UNIT - 4
LIMIT STATE DESIGN OF COLUMNS

Types of columns - Braced and unbraced columns - Design of short column for axial, uniaxial and biaxial bending - Design of long columns.

Limit State Design of columns :-

Column transmits load coming from beam/slab and distributes to the foundation.

Usually columns are square, rectangle and circular, L - shaped in cross - section.

It is reinforced with longitudinal and lateral ties.

Load carrying capacity of columns is depending upon longitudinal steel and c/s size of the column.

Lateral ties are giving lateral support to the longitudinal steel. The columns are analysed for axial force and moments.

Types of columns :-

Based on type of lateral ties
⇓
Based on materials used in c/s.
⇓
Based on slenderness ratio
⇓

Based on load

⇓

Based on sideway.

Based on type of lateral ties :-

i) Tied column

ii) Spiral columns.

Based on materials used in c/s :

- Composite columns
- Concrete filled steel tubes.

Based on slenderness ratio :-

- Long Column
- Short column
- Pedestal.

Based on load :-

- Axially loaded column
- Uniaxial Eccentrically loaded Column.
- Biaxial Eccentrically loaded column.

Based on Side way :-

- Braced columns
- Unbraced columns.

Limit State of collapse : Compression

Assumptions :-

(i) Plane sections normal to the axis remain plane after bending.

(ii) The maximum strain in concrete at the outermost compression fiber is taken as 0.0035 in bending.

(iii) For design purpose, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor for concrete, $\gamma_m = 1.5$ shall be applied in addition to this.

(iv) The tensile strength of the concrete is ignored.

(v) The maximum compressive strain in concrete in axial tension is taken as 0.002.

Design of short column for axial, uni-axial and bi-axial bending :-

Minimum Eccentricity :-

All columns should be designed for min. eccentricity, e_{min} due to the following reasons.

i) Lateral loads such as wind and seismic loads are not considered in design.

ii) Mis alignment in construction.

iii) Slenderness effects not considered in design.

iv) Accidental lateral / eccentric loads.

$$\text{Minimum eccentricity, } e_{min} = \frac{L}{500} + \frac{D}{30} \nless 20\text{mm}.$$

Short axially loaded column in compression :-

Load is acting exactly at the centroid of the column is called as axially loaded column.

As per clause 39.3, IS 456 - 2000, when the minimum eccentricity as per the previous equation does not exceed 0.5 times the lateral dimension, the member shall be designed as an axially loaded column.

Ultimate load carrying capacity of a short axially loaded column,

$$P_u = 0.45 f_{ck} A_c + 0.67 f_y A_{sc}.$$

Compression member with helical reinforcement :-

$$\frac{\text{Volume of helical rft}}{\text{Volume of core}} \nless 0.36\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y}$$

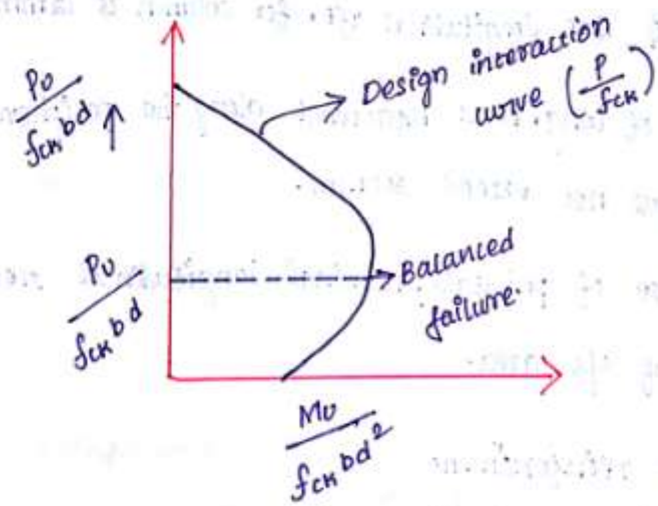
Members subjected to combined axial load and uni-axial bending :-

Axial load and bending moment along one direction (M_x/M_y) are applied simultaneously on the column is known as uni-axial eccentrically loaded column.

The design of column subjected to combined axial load and uniaxial bending will involve lengthy calculation by trial and error method.

In order to overcome these difficulties interaction diagrams are used.

Typical column interaction diagram :-



Members subjected to combined axial load and biaxial bending :-

$$\left[\frac{M_{ux}}{M_{ux_1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy_1}}\right]^{\alpha_n} \leq 1$$

Longitudinal reinforcement :-

a) Minimum area of longitudinal steel = 0.8 %

Maximum area of longitudinal steel = 6 % of G/A of column.

To avoid practical difficulties such as placing and compacting of concrete, generally 4 % of max. rft is provided.

b) The minimum number of longitudinal bars provided in a column shall be,

i) Rectangular column - 4 Nos

ii) Circular column - 6 Nos

c) Min. φ for longitudinal rft. for column is 12 mm.

d) Spacing of long. bars measured along the periphery of the column shall not exceed 300mm.

e) In case of pedestals, nominal longitudinal reinforcement ≮ 0.15 % of c/s area.

Transverse reinforcement

Lateral ties are provided in RC columns for the following purposes.

i) Lateral ties hold the main/longitudinal steel bars in position.

ii) It provides lateral support to main reinforcements against buckling.

Pitch and diameter of lateral ties :-

The pitch of transverse reinforcement shall not be more than the least of the following distances.

i) The least lateral dimension of the compression members.

ii) 16 times the smallest diameter of the long. reinforcement bar to be tied.

iii) 300mm.

The diameter of lateral ties shall not be less than,

i) Diameter of the long. reinforcement / 4.

ii) 5mm.

Helical Reinforcements :-

i) Spacing/pitch ≯ 75mm.

ii) Spacing ≯ diameter of core/6

iii) Spacing ≮ 25mm.

iv) Spacing ≯ 3 x diameter of helical rft.

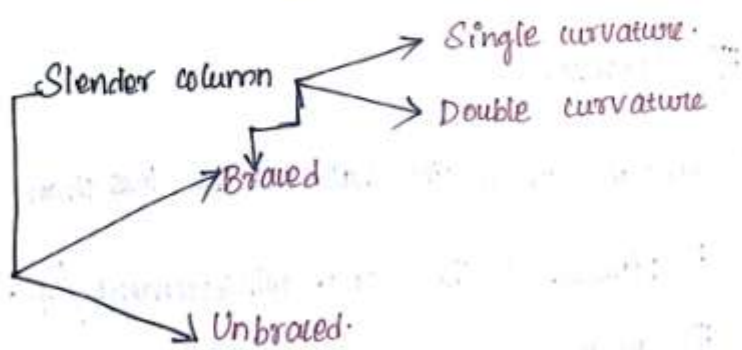
v) $p_s = \frac{\text{Volume of helical reinforcement}}{\text{Volume of core}} \not< 0.36\left(\frac{A_g}{A_{core}} - 1\right)\frac{f_{ck}}{f_y}$

A_g = Gross area of the section.

Slender Column / Long Column :-

If the slenderness ratio of the column about either axis is greater than 12, is classified as long column.

i) $\frac{l_{ex}}{D_x}$ (or) $\frac{l_{ey}}{D_y} > 12$.



Problems :-

(1) Design a square column subjected to an ultimate axial load of 1000 kN. Consider concrete grade M_{20} and steel grade Fe-415.

Given Data :-

Ultimate axial load, $P_u = 1000 \text{ kN} = 1000 \times 10^3 \text{ N}$.

Shape of column, [Square in c/s].

Grade of concrete - M_{20}, $f_{ck} = 20 \text{ N/mm}^2$.

Grade of steel - Fe-415, $f_y = 415 \text{ N/mm}^2$.

Solution :- $P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc} \longrightarrow (1)$

(a) Design of longitudinal reinforcement.

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}.$$

Assume, A_{sc} = 2% of gross area of column.

$$A_{sc} = 0.02 \times A_g.$$

$$A_{sc} = A_s + A_c = A_g.$$

$$A_c = A_g - A_{sc}$$

$$= A_g - 0.02 A_g$$

$$= 0.98 A_g.$$

$$1000 \times 10^3 = (0.4 \times 20 \times 0.98 A_g) + (0.67 \times 415 \times 0.02 A_g)$$

$$1000 \times 10^3 = 13.401 A_{sc}$$

$$A_g = 74621.30 \text{ mm}^2$$

$$\text{Size of column} = \sqrt{74621.30}$$

$$= 273.17 \text{ mm}$$

Provide 275mm x 275mm size of column.

Subs $A_g = 275 \times 275$ mm, $A_c = 0.98 A_g$ in (1)

$$1000 \times 10^3 = (0.4 \times 20 \times 0.98 \times 275 \times 275) + (0.67 \times 415 \times A_{sc})$$

$$A_{sc} = 1464.125 \text{ mm}^2.$$

Provide 16mm ϕ bars,

$$N = \frac{1464.125}{201} = 7.28 \simeq 8.$$

Provide 8 Nos - 16mm ϕ longitudinal reinforcement.

(b) Design of ties

ϕ of ties $> \frac{1}{4} \times \phi$ of main bar $= \frac{1}{4} \times 16$ $= 4$mm (or) 5mm } whichever is more.

Provide 6mm φ ties.

Spacing b/n ties can be taken from the least value of the following 4 cases,

i) Spacing = 48 x diameter of ties = 48 x 6 = 288mm.

ii) Spacing = 16 x diameter of long. rft = 16x16 = 256mm.

iii) Spacing = 300mm.

iv) Spacing = Least lateral dimension = 215mm.

Provide 6mm φ ties @ 255mm c/c.

Design a circular column subjected to an ultimate axial load of 800kN. Consider concrete grade of M_{25} and steel grade of Fe-415.

Given Data :-

Ultimate axial load (P_u) = 800 kN = 800×10^3 N.

Shape of column : circular in c/s.

Grade of concrete : $f_{ck} = 25 N/mm^2$.

$f_y = 415 N/mm^2$.

Solution :-

(a) Longitudinal reinforcement calculation :-

$$P_u = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_{sc}) \rightarrow (1)$$

$$A_{sc} + A_c = A_g.$$

$$A_{sc} = 0.02 \times A_g.$$

$$A_c = 0.98A_g$$

$$800\times10^3 = 1.05\left[0.4\times25\times0.98A_g\right) + \left(0.67\times415\times0.02A_g\right)$$

$$800\times10^3 = 1.05\left(9.8A_g + 5.561A_g\right).$$

$$A_g = 49600\,mm^2.$$

$$D \Rightarrow \frac{\pi}{4}\times D^2 = 49600$$

$$D = 251.20\,mm.$$

Provide 255mm ϕ column,

$$A_g = \frac{\pi}{4}(255)^2,\quad A_c = 0.98A_g.$$

$$800\times10^3 = 1.05\left\{\left(0.4\times25\times0.98\times\pi\times\frac{255^2}{4}\right) + 0.67\times415\times A_{sc}\right\}$$

$$800\times10^3 = 525515.61 + 291.95A_{sc}.$$

$$A_{sc} = 940.17\,mm^2.$$

Provide 16mm ϕ bars,

$$No = \frac{940.17}{201} = 4.67 \simeq 5.$$

Provide 6 Nos 16mm ϕ bars

(b) Design of ties :-

Provide 6mm ϕ spiral reinforcement.

i) Spacing / pitch $\not>$ 75mm.

ii) spacing $\not>$ ϕ of core/6.

Diameter of core = outer to outer distance b/n spirals.

$= D - 2\,(\text{clear cover})$.

$$\text{Spacing} = (D - 2 \times \text{Clear cover to the main rft}) / 6$$

$$\text{Spacing} = \frac{(255 - 2 \times 40)}{6} = 175/6.$$

$= 29.17\text{mm}.$

(iii) spacing $\not<$ 25 mm,

(iv) $\not<$ $3 \times \phi$ of helical rft. $= 3 \times 6 = 18\text{mm}$.

(v) $p_s \not< 0.36 \left(\frac{A_g}{A_c} - 1\right) \frac{f_{ck}}{f_y} \rightarrow (1)$.

$$P_s = \frac{(\pi \times 169) \times \left(\frac{\pi}{4} \times 6^2\right)}{\left(\frac{\pi}{4} \times (175)^2\right) \times \text{pitch}}$$

$$= \frac{0.62}{\text{pitch}} \nless 0.36 \left(\frac{A_g}{A_c} - 1\right) \frac{f_{ck}}{f_y}$$

$$\frac{A_g}{A_c} = \frac{\left(\pi/4 \times 225^2\right)}{\left(\pi/4 \times 175^2\right)} = 2.12.$$

Subs P_s & $\frac{A_g}{A_c}$ in equn ① :

$$\frac{0.62}{S_v} \nless 0.36 (2.12 - 1) \frac{25}{415}$$

$$\frac{0.62}{S_v} \nless 0.0243$$

$$S_v \ngtr 21.30 .$$

Provide pitch of spiral = 25mm (minimum pitch).

A short circular axially loaded column is effectively held in position @ both ends, restrained against rotation at one end. Length of the column is 6m and is to carry a characteristic load of 3000 kN. Assuming that the ends of the column are fully restrained, design the column, if it is to be made as a spirally reinforced column. Use M.. grade concrete and Fe-415 grade steel.

Given Data :-

Type of column : short circular column.

End conditions : Effectively held in position @ both ends, restrained against rotation at one end (one end fixed and the other end hinged).

Characteristic load, $P = 3000$ kN.

$P_u = 4500$ kN.

Unsupported length of column $L = 6m$.

$f_{ck} = 20 N/mm^2$.

$f_y = 415 N/mm^2$.

Solution :-

(a) Size of column calculation,

Based on short column criteria,

$$\frac{l_e}{D} < 12$$

Effective length of column, $l_e = \frac{L}{\sqrt{2}} = 0.707 \times 6000$

$= 4242.6mm$.

$$D > \frac{4242.6}{12} > 353.55 mm.$$

b) check for eccentricity :-

i) $e_{min} = \frac{L}{500} + \frac{D}{30} = \frac{6000}{500} + \frac{360}{30}$

$= 24mm \nless 20mm.$

Hence $e_{min} = 24mm.$

ii) $e_{min} \leq 0.05D = 0.05 \times 360$

$= 180 mm.$

By comparing conditions i) and ii)

$e_{min} = 24mm \nless 0.05D.$

It is not satisfied. Hence it is not axially loaded column.

Try 750 mm size column ϕ.

$e_{min} = \frac{L}{500} + \frac{D}{30} = \frac{6000}{500} + \frac{750}{30} = 37 \nless 20mm$

Hence $e_{min} = 37mm.$

Also check, $e_{min} \leq 0.05D = 0.05 \times 750$

$= 37.5 mm$

$e_{min} = 37mm \nless 37.5mm$

It is axially loaded columns.

$$\frac{le}{D} = \frac{4842.6}{50} = 5.65 < 12,$$

It is a short column.

c) Longitudinal reinforcement calculation.

$$P_U = 1.05\left(0.4 f_{ck} A_c + 0.67 f_y A_{sc}\right).$$

$$A_g = \frac{\pi}{4}(750)^2 = 441786.47 mm^2.$$

$$= 1.05\{0.4 \times 20 \times (441786.47 - A_{sc}) + 0.67 \times 415 \times A_{sc})\}$$

$$\Rightarrow \frac{4500 \times 10^3}{1.05} - 3534291.76 = 270.05 A_{sc}$$

$$A_{sc} = 2782.53 mm^2.$$

Provide 25mm ϕ bars,

$$No = \frac{2782.53}{491} = 5.67 \simeq 6.$$

Provide minimum A_{sc} = 6 # 25mm ϕ bars.

d) Design of ties :-

Provide 8mm ϕ spiral for ties.

i) spacing $\ngtr$ diameter of core / 6 :

$$c = \frac{(750 - 2 \times 40)}{} = 670/6 = 111.67 mm.$$

iii) Spacing ≮ 25 mm

iv) Spacing ≮ 3 × φ of helical rft = 3×8 = 24 mm

v) $P_s \nless 0.36\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y} \longrightarrow (1)$

$P_s = \frac{\text{Volume of spiral in one loop}}{\text{Volume of core of concrete in one loop}}$

Volume of spiral = perimeter of spiral × Area of spiral rft

Perimeter = $\pi \times D_s$

D_s = Diameter of core − φ of helical rft

$D_s = 670 - 8 = 662$ mm

$$P_s = \frac{(\pi \times 662) \times \left(\frac{\pi}{4} \times 8^2\right)}{\left(\frac{\pi}{4} \times 670^2\right) \times \text{pitch}}$$

$$P_s = \frac{0.30}{\text{pitch}} = \frac{0.30}{S_v}$$

$$P_s \nless 0.36\left(\frac{A_g}{A_{core}} - 1\right)\frac{f_{ck}}{f_y}$$

$$\frac{A_g}{A_c} = \frac{\frac{\pi}{4} \times 750^2}{\frac{\pi}{4} \times 670^2} = 1.25$$

$+ 0.30 (1.25-1) \frac{20}{415} = 0.0043.$

Subs p_s & $\frac{A_g}{A_{core}}$ in eqn ①

$$\frac{0.30}{S_v} = 0.0043$$

$$S_v = 69.77mm.$$

Provide 8mm φ spiral @ 65mm pitch.

Design an uniaxially eccentrically loaded braced □ lr column for the following data.

Factored load P_U = 1500kN.

Factored uniaxial moment, M_{Ux} = 850 kN-m.

Unsupported length, L = 3.5m.

Eff. length, l_{ex} = 3m, L_{ey} = 2.75m.

Column size = 300 x 500 mm.

Assume M25 & Fe 415 combinations used.

Given Data :-

width of beam (b / Dx) = 500mm.

Overall depth of beam (D/Dy) = 300mm.

Ultimate axial load, P_U = 1500 kN.

Factored uniaxial moment, M_{ux} = 250 kN-m.

M_{uy} = 0 kN-m.

Eff. length, $l_{ex} = 3m$, $l_{ey} = 2.75m$.

$f_{ck} = 25 N/mm^2$, $f_y = 415 N/mm^2$.

Solution :-

ⓐ Check for eccentricity.

$$\frac{l_{ex}}{D_x} = \frac{3000}{500} = 6 < 12.$$

$$\frac{l_{ey}}{D_y} = \frac{2750}{300} = 9.17 < 12.$$

Additional moment calculations.

(i)

$$e_{min} = \frac{L}{500} + \frac{D_x}{30} = \frac{3500}{500} + \frac{500}{30}$$

$$= 23.67 \text{ mm} \nless 20 \text{ mm}$$

$$e_{x min} = 23.67 \text{ mm}$$

$$M_{ux} = P_u \cdot e_{x min} = 1500 \times \frac{23.67}{1000}$$

$$= 35.50 \text{ kN-m} < M_{ux}$$

Hence final $M_{ux} = 250$ kN-m.

ii) $$e_{y min} = \frac{L}{500} + \frac{b}{30} = \frac{3500}{500} + \frac{300}{30}$$

$$= 17 \text{mm} \nless 20 \text{mm}.$$

Hence $e_{y\,min} = 20mm$.

$$M_{uy_e} = P_u \cdot e_{y\,min} = 1500 \times \frac{20}{1000}$$

$$= 30\ kN \cdot m > 0kN\text{-}m.$$

$$M_{uy_e} = P_u \cdot e_{y\,min} = 1500 \times \frac{20}{1000}$$

$$= 30kN\text{-}m$$

$$\therefore M_{uy_e} = M_{uy} = 30kN\text{-}m.$$

b) Longitudinal reinforcement calculation :-

Provide nominal / clear cover = 40mm.

Eff. cover, $d' = \text{clear cover} + \frac{\phi}{2}$

$$= 40 + \frac{25}{2} = 52.5mm$$

$$\frac{d'}{D} = \frac{52.5}{5000} = 0.105 \simeq 0.1$$

$$\frac{P_u}{f_{ck}bD} = \frac{1500 \times 10^3}{25 \times 300 \times 500} = 0.4$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{250 \times 10^6}{25 \times 300 \times (500)^2} = 0.133$$

$$\frac{P}{f_{ck}} = 0.11$$

$= 2.75\%$

$$p = \frac{100 A_s}{bD}$$

$$A_{sc\ reqd} = \frac{pbD}{100} = \frac{2.75}{100} \times 300 \times 500$$

$$= 4125\ mm^2.$$

Provide 8 Nos - 25 mm ϕ and 4 Nos - 20 mm ϕ.

$$A_{sc\ (provided)} = (8 \times 491) + (4 \times 314)$$

$$= 5184\ mm^2.$$

c) Check for longitudinal reinforcement :

$$\left[\frac{M_{ux}}{M_{ux_1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy_1}}\right]^{\alpha_n} \leq 1.$$

Here

$M_{ux} = 250$ kN-m.

$M_{uy} = 30$ kN-m.

$$P_{prov} = \frac{100 A_s}{bD} = \frac{100 \times 5184}{300 \times 500}$$

$$= 3.46\%$$

i) $\frac{P_u}{f_{ck}} = \frac{3.46}{20} = 0.17$,

$\frac{d'}{D} = \frac{50}{500} = 0.10 \quad \left[d' = 40 + \frac{20}{2} = 50mm \right]$

$\frac{P_u}{f_{ck} bD} = 0.4, \quad \frac{M_{ux}}{f_{ck} b D^2} = 0.19.$

$M_{ux_1} = 0.19 \times 25 \times 300 \times 500^2 = 356.25 \times 10^6$ N-mm.

$= 356.25$ kN-m.

(ii) $\frac{P}{f_{ck}} = 0.17; \quad \frac{d'}{B} = \frac{50}{300} = 0.167.$

$\frac{M_u}{f_{ck} bD^2} = 0.168, \quad \frac{P_u}{f_{ck} bD} = 0.4$

$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$

$P_{uz} = \left[0.45 \times 20 \times (300 \times 500 - 5184)\right] + 0.75 \times 415 \times 5184$

$P_{uz} = 2916.86$ kN

$\frac{P_u}{P_{uz}} = \frac{1500}{2916.86} = 0.51$

$\alpha_n = 1.52$.

$$\left[\frac{250}{356.25}\right]^{1.52} + \left[\frac{30}{133.2}\right]^{1.52} = 0.687 \leq 1$$

Hence safe.

b) Design of ties

ϕ of ties $> \frac{1}{4} \times \phi$ of largest main bar $= \frac{1}{4} \times 25 = 6.25mm$

(or)

5mm

} whichever is max.

Provide 8mm ϕ ties,

spacing b/n ties can be taken from the least value of the following 4 cases.

(i) Spacing = $48 \times \phi$ of ties $= 48 \times 8 = 384mm$.

(ii) S = $16 \times \phi$ of long. rft = $16 \times 20 = 320mm$.

(iii) S = 300mm.

(iv) S = Least lateral dimension = 300mm.

Provide 8mm ϕ ties at 300mm c/c.