B according to straight line law.

Problem 6.11. *Draw the S.F. and B.M. ...*