

UNIT II KINEMATICS OF LINKAGE MECHANISMS

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Displacement, velocity and acceleration analysis of simple mechanisms – Graphical method – Velocity and acceleration polygons – Velocity analysis using instantaneous centres – kinematic analysis of simple mechanisms – Coincident points – Coriolis component of Acceleration – Introduction to linkage synthesis problem.

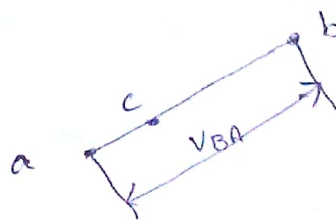
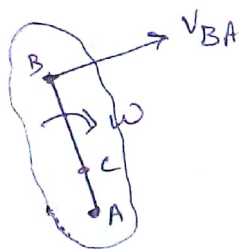
Kinematics of linkage mechanisms :-

- * The relative velocity method is for determining the velocity of different Points in the mechanism.

Relative velocity of two bodies moving in straight lines :-

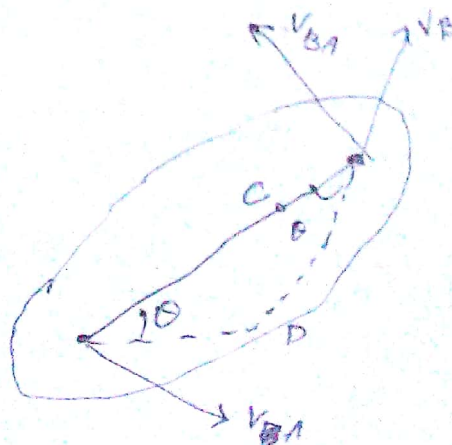
Nature of a link :-

- * velocity of any Point on a link w.r.t another Point on the same link is always $\perp$ to the line joining these Points on the configuration diagram.



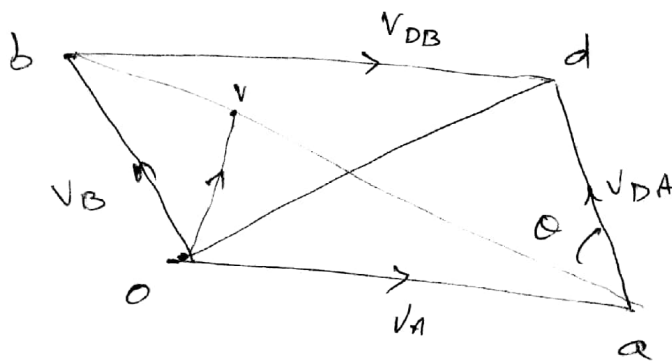
velocity of a Point on a link by relative velocity method :-

- * Consider two Points A & B on a link.



Procedure :-

- * Take some Convenient Point O , known as the pole.
- * Through O , draw a Parallel & equal to V_A to some suitable scale.
- * Through a , draw a line $\perp$ to AB , this line will represent the velocity of B w.r.t A (i.e.) V_{BA} .
- * Through O draw a line $\parallel$ to V_{BA} intersecting the line V_{OA} at b .



velocity diagram.

Centrode :-

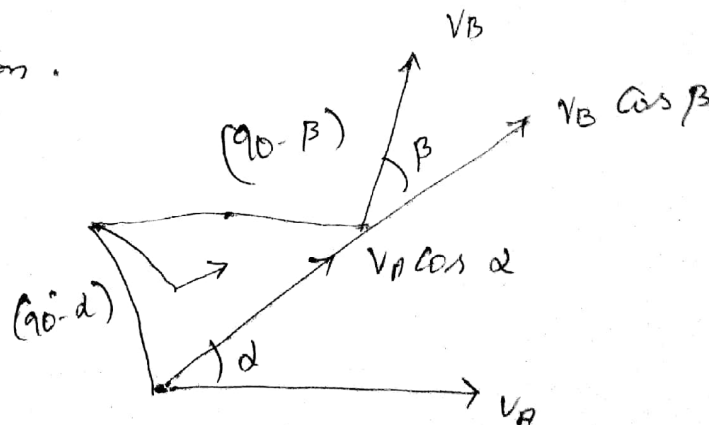
- * The instantaneous centre of a moving body may be defined as that centre which goes on changing from one instant to another.
- * The locus of all such instantaneous centre is known as centrode.

Methods for determining the velocity of a Point on a link

- * Instantaneous Centre method.
- * Relative velocity method.

Instantaneous Centre method:-

- * The instantaneous Centre method of analysing the motion in a mechanism is based upon the concept that any displacement of a body having motion in one plane can be considered as a pure rotational motion of a rigid link as a whole about some centre known as Instantaneous Centre (or) Virtual Centre of rotation.



Now resolving the velocities along AB

$$V_A \cos \alpha = V_B \cos \beta$$

$$\frac{V_A}{V_B} = \frac{\cos \beta}{\cos \alpha} = \frac{\sin (90^\circ - \beta)}{\sin (90^\circ - \alpha)}$$

by lamis theorem ,

$$\frac{AI}{\sin(90^\circ - \beta)} = \frac{BI}{\sin(90^\circ - \alpha)}$$

$$\frac{V_A}{V_B} = \frac{AI}{BI} \quad (\text{or}) \quad \frac{V_A}{AI} = \frac{V_B}{BI} = \omega$$

ω - angular velocity of rigid link.

Properties of Instantaneous Centre,

- * A rigid link rotates instantaneously relative to another link at the Instantaneous Centre for the Configuration of the mechanism considered.
- * The velocity of instantaneous Centre relative to any third rigid link will be same whether the instantaneous Centre is regarded as a Point on the first rigid link or on the second rigid link.

No. of Instantaneous Centre in mechanism :-

- * Constrained (kinematic) Chain is equal to the no. of Possible Combination of two links.

$$N = \frac{n(n-1)}{2}, \quad n \rightarrow \text{no. of links.}$$

Types:-

- * 1. Fixed instantaneous centres
- 2. Permanent instantaneous centre
- 3. Neither fixed nor Permanent IC.

Arnold Kennedy theorem:-

- * It states that if three bodies move relatively to each other the have three instantaneous centre lie on a straight line.

Method for locating IC.

- * 1st of all, determine the no. of instantaneous centres by using relation

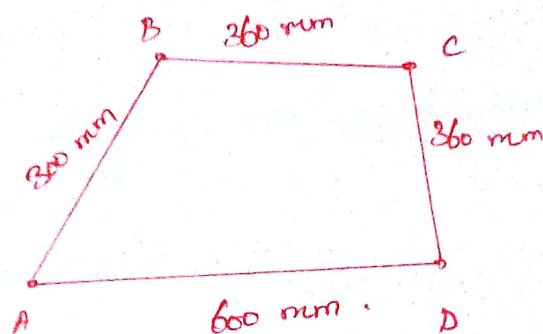
$$N = \frac{n(n-1)}{2}$$

- * Make a list of all the instantaneous centres in a mechanism. Since for a four bar mechanism there are Instantaneous centres.
- * Locate the fixed & Permanent Instantaneous centre by inspection.

- * locate the remaining neither fixed nor Permanent instantaneous centres by Kennedy's theorem
- * Join the Points by solid line to show that these centres are already found.
- * In order to find the two Instantaneous centres join two such points that the line joining them form two adjacent triangles in the circle diagram.

Ex:-

In a Pin jointed four bar mechanism as shown in fig.
 $AB = 300 \text{ mm}$, $BC = CD = 360 \text{ mm}$ & $AD = 600 \text{ mm}$.
the angle $\angle BAD = 60^\circ$. The Crank AB rotates uniformly
at 100 rpm. locate all the instantaneous centre &
find the velocity of the link BC.



Given:-

$$N_{AB} = 100 \text{ r.p.m}$$

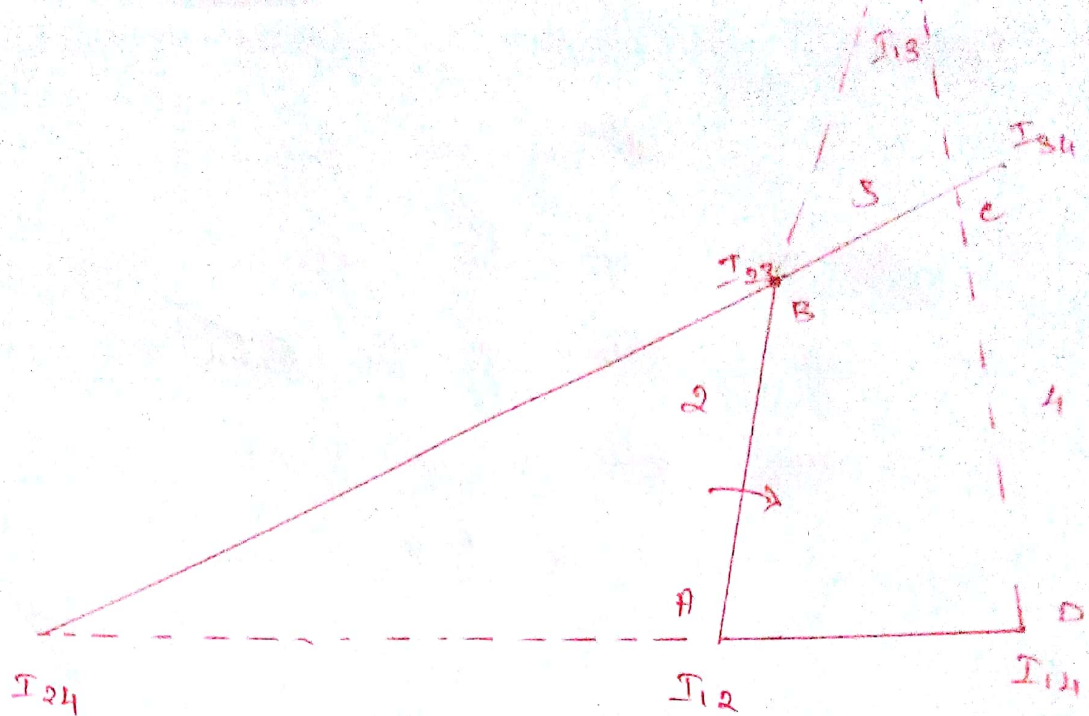
$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60}$$
$$= 10.47 \text{ rad/s}$$

$$\text{length of Crank } AB = 300 \text{ mm} = 0.3 \text{ m}$$

$$V_B = \omega_{AB} \times AB = 10.47 \times 0.3$$
$$= 3.141 \text{ m/s.}$$

location of I.C

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$



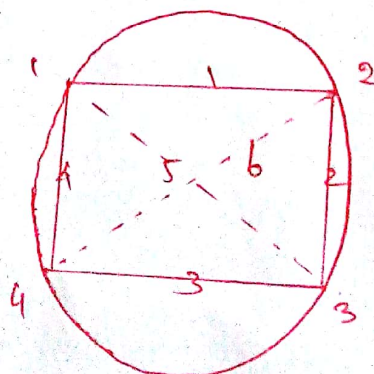
Angular Velocity of the link BC

ω_{BC} = Angular velocity of the link BC

$$V_B = \omega_{BC} \times I_{13} B$$

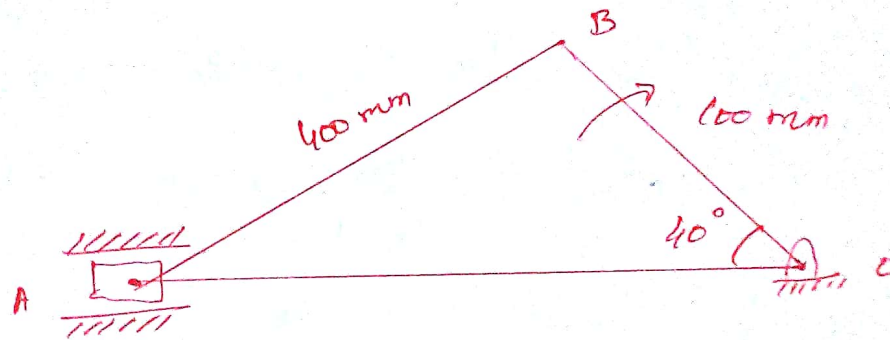
$$I_{13} B = 500 \text{ mm} = 0.5 \text{ m}$$

$$\omega_{BC} = \frac{V_B}{I_{13} B} = \frac{3.141}{0.5} = 6.28 \text{ rad/s}$$



Ex:-

locate all the IC of the slider crank mechanism as shown in fig. The lengths of crank AB & connecting rod AB are 100 mm & 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s. find
1. velocity of the slider. 2. Angular velocity of the connecting rod AB.



Given:-

$$\omega_{OB} = 10 \text{ rad/s}$$

$$OB = 100 \text{ mm} = 0.1 \text{ m}$$

$$V_{OB} = V_B = \omega_{OB} \times OB$$

$$= 10 \times 0.1 = 1 \text{ m/s}$$

Location of IC.

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

Velocity of slider.

$$I_{13} A = 460 \text{ mm} = 0.46 \text{ m}$$

$$I_{13} B = 560 \text{ mm} = 0.56 \text{ m}$$

WKT

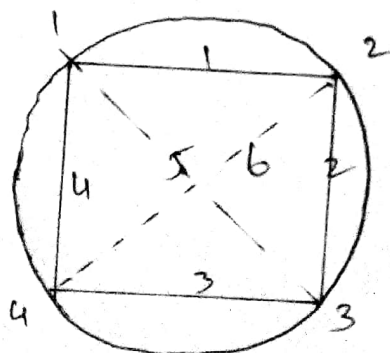
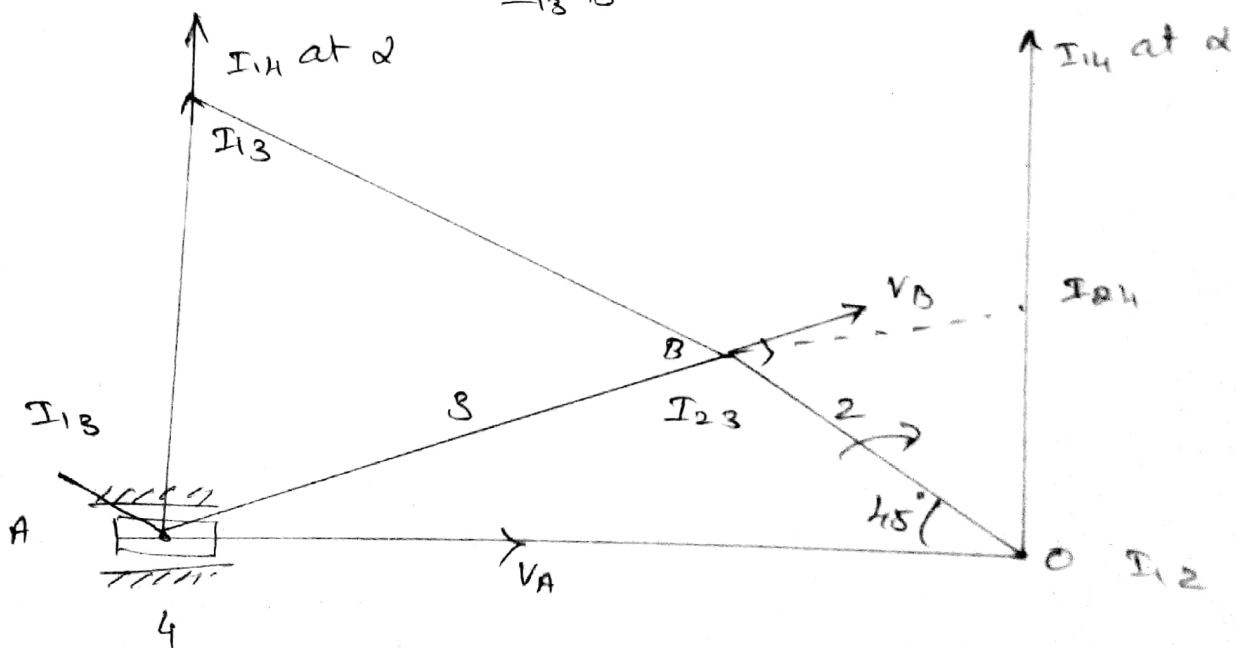
$$\frac{V_A}{I_{13} A} = \frac{V_B}{I_{13} B}$$

$$\begin{aligned} V_A &= V_B \times \frac{I_{13} A}{I_{13} B} \\ &= 1 \times \frac{0.46}{0.56} \\ &= 0.82 \text{ m/s} \end{aligned}$$

Angular velocity of connecting rod AB :- (ω_{AB})

$$\omega_{AB} = \frac{V_A}{I_{13} A} = \frac{V_B}{I_{13} B} = \omega_{AB}$$

$$\omega_{AB} = \frac{V_B}{I_{13} B} = \frac{1}{0.56} = 1.78 \text{ rad/s}$$



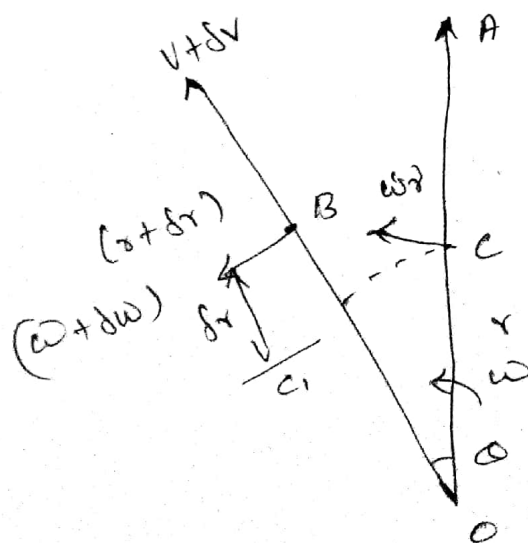
Coriolis Component of acceleration:-

- * When a Point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the Coriolis Component of the acceleration must be calculated.
- * Consider a link OA & a slider B as shown in fig.
- * The slider B moves along the link OA . The Point C is the Coincident Point on the OA .

ω - angular velocity of link OA at time t

v - velocity of the slider B along the link OA .

$\omega \cdot r$ - velocity of the slider B .



- * This tangential Component of acceleration of the slider B w.r.t. Coincident Point C on the link

Introduction of linkages:-

- * A Mechanical linkage is an assembly of bodies connected to manage forces & movement
- * The movement of a body or link is studied using geometry so the link is considered to be rigid.
- * The connections b/w links are modeled as providing ideal movement, Pure rotation or Sliding for eg. and are joints.
- * A linkage modeled as network of rigid links & ideal joints is called kinematic chain
- * Linkages may be constructed from open chains, closed chains or combination of open & closed chains.