UNIT II Stiffness matrix method

Stiffness method (or) displacement method (or) equilibrium method

Translation (Δ) } are unknowns.
rotation (θ)

Rotation – Angular displacement.

Translation – Linear displacement.

Equilibrium equation:

$$K\Delta = P - P_L$$

This method is based on slope deflection method.

Equilibrium equation $P = P_L + K\Delta$

P = Final moment.

P_L = fixed end moment.

[KI (or) Degree of freedom]

K_{ij} = Stiffness co-efficient
(or)
K_{ij} = Stiffness influence co efficient.

K_{ij} :-

① Force developed "i" due to unit displacement at "j"

② moment developed i due to unit rotation at "j"

Final moment = fixed end moment + K Δ [θ_B, δ]

Step:1

K.I (or) Degree of freedom.

Step:2

Assign the co-ordinates fully restrained structure [support (Δ), moment (θ)].

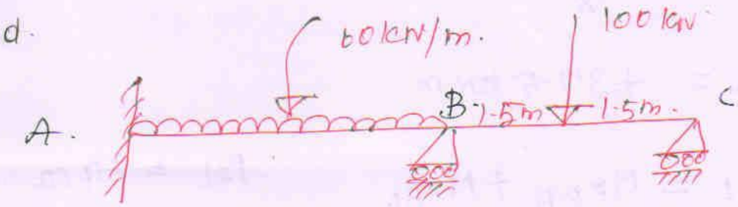
Kinematic indeterminancy (or) Degree of freedom:-

The number of independent joint displacement [both rotation & translation] in a structure is known as degree of freedom (or) kinematic indeterminancy.

Stiffness influence co-efficient (K_{ij})

It is defined as the force developed at i^{th} degree of freedom due to unit displacement impressed at j degree of freedom, all other degrees of freedom except "j" being kept arrested.

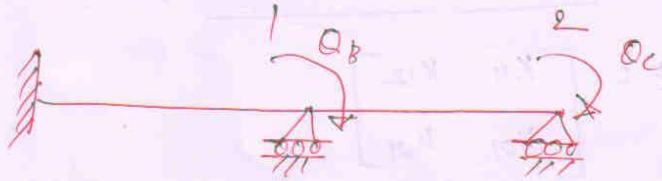
Pbm Analyse the continuous beam by displacement method.



Step:1 Determination of KI (or) DOF

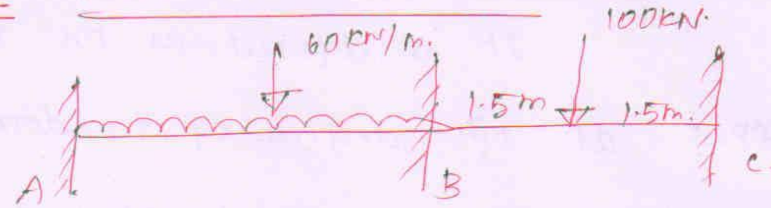
$KI = 2 \quad [\theta_B \;\&\; \theta_C]$

Step:2 Assigning co-ordinate direction



Step:3 Determination of $[P_L]$

Step 3 [Determination of (P_L)].



$$M_{FAB} = \frac{-wl^2}{12} = \frac{-60 \times 4^2}{12} = -80 \text{ kN·m}.$$

$$M_{FBA} = \frac{+wl^2}{12} = \frac{+60 \times 4^2}{12} = +80 \text{ kN·m}.$$

$$M_{FBC} = \frac{-wl}{8} = -37.5 \text{ kN·m}.$$

$$M_{FCB} = +37.5 \text{ kN·m}$$

$$P_{1L} = M_{FBA} + M_{FBC} \qquad P_{2L} = M_{FCB}$$

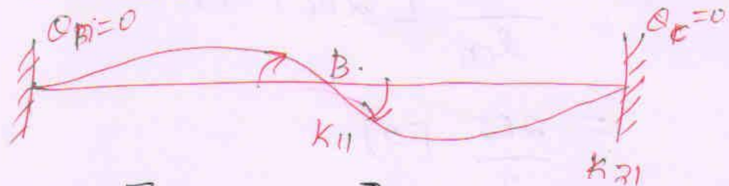
$$\boxed{P_{1L} = 42.5} \qquad \boxed{P_{2L} = 37.5}$$

$$[P_L] = \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix}$$

Step 4 Stiffness matrix [k]

$$k = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{21} \end{bmatrix}$$

Case (a) apply unit rotation at B.



$$K_{11} = \frac{2EI}{l_{AB}}\left[2\theta_B + \theta_A\right] + \frac{2EI}{l_{BC}}\left[2\theta_B + \theta_C\right]$$

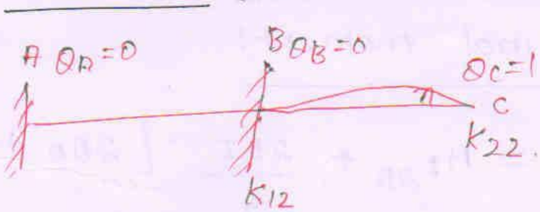
$$= \frac{2EI}{4}[2] + \frac{2EI}{3}[2]$$

$$K_{11} = 2.33\,EI$$

$$K_{21} = \frac{2EI}{3}\left[2\theta_C + \theta_B\right]$$

$$K_{21} = 0.67\,EI$$

Case (b) apply unit rotation at C



$$K_{12} = \frac{2EI}{l_{BC}}\left[2\theta_B + \theta_C\right]$$

$$K_{12} = 0.67\,EI$$

$$k_{22} = \frac{2EI}{l_{CB}}\left[2\theta_c + \theta_B\right]$$

$$= \frac{2EI}{3}\left[2\right]$$

$$K_{22} = 1.33\,EI$$

Step:5 Solving equilibrium equation

$$P - P_L = K\Delta.$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix} = EI\begin{bmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix}$$

$$\theta_B = -11.84/EI$$

$$\theta_C = -22.22/EI$$

Step:6 Final moment

$$M_{AB} = M_{F_{AB}} + \frac{2EI}{4}\left[2\theta_A + \theta_B\right]$$

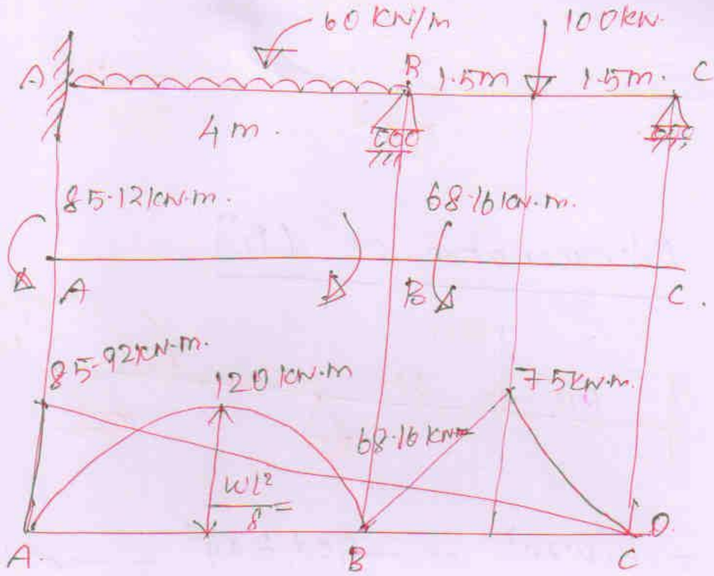
$$M_{AB} = -85.92 \text{ kN.m.}$$

$$M_{BA} = 68.16 \text{ kN.m.}$$

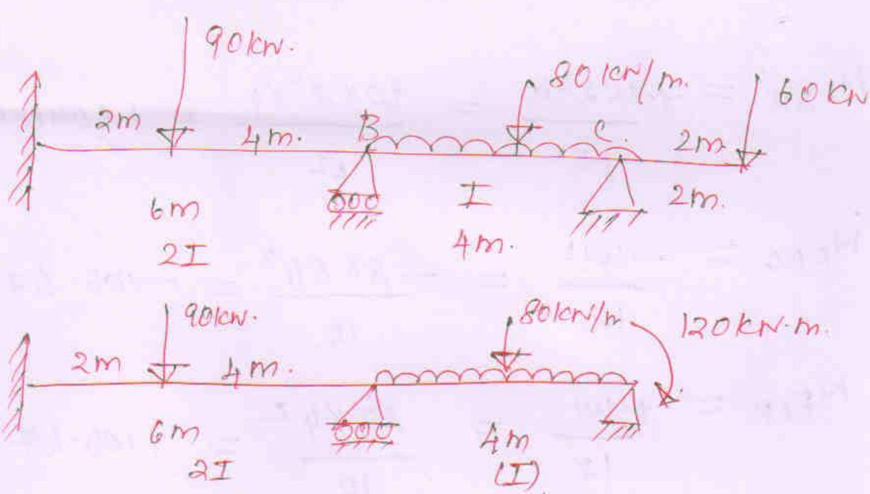
$$M_{BC} = -68.16 \text{ kN.m}$$

$$M_{CB} = 0.$$

Step 7 Bending moment diagram!



(2).

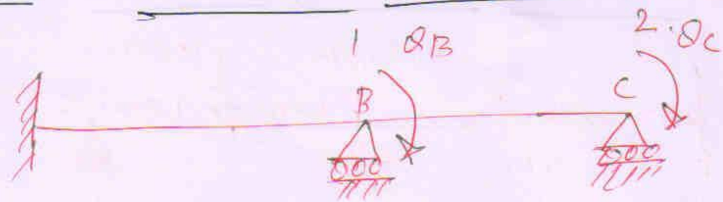


Step:1 kinematic indeterminancy (or) DOF

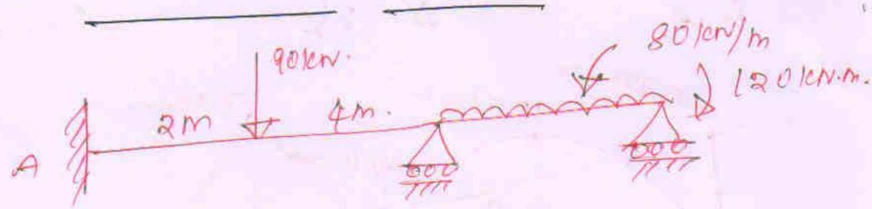
KI = 2 [θ_B & θ_C]

(1)

Step 12 Assign the co-ordinate direction



Step 12 Determination of [PL]



$$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-90 \times 2 \times 4^2}{6^2} = -80 \text{ kN.m.}$$

$$M_{FBA} = \frac{+wa^2b}{l^2} = \frac{90 \times 2^2 \times 4}{6^2} = +40 \text{ kN.m.}$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-80 \times 4^2}{12} = -106.67 \text{ kN.m.}$$

$$M_{FCB} = \frac{+wL}{12} = \frac{80 \times 4^2}{12} = +106.67 \text{ kN.m}$$

$$P_{1L} = M_{FBC} + M_{FCB}$$

$$\boxed{P_{1L} = -66.67 \text{ kN.m.}}$$

$$P_{2L} = M_{FCB}$$

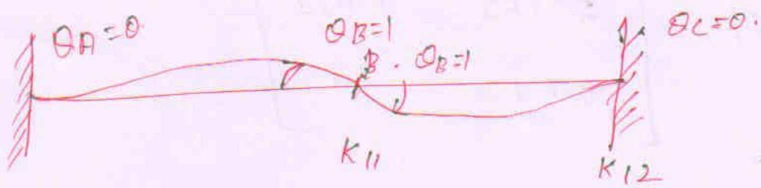
$$\boxed{P_{2L} = 106.67 \text{ kN.m}}$$

$$P_L = \begin{bmatrix} -66.67 \\ +106.67 \end{bmatrix}$$

Step 14 stiffness matrix [k]

$$[k] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

Case (a) apply unit rotation at B



$$k_{11} = \frac{2EI}{l_{BA}}[2\theta_B + \theta_C] + \frac{2EI}{l_{BC}}[2\theta_B + \theta_C]$$

$$= \frac{4EI}{6}[2\times 1] + \frac{2EI}{4}[2]$$

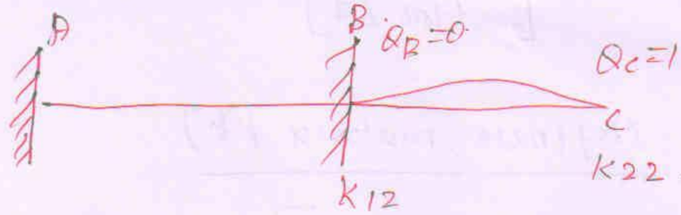
$$\boxed{k_{11} = 2.33\,EI}$$

$$k_{21} = \frac{2EI}{l_{CB}}[2\theta_C + \theta_B]$$

$$k_{21} = \frac{2EI}{4}[1]$$

$$\boxed{k_{21} = 0.5\,EI}$$

Case (b) apply unit rotation at C



$K_{12} = 0.5\,EI$

$K_{22} = \frac{2EI}{4}[2\theta_C]$ $\quad$ $K_{22} = EI$

$$[K] = \begin{bmatrix} 2.33EI & 0.5EI \\ 0.5EI & EI \end{bmatrix}$$

$$K\Delta = P - P_L$$

$$EI \begin{bmatrix} 2.33 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 \\ 120 \end{bmatrix} - \begin{bmatrix} -66.67 \\ 106.67 \end{bmatrix}$$

$\theta_B = 28.84/EI$

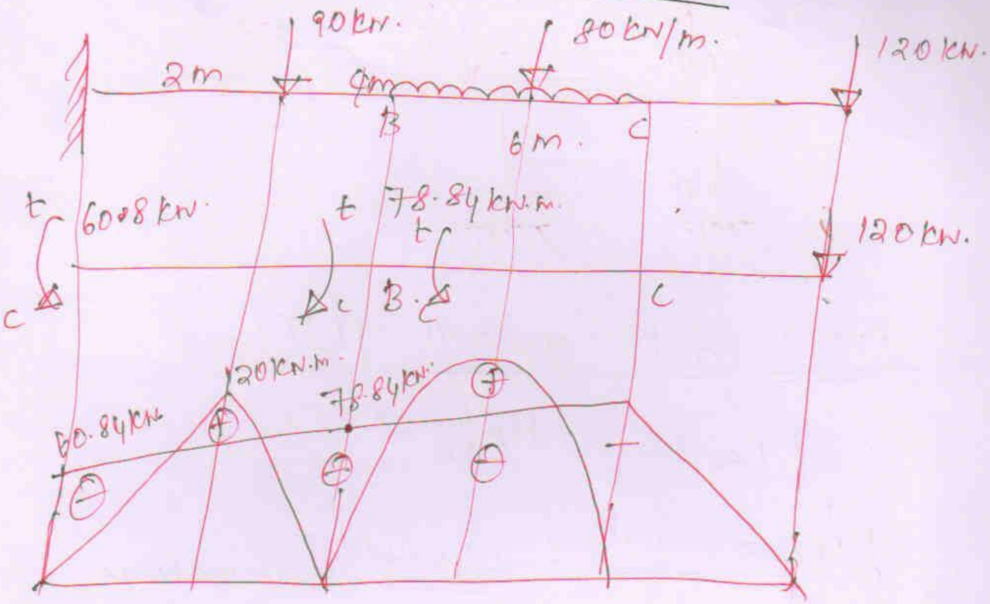
$\theta_C = -1.09/EI$

Step:6 final moment

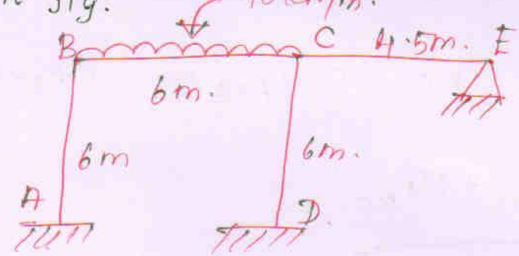
$M_{AB} = -60.8$ kN.m. $\quad$ $M_{BC} = -78.84$ kN.m.

$M_{BA} = 78.4$ kN.m $\quad$ $M_{CB} = 0$.

Step:6 Bending moment diagram.



(5) Using displacement mtd analyse the frame shown in fig.

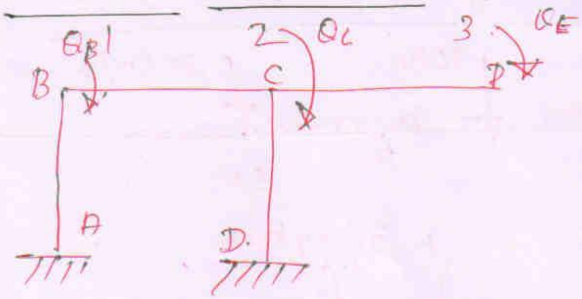


Step:1) Kinematic indeterminancy (or) DOF

$KI = 3$ ~~(or)~~ $[\theta_B, \theta_C, \theta_E]$

$$\Delta = \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_E \end{bmatrix}$$

Step 2 Assign the co-ordinate direction.



Step 3 Determination of $[P_L]$

$M_{FAB} = 0 \quad M_{FBA} = 0$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-40 \times 6^2}{12} = -120 \text{ kN·m.}$$

$$M_{FCB} = \frac{+wl^2}{12} = \frac{+40 \times 6^2}{12} = +120 \text{ kN·m.}$$

$M_{FCD} = M_{FDC} = 0 \quad M_{FCE} = 0 \quad M_{FEC} = 0.$

$P_{1L} = M_{FBA} + M_{FBC}$

$\boxed{P_{1L} = -120 \text{ kN}}$

$P_{2L} = M_{FCB} + M_{FCE} + M_{FCD}$

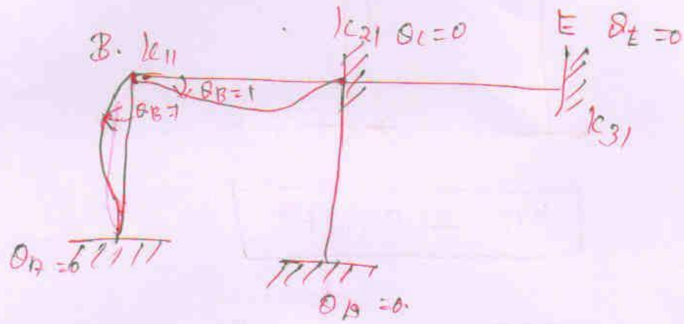
$P_{2L} = +120 \text{ kN.}$

$P_{3L} = M_{FEC}$

$P_{3L} = 0 \qquad [P_L] = \begin{bmatrix} -120 \\ 120 \\ 0 \end{bmatrix}$

Step: 4 ~~Determination of [P]~~ Stiffness matrix [K]

case (a) apply unit rotation at B



$$k_{11} = \frac{2EI}{l_{AB}}[2\theta_B + \theta_C] + \frac{2EI}{l_{BC}}[2\theta_B + \theta_C]$$

$$K_{11} = \frac{2EI}{6}[2] + \frac{2EI}{6}[2]$$

$$\boxed{k_{11} = 1.33\,EI}$$

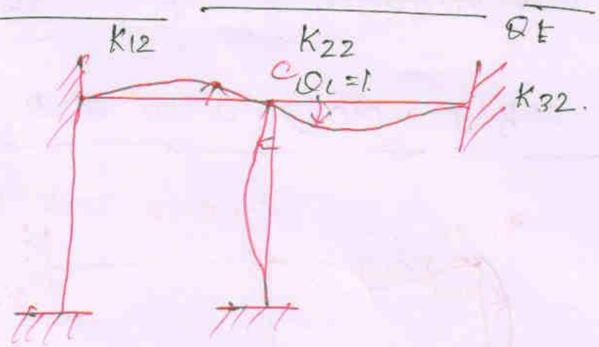
$$K_{21} = \frac{2EI}{l_{CB}}[2\theta_C + \theta_B]$$

$$K_{21} = \frac{2EI}{6}[1]$$

$$\boxed{k_{21} = 0.33\,EI}$$

$$\boxed{k_{31} = 0}$$

Case (b) apply unit rotation at C



$$K_{12} = 0.33EI$$

$$K_{22} = \frac{2EI}{6}\left[2\theta_c + \theta_B\right] + \frac{2EI}{4.5}\left[2\theta_c\right] + \frac{2EI}{6}\left[\;\right]$$

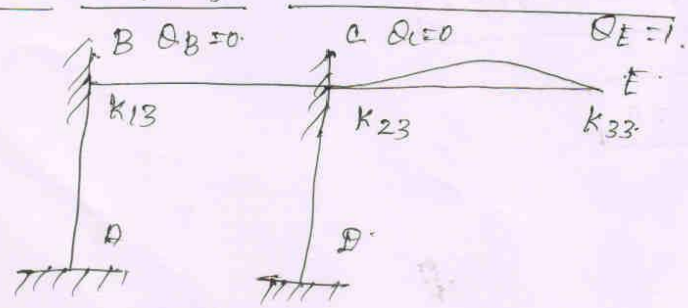
$$K_{22} = 2.22EI$$

$$K_{32} = \frac{2EI}{l_{EC}}\left[2\theta_E + \theta_c\right]$$

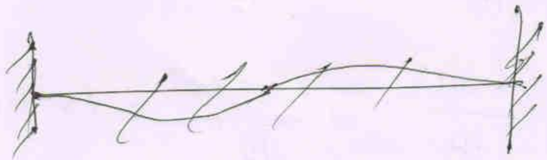
$$K_{32} = \frac{2EI}{4.5}\left[1\right]$$

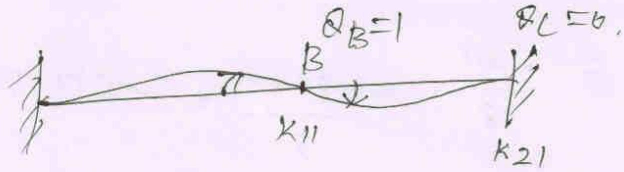
$$K_{32} = 0.44\,EI$$

Case (c) apply unit rotation at E



Case (a) apply unit rotation at B





$$k_{11} = \frac{2EI}{l_{AB}}[2\theta_B + \theta_A] + \frac{2EI}{l_{BC}}[2\theta_B + \theta_C]$$

$$= \frac{2EI}{6}[2 \times 1] + \frac{2EI}{4}[2 \times 1].$$

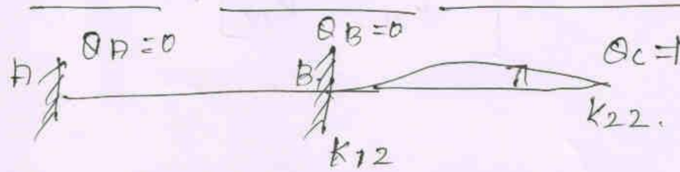
$$\boxed{k_{11} = 1.66\ EI}$$

$$k_{21} = \frac{2EI}{4}[2\theta_C + \theta_B]$$

$$k_{21} = \frac{2EI}{4}[1]$$

$$\boxed{k_{21} = 0.5\ EI}$$

Case 1(b) apply unit rotation at C



$$M_{FAB} = \frac{-wl^2}{12} = \frac{-20 \times 6^2}{12} = -60 \text{ kN.m}$$

$$M_{FBA} = \frac{+wl^2}{12} = \frac{+20 \times 6^2}{12} = +60 \text{ kN.m}$$

$$M_{FBC} = \frac{-wl}{8} = \frac{-20 \times 4}{8} = -10 \text{ kN.m}$$

$$M_{FCB} = \frac{+wl}{8} = \frac{+20 \times 4}{8} = +10 \text{ kN.m}$$

$$P_{1L} = M_{FBA} + M_{FBC}$$

$$\boxed{P_{1L} = 60 - 10 = 50 \text{ kN.m}}$$

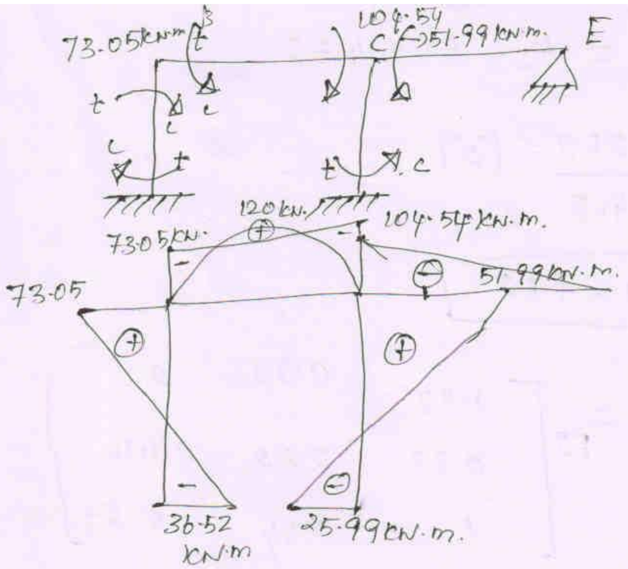
$$P_{2L} = M_{FCB} = +10 \text{ kN.m}$$

$$\boxed{P_{2L} = 10 \text{ kN.m}}$$

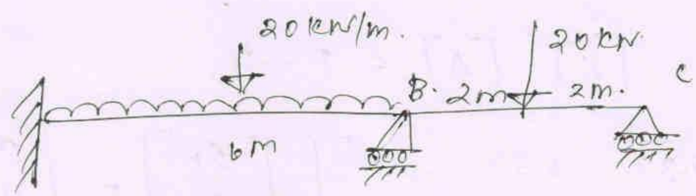
$$[P_L] = \begin{bmatrix} 50 \\ 10 \end{bmatrix}$$

Step: 4 Stiffness matrix $[K_L]$.

$$[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$



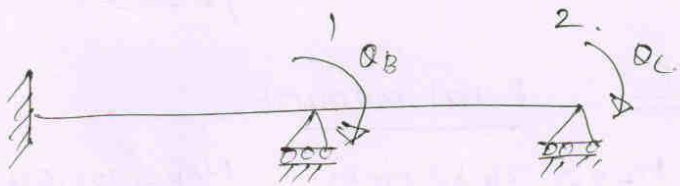
4.



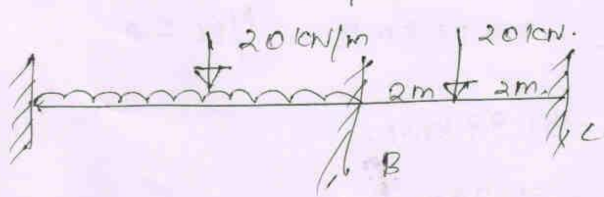
Step I: Kinematic indeterminacy (or) DOF.

$KI = 2 \quad [\theta_B, \theta_C]$

Step II: Assign the co-ordinate direction



Step 2: Determination of $[P_L]$



$k_{13} = k_{23} = 0.44 EI$

$k_{33} = \frac{2EI}{4.5} [2]$

$k_{33} = 0.89 EI$

$$[k] = EI \begin{bmatrix} 1.33 & 0.33 & 0 \\ 0.33 & 2.22 & 0.44 \\ 0 & 0.44 & 0.89 \end{bmatrix}$$

Step: 5 $[k][\Delta] = P - P_L$

$$EI \begin{bmatrix} 1.33 & 0.33 & 0 \\ 0.33 & 2.22 & 0.44 \\ 0 & 0.44 & 0.89 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -120 \\ 120 \\ 0 \end{bmatrix}$$

$\theta_B = 109.57/EI$

$\theta_C = -77.98/EI$

$\theta_E = 38.55/EI$

Step: 6 Final moment

$M_{AB} = 36.52$ kN.m.

$M_{BA} = 73.05$ kN.m.

$M_{BC} = -73.05$ kN.m.

$M_{CB} = 104.54$ kN.m.

$M_{CE} = -51.98$ kN.

$M_{EC} = 0$.

$M_{CD} = -51.99$ kN.m.

$M_{DC} = -25.99$ kN.m.

$$K_{12} = \frac{2EI}{l_{BC}} [2\theta_B + \theta_C]$$

$$= \frac{2EI}{4} [1] = \boxed{K_{12} = 0.5EI} \theta_C$$

$$K_{22} = \frac{2EI}{l_{CB}} [2\theta_C + \theta_B]$$

$$K_{22} = \frac{2EI}{4} [2 \times 1]$$

$$\boxed{K_{22} = EI}$$

Step:5 Solving equilibrium equation

$$P - P_L = K\Delta.$$

$$EI \begin{bmatrix} 1.66 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 50 \\ 10 \end{bmatrix}$$

$$\boxed{\begin{aligned} \theta_B &= -31.91/EI \\ \theta_C &= 5.95/EI \end{aligned}}$$

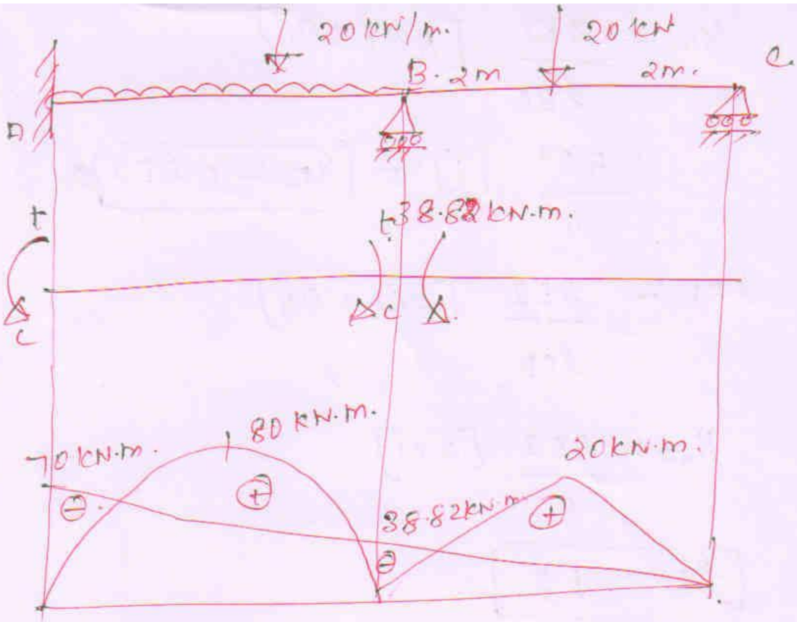
Step:6 Final moment

$M_{AB} = -70$ kN.m.

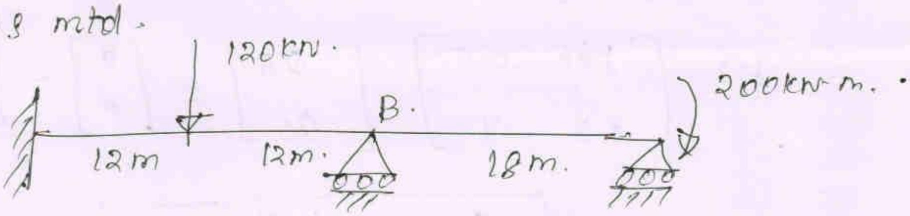
$M_{BA} = 38.82$ kN-m

$M_{BC} = -38.82$ kN.m.

$M_{CB} = 0.$



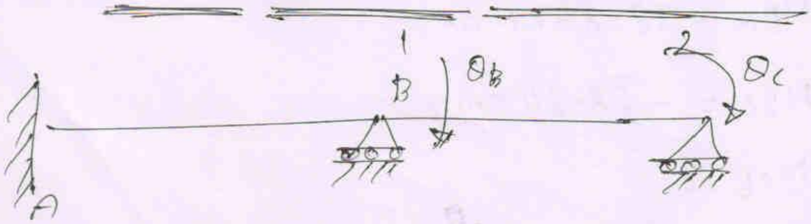
6. Analyse the continuous beam by matrix stiffness mtd.



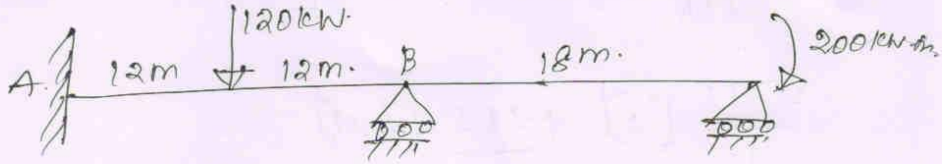
Step 1 Kinematic indeterminacy (or) DOF

KI = 2 (θ_B & θ_C)

Step 2 Assign the co-ordinates direction



Step: 3 Determination of [PL]



$$M_{FAB} = \frac{-wl}{8} = \frac{-120 \times 24}{8} = -360 \text{ kN.m}$$

$$M_{FBA} = \frac{+wl}{8} = +360 \text{ kN.m}$$

$M_{FBC} = 0 \qquad M_{FCB} = 0$

$P_{1L} = M_{FBA} + M_{FBC}$

$$\boxed{P_{1L} = 360 \text{ kN.m}}$$

$P_{2L} = M_{FCB}$

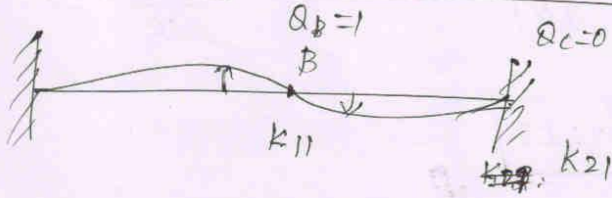
$$\boxed{P_{2L} = 0}$$

$$[P_L] = \begin{bmatrix} 360 \\ 0 \end{bmatrix}$$

Step: 4 Stiffness matrix $[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$

Apply unit rotation

case (a) Apply unit rotation at B



$$k_{11} = \frac{2EI}{l_{BA}}[2\theta_B + \theta_A] + \frac{2EI}{l_{BC}}[2\theta_B + \theta_C]$$

$$= \frac{2EI}{12}[2] + \frac{2EI}{18}[2 \times 1]$$

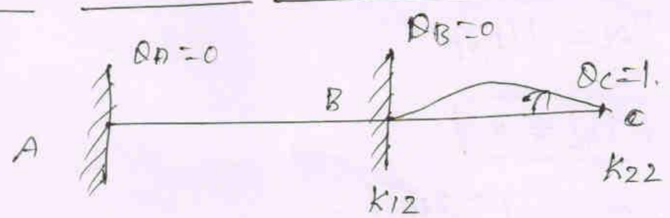
$$\boxed{k_{11} = 0.388\,EI}$$

$$k_{21} = \frac{2EI}{l_{CB}}[2\theta_C + \theta_B]$$

$$= \frac{2EI}{18}[1]$$

$$\boxed{k_{21} = 0.11EI}$$

case (b) apply unit rotation at C



$$\boxed{k_{12} = 0.11EI}$$

$$k_{22} = \frac{2EI}{18}[2\theta_C + \theta_B] \qquad \frac{2EI}{l_{CB}}[2\theta_C + \theta_B]$$

$$= \frac{2EI}{18}[2] \qquad \frac{2EI}{18}[2]$$

$$\boxed{k_{22} = 0.22EI}$$

Step 15 Solving equilibrium equation

$$P - P_L = K\Delta.$$

$$EI\begin{bmatrix}0.388 & 0.11\\ 0.11 & 0.22\end{bmatrix}\begin{bmatrix}\theta_B\\ \theta_C\end{bmatrix} = \begin{bmatrix}0\\ 200\end{bmatrix} - \begin{bmatrix}380\\ 0\end{bmatrix}$$

$$\theta_B = -1380/EI$$
$$\theta_C = 1590/EI$$

Final moment:

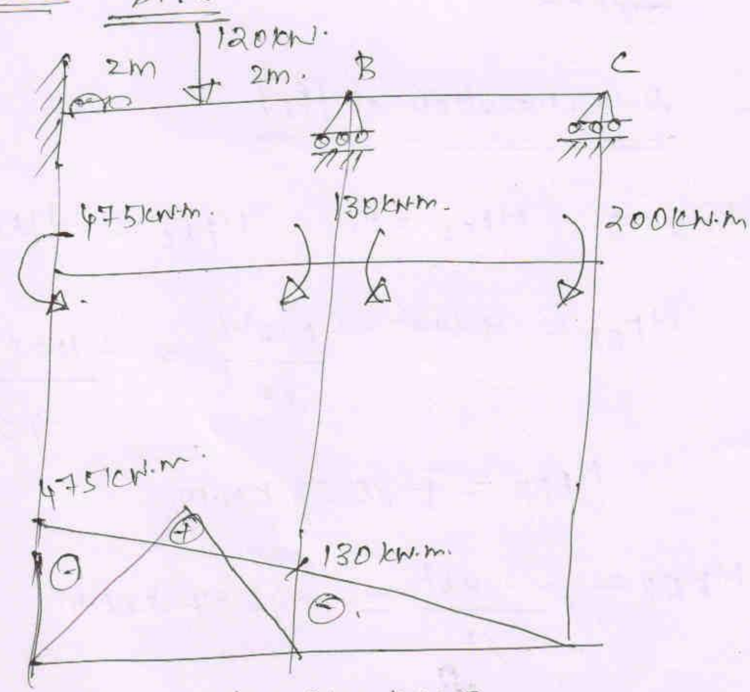
$M_{AB} = -475$ kN.m.

$M_{BA} = 130$ kN.m.

$M_{BC} = -130$ kN.m.

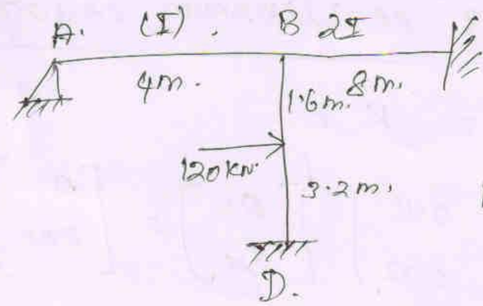
$M_{CB} = 200$ kN.m

Step 17 BMD



FINAL BMD.

7



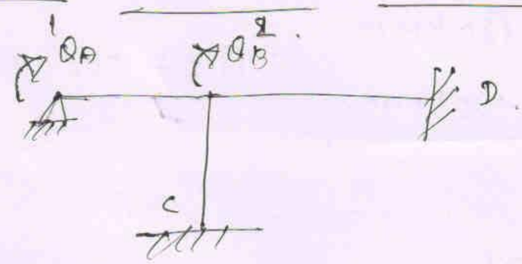
Analyse the frame by stiffness method.

Step:1 Kinematic indeterminancy (or) DOF

$KI = 2 \; [\theta_A, \theta_B]$.

$$\Delta = \begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix}$$

Step:2 Assign the co-ordinate direction



Step:3 Determination of [PL]

$M_{FAB} = 0 \quad M_{FBA} = 0. \quad M_{FBC} = 0 \quad M_{FCB} = 0.$

$$M_{FBD} = + \frac{wa^2b}{l^2} = + \frac{120 \times 3.2^2 \times 1.6}{4.8^2}$$

$M_{FBD} = +85.33$ kN.m.

$$M_{FDB} = \frac{-wab^2}{l^2} = -42.67 \text{ kN.m.}$$

$P_{1L} = M_{FAB} = 0$

$P_{2L} = M_{FBA} + M_{FBC} + M_{FBD}$

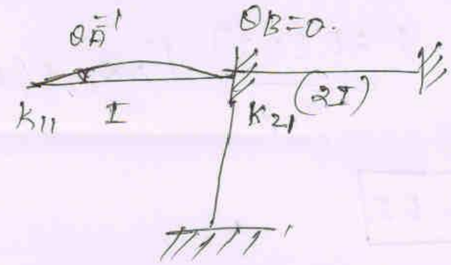
$= 0 + 0 + 85.33$

$P_{2L} = 85.33$ kN·m

$$[P_L] = \begin{bmatrix} 0 \\ 85.33 \end{bmatrix}$$

Step 4 Stiffness matrix (k)

case (a) apply unit rotation at A



$K_{11} = \frac{2EI}{l_{AB}} [2\theta_A + \theta_B]$

$= \frac{2EI}{4} [2 \times 1]$

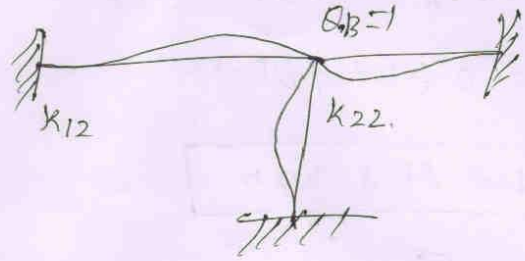
$K_{11} = EI$

$K_{21} = M_{BA}$

$= \frac{2EI}{l_{BA}} [2\theta_B + \theta_A]$

$= \frac{2EI}{4} [1] = K_{21} = 0.5EI$

Case (b) Apply unit rotation at B



$K_{12} = 0.5\,EI$

$K_{22} = M_{BA} + M_{BC} + M_{BD}$

$= \frac{2EI}{4}[2\theta_B + \theta_A] + \frac{2E(2I)}{8}[2\theta_B + \theta_C] + \frac{2E(3I)}{4.8}[2\theta_B + \theta_D]$

1.6
2.2
1.8

$$\boxed{K_{22} = 4.5\,EI}$$

$$[K] = \begin{bmatrix} EI & 0.5EI \\ 0.5EI & 4.5EI \end{bmatrix}$$

Step: 5 Equilibrium equation.

$[K][\Delta] = P - P_L$

$$EI\begin{bmatrix} 1 & 0.5 \\ 0.5 & 4.5 \end{bmatrix}\begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 85.33 \end{bmatrix}$$

$$\theta_A = 10.03/EI$$

$$\theta_B = -20.08/EI$$

Step: 6 Final moment

$M_{AB} = 0$

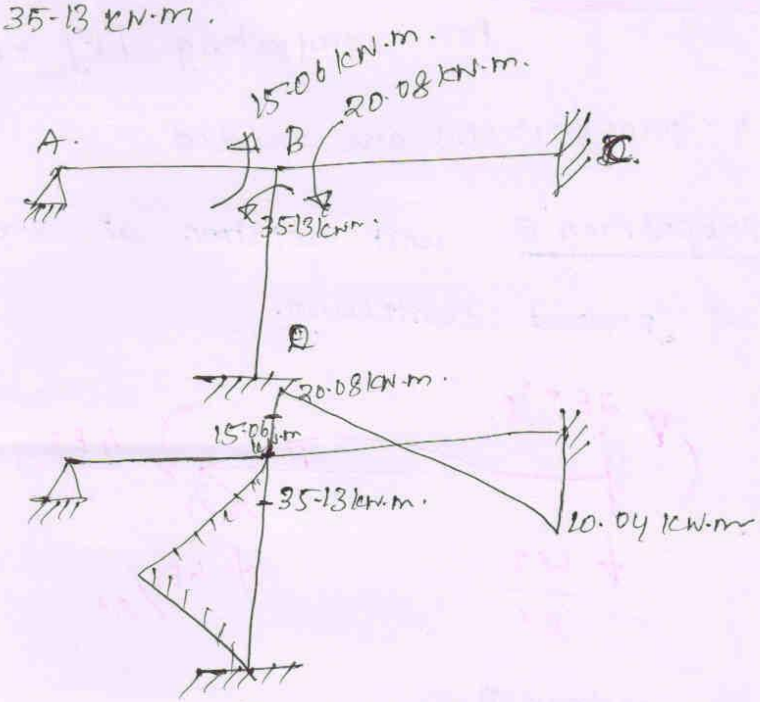
$M_{BA} = -15.06$ kN.m.

$M_{BC} = -20.08$ kN.m.

$M_{CB} = -10.04$ kN.m.

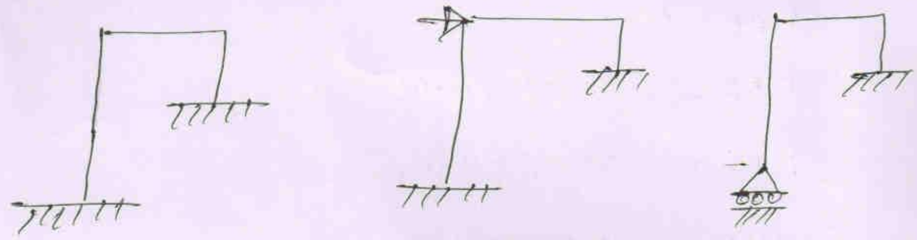
$M_{BD} = 35.13$ kN.m.

Step 7



FINAL BMD.

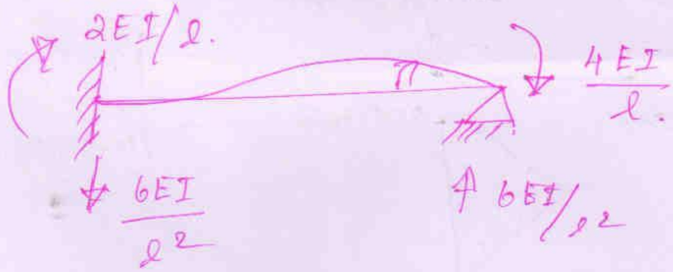
FRAMES WITH SWAY



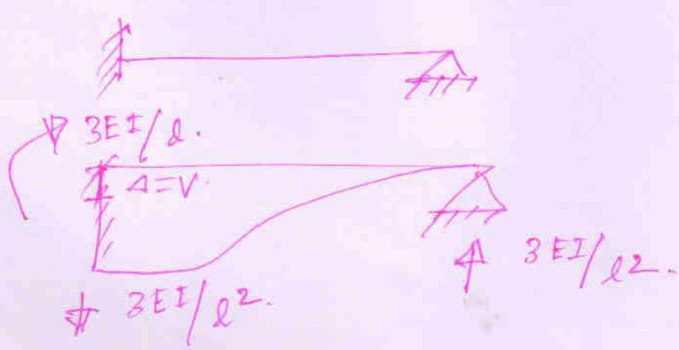
Proposition:

For computing [K] knowledge of 4 propositions are needed.

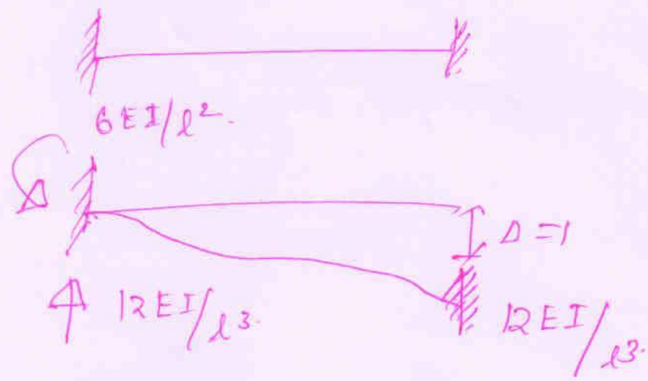
Proposition ① unit rotation at simply supported of proped cantilever.



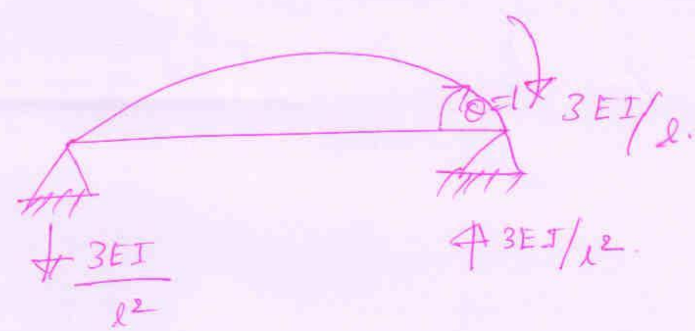
Proposition ② unit rotation (or) displacement at fixed end of propped cantilever

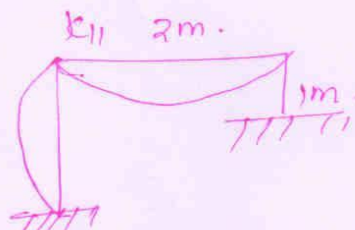


Proposition: unit translation at one end of the fixed beam.



② unit translation in simply supported beam:-

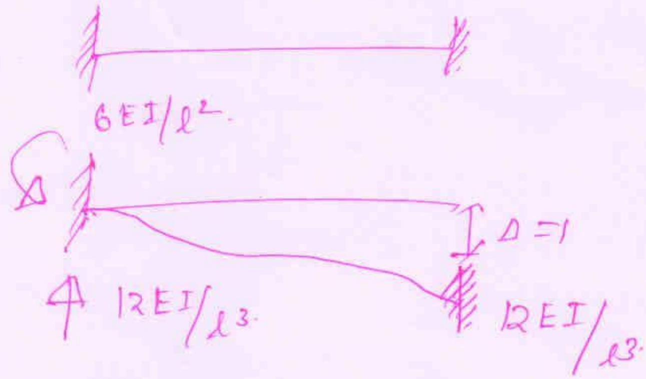




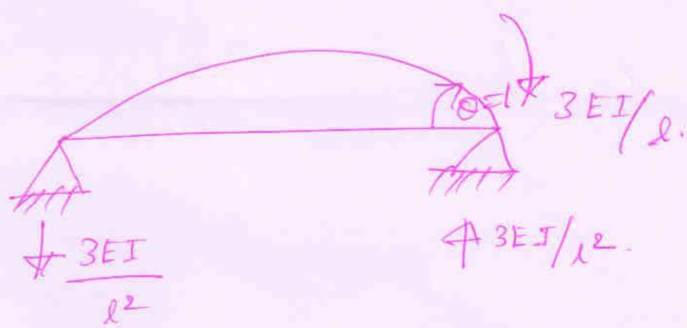
$$K_{11} = \frac{2EI}{l}\left[2\theta_B + \theta_A\right] + \frac{2EI}{l}\left[2\theta_B + \theta_C\right].$$

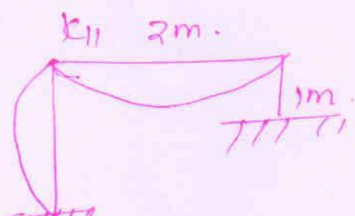
$$K_{11} = 8EI/l$$

Proposition: unit translation at one end of the fixed beam.



② unit translation in simply supported beam:-



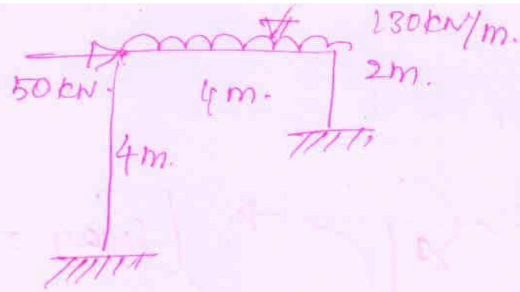


$K_{11} = \frac{2EI}{l}\left[2\theta_B + \theta_A\right]$

$+ \frac{2EI}{l}\left[2\theta_B + \theta_C\right]$.

$K_{11} = 8EI/l$

Pbm ①



Step:1 Sway θ_B, θ_C

$KI = 3$

Step:2 Fixed end moment

$M_{FAB} = 0 \quad M_{FBA} = 0$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-30 \times 4^2}{12} = -40 \text{ kN·m.}$$

$$M_{FCB} = \frac{+wl^2}{12} = \frac{30 \times 4^2}{12} = +40 \text{ kN·m.}$$

$M_{FCD} = 0 \quad M_{FDC} = 0.$

Step:3 $[P_L]$ Matrix.

$P_{1L} = 0.$

$P_{2L} = M_{FBA} + M_{FBC}$

$P_{2L} = -40.$

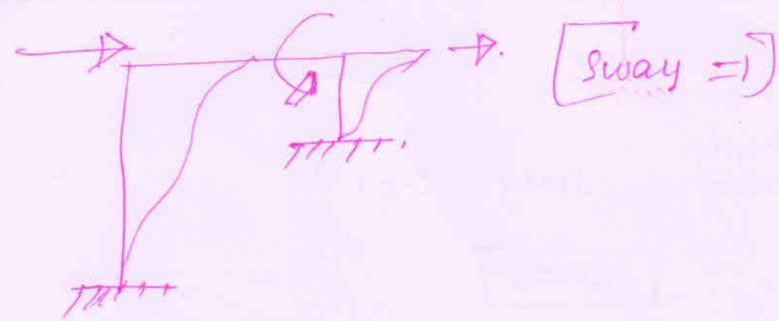
$P_{3L} = M_{FCB} + M_{FCD}$

$P_{3L} = 40 \text{ kN}.$

$$[P_L] = \begin{bmatrix} 0 \\ -40 \\ -40 \end{bmatrix}.$$

Step:4 [k].

(I) unit rotation at B

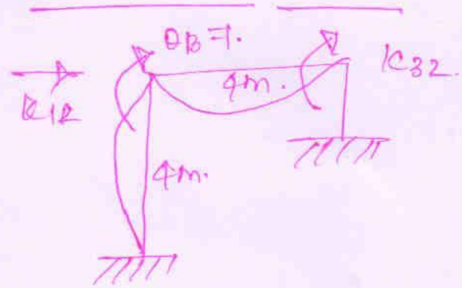


$$K_{11} = \frac{12EI}{L_1^3} + \frac{12EI}{L_2^3} = \frac{12EI}{4^3} + \frac{12EI}{2^3}.$$

$$K_{21} = \frac{-6EI}{l_1^2} = \frac{-6EI}{4^2}$$

$$K_{31} = \frac{-6EI}{l_2^2} = \frac{-6EI}{2^2}.$$

II unit rotation at B

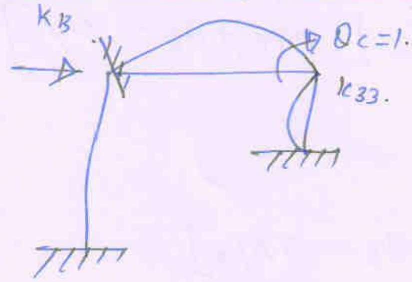


$$K_{12} = K_{21} = -0.375EI$$

$$K_{22} = \frac{2EI}{L}[2\theta_B+\theta_A] + \frac{2EI}{L}[2\theta_B+\theta_C]$$

$$K_{32} = \frac{1}{2}EI$$

III unit rotation at $[\theta_c = 1]$



$$K_{13} = K_{31} = -1.5 EI$$

$$K_{23} = \frac{2EI}{4}[2\theta_B + \theta_c]$$

$$= 1/2\, EI$$

$$K_{33} = \frac{2EI}{4}[2\theta_c + \theta_B] + \frac{2EI}{2}[2\theta_c + \theta_A]$$

$$K_{33} = 3EI$$

$$[K] = EI \begin{bmatrix} 1.687 & -0.375 & -1.5 \\ -0.375 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}$$

$$[K][\Delta] = [P] - [P_L]$$

$$[K] \begin{bmatrix} \Delta_1 \\ \theta_B \\ \theta_c \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -40 \\ 40 \end{bmatrix}$$

$$\Delta_1 = 35.57/EI$$

$$\theta_B = 26.67/EI$$

$$\theta_c = 0.$$

Final moment:-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - 3\Delta/l\right]$$

$$M_{AB} = 0 \text{ kN·m.}$$

$$M_{BA} = 0 + \frac{2EI}{4}\left[2\theta_B - 3\Delta/l\right].$$

$$M_{BA} = 13.33 \text{ kN·m.}$$

$$M_{BC} = -13.33 \text{ kN·m.}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - 3\Delta/l\right]$$

$$M_{CB} = 40 + \frac{2EI}{4}\left[(2\times 0) + 26.67 - 3\times 0\right]$$

$$\boxed{M_{CB} = 53.33 \text{ kN·m}}$$

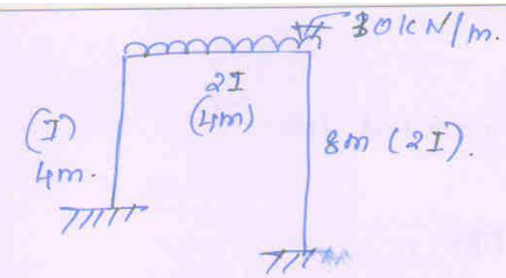
$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - 3\Delta/l\right]$$

$$M_{BC} = -40 + \frac{2EI}{l}\left[2\times 26.67\right]$$

$$\boxed{M_{BC} = -13.33 \text{ kN·m}}$$

$$\boxed{M_{CD} = -53.33 \text{ kN·m}}$$

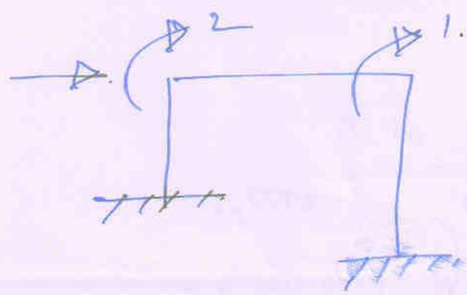
Pbm (2)



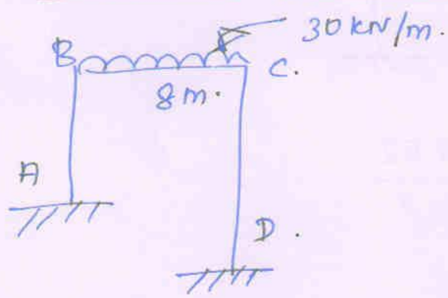
Step 1 unknowns (θ_B, θ_C, Δ)

KI = 3

Step 2 Assigned co-ordinates.



Step 3 Restrained structure



$M_{FAB} = 0.$ $M_{FBA} = 0.$

$M_{FBC} = -160 \text{ kN·m.} \left(-wl^2/12 = \frac{-30 \times 8^2}{12}\right)$

$M_{FCB} = \frac{+wl^2}{12} = \frac{+30 \times 8^2}{12} = +160 \text{ kN.m.}$

Step:4 $[P_L]$.

$P_{1L} = 0$ [No Horizontal force].

$P_{2L} = M_{FBA} + M_{FBC}$.

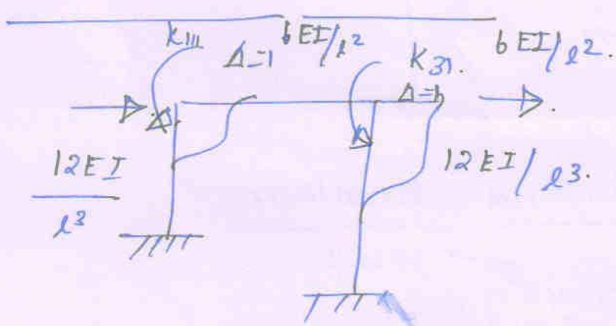
$P_{2L} = -160\,kN$.

$P_{3L} = M_{FCB} + M_{FCD} = +160\,kN.m$.

$$[P_L] = \begin{bmatrix} 0 \\ -160 \\ +160 \end{bmatrix}$$

Step:5 $[K]$.

① unit translation at ①



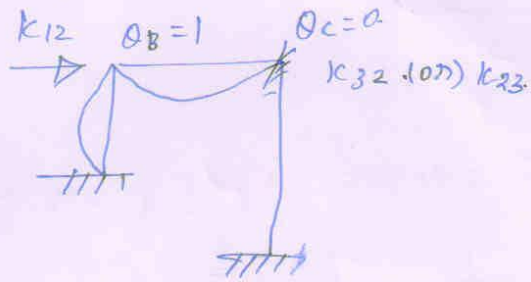
$$k_{11} = \frac{12EI}{L_1^3} + \frac{12EI}{L_2^3}.$$

$$k_{11} = \frac{12EI}{4^3} + \frac{12EI\,(2I)}{8^3}.$$

$$\boxed{k_{11} = 0.234\,EI}$$

$$k_{21} = \frac{-6EI}{l_1^2} = -0.375\,EI$$

(II) unit rotation at 2



k_{21} (or) $k_{12} = 0.375\,EI = -6EI/L_1^2$

$$\boxed{k_{21} = -0.375\,EI}$$

$k_{22} = M_{BC} + M_{BA}$

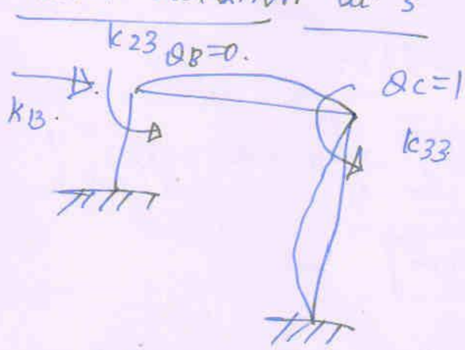
$$= \frac{2E(2I)}{8}\left[2\theta_B + \theta_C\right] + \frac{2EI}{4}\left[2\theta_B + \theta_A\right]$$

$$\boxed{k_{22} = 2EI}$$

$$k_{32} = \frac{2EI_2}{L_2}\left[2\theta_C + \theta_B\right]$$

$$\boxed{k_{32} = 0.5EI}$$

(III) unit rotation at 3



$$k_{13} \text{ (or) } k_{31} = \frac{-6E(2I)}{8^2} = -0.187\,EI$$

$$k_{23} = 0.5EI$$

$$k_{33} = M_{CB} + M_{CD}$$

$$= \frac{4EI}{8}[2\theta_C + \theta_B] + \frac{4EI}{8}[2\theta_C + \theta_D]$$

$$k_{33} = 2EI$$

$$[K] = \begin{bmatrix} 0.234 & -0.375 & -0.1875 \\ -0.375 & 2 & 0.5 \\ -0.1875 & 0.5 & 2 \end{bmatrix}$$

$$[K][\Delta] = [P] - [P_L]$$

$$\begin{bmatrix} 0.234 & -0.375 & -0.1875 \\ -0.375 & 2 & 0.5 \\ -0.1875 & 0.5 & 2 \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -160 \\ 160 \end{bmatrix}$$

$$\Delta_1 = 125.78/EI$$

$$\theta_B = 128.68/EI$$

$$\theta_C = -100.38/EI$$

$$M_{AB} = M_{FAB} + \frac{2EI}{4}[2\theta_A + \theta_B - 3\Delta/l]$$

$$M_{AB} = 17.17 \text{ kN·m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{4}[2\theta_B + \theta_A - 3\Delta/l]$$

$$M_{BA} = 81.51 \text{ kN·m}$$

$M_{BC} = -160 + \frac{4EI}{8EI}\left[2 \times 128.68 - 100.38\right]$

$\boxed{M_{BC} = -81.51 \text{ kN.m}}$

$M_{CB} = +123.96 \text{ kN.m.}$

$M_{CD} = -123.96 \text{ kN.m.}$ $M_{DC} = -73.77 \text{ kN.m.}$