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UNIT-3
LIMIT STATE DESIGN FOR BOND ANCHORAGE, SHEAR & TORSION.

Behaviour of RC members in bond and Anchorage - Design requirements as per current code - Behaviour of RC beams in shear, and torsion - Design of RC members for combined bending shear and torsion.

Behaviour of RC members in bond and anchorage :-

RC. beam is a composite material which comprises of reinforcing steel and concrete with proper bond.

If this bond is inadequate, slipping of the reinforcing bar will occur, which destroy the composite action.

Bond between steel and concrete can be achieved by the following methods.

i) chemical adhesion - Cement is used as a binder in concrete, which sets and hardens independently, and can bind other materials such as aggregate and steel together.

ii) Frictional resistance - Friction b/n surface roughness of the reinforcement and grip exerted by the concrete.

iii) Mechanical interlock - Twisted (or) deformed bars can be used to increase the bond.

Bond stress :-

Stress developed @ the interface of steel bar and the surrounding is known as bond stress.

Factors affecting bond strength :-

(i) Plain bars are used for long. rft and stirrups.

(ii) Larger ϕ bars give less bond than smaller one.

(iii) Inadequate cover thickness and development length.

(iv) Rft. without L & U hooks.

(v) Lesser concrete grade.

Splices :-

When the available length of a bar is less than the length required, splices are provided. Splicing can be done by one of the following cases.

i) Lapping of bars

ii) Welding of bars - bars larger than 36mmφ should be avoided.

iii) Mechanical connections.

Shear Design :-

RC beams are designed to resist the shear forces resulting from external loads, after determination of bending reinforcement.

In beams, combined action of flexure and shear produces principal tensile and compressive stresses.

When the principal tensile stress exceeds tensile stress of concrete, formation of crack occurs.

Consider a 'ss' beam carrying UDL throughout its length, max. B.M occurs at mid-span where S.F is small. (3)

At mid-span; principal tensile stress is equal to the flexural tensile stress. Because of that flexural cracks will appear ⊥r to the axis of the beam at mid-span.

At the same case, B.M is minimum @ support where S.F is maximum. Near to the support, principal tensile stress is nearly equal to the shear stress and is acting 45° inclined to the axis of the beam.

It is called as diagonal tension and responsible for the development of inclined cracks.

Shear reinforcements are required near to the support to resist inclined cracks due to shear stress.

As per code, critical section located at a distance 'd' from the face of the support.

For a flexural member, max. S.F. occurs at face of the support and progressively reduces with increasing distance from the support.

Inclined cracks do not develop near the face of the support and progressively reduces with increasing distance with the support.

Gen. inclined crack appears at a distance 'd' from the face of the support.

Based on experimental results on RC ▭lr beam.

(i) 20 to 40% shear resisted by uncracked concrete.

(ii) 33 to 50% shear resisted by aggregate interlock.

(iii) 15 to 25% shear resisted by flexural reinforcement.

Torsional reinforcement in design:-

RC members may be subjected to torsion in combination with bending and shear.

Longitudinal and transverse reinforcement shall be provided for RC beams to resist torsion.

Ex :- curved beams.

Types of torsion :-

i) Statically determinate / Equilibrium torsion.

ii) Statically indeterminate / compatibility torsion.

Problem on Bond & Anchorage length :-

check for bond stress at the point of inflection of a continuous beam. If it is subjected to ultimate shear force of 300 kN at the point of inflection. Consider concrete of grade M_{20} and steel of grade Fe-415.

Given data :-

(i) Ultimate shear force, $V_U = 300$ kN.

(ii) Grade of concrete M_{20}, $f_{ck} = 20\ N/mm^2$.

(iii) Grade of steel, Fe-415, $f_y = 415\ N/mm^2$.

(8)

For anchorage bond stress not to exceed its value, the following condition @ the point of inflection should be satisfied.

Anchorage length of bar for continuous beam $\} = L_d \leq \frac{M_U}{V_U} + l_d$.

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{\phi \times 0.87 fy}{4 \tau_{bd}}$$

$$L_d = \frac{16 \times 0.87 \times 415}{4 \times 1.92} = 752.19 \text{ mm}.$$

(For deformed bars, $\tau_{bd} = 1.92 \ N/mm^2$).

M_U = Ultimate moment of resistance of the section.

$$P_t \text{ provided} = \frac{100 A_s}{bd} = \frac{100 \times 3 \times 201}{230 \times 450}$$

$$= 0.583 \ \%.$$

From Table 2, Sp-16

$$\frac{M_U}{bd^2} = 1.85$$

$$M_U = 1.85 \times 230 \times (450)^2$$

$$= 86.16 \times 10^6 \text{ N-mm}.$$

$$L_d = 450 \text{ mm} / 12 \times 16 = 192 \text{ mm}.$$

$$l_d = 450 \text{ mm}$$

$$\frac{M_U}{V_U} + L_d = \frac{86.16 \times 10^6}{250 \times 10^3} + 450$$

$$= 794.64 \text{ mm} > L_d.$$

∴ Point of deflection is within the safe limit.

A cantilever beam of span 1.5m subjected to UDL (factored load) of 60 kN/m including self weight. The c/s and longitudinal section of the beam. Assume M20 concrete and Fe-415 steel combination is used. Calculate development length for tension and compression zone.

Given Data :-

At free end,

Width of beam, b = 200mm.

Overall depth, D = 200mm.

At face of the support,

Width of beam, b = 200mm.

Overall depth, D = 300 mm.

L = 1.5m.

w_U = 60 kN/m.

f_{ck} = 20 N/mm², f_y = 415 N/mm².

Solution :-

For tension bars,

$$\text{Development length, } L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}.$$

$\tau_{bd} = 1.2 \times 1.6 = 1.92\ N/mm^2$. (4)

Design stress, $\sigma_s = 0.87 f_y$.

$\phi = 20mm$.

$$L_{d\,tension\ bars} = \frac{20 \times 0.87 \times 415}{4 \times 1.92}$$

$= 940.23 mm$.

(ii) For tension bars,

For compression zone, τ_{bd} value increased by 25%.

and for deformed bars, τ_{bd} value increased by 60%.

$$\tau_{bd} = 1.2 \times 1.6 \times 1.25 = 2.40 N/mm^2$$

Design stress, $\sigma_s = 0.87 f_y$.

$\phi = 16mm$.

$$L_{d(comp)} = \frac{16 \times 0.87 \times 415}{4 \times 2.40}$$

$= 601.75 mm$.

$$\text{Actual anchorage length} = \frac{A_{s\,req}}{A_{s(pro)}} \times L_d$$

$$M_{max} = \frac{wl^2}{2}$$

$$M_{max} = \frac{60 \times 1.5^2}{2} = 67.5 kN\cdot m = 67.5 \times 10^6 N\cdot mm$$

Width of beam, $b = 200mm$.

$D = 300mm$.

... = 250mm.

Eff. depth, $d = 300 - 25 - \frac{20}{2} = 265mm$.

$d' = 25 + \frac{16}{2} = 33mm$.

$\frac{M_u}{bd^2} = \frac{67.50 \times 10^6}{200 \times (265)^2} = 4.80\ N/mm^2$.

$\frac{d'}{d} = \frac{33}{265} = 0.125$

$\therefore p_t = \frac{1.583 + 1.620}{2} = 1.60\%$.

$p_c = \frac{0.66 + 0.719}{2} = 0.69\%$.

$\therefore Ast_{(rqd)} = p_t \times bd = \frac{1.60}{100} \times 200 \times 265$

$= 848mm^2$.

$Ast_{(prov)}$ = 3 NOS - 20mmφ @ tension zone.

Actual development length, $L_d = \frac{848}{3 \times 314} \times 940.23$

$= 846.41mm$.

Provide development length of 850mm @ tension zone.

$Asc_{(req)} = p_c \times bd = \frac{0.69}{100} \times 200 \times 265 = 365.7mm^2$.

$Asc_{(prov)}$ = 2 NOS - 16mmφ @ compression zone.

$L_d = \frac{365.7}{2 \times 201} \times 601.75$

$L_d = 547.41mm$.

Provide development length of 550mm @ compression zone.

Determine the anchorage length of bars @ the ss end of reinforced concrete beam of overall size 300 x 450mm with 3 # 16mm ϕ at tension zone. The beam is subjected to an ultimate shear force of 200 kN @ the centre of the support. Consider concrete grade M20 and steel of grade Fe-415. Width of support = 300mm. (5)

Given Data :-

$b = 300mm$.

$D = 450mm$.

$A_{st} = 3 \# 16mm \phi$

$V_u = 200 kN$.

Width of support = 300mm.

$f_{ck} = 20N/mm^2$, $f_y = 415N/mm^2$.

Solution :-

i) Anchorage length of bar @ simply supported end of beam,

$$L_{d_1} \leq 1.3 \frac{M_u}{V_u} + L_d.$$

$$L_{d_1} = \frac{\phi \sigma_s}{4\tau_{bd}} = \frac{16 \times 0.87 \times 415}{4 \times 1.92} = 752.19.$$

$$L_{d_1} = 753mm.$$

$$d' = 25 + \frac{16}{2} = 33mm.$$

$$d = 450 - 33 = 417mm.$$

$$p_t = \frac{100 A_s}{bd} = \frac{100 \times 3 \times 201}{300 \times 417} = 0.482\%.$$

$$y = y_1 + \frac{(y_2-y_1)}{(x_2-x_1)}(x-x_1)$$

$$= 1.55 + \frac{(1.60-1.55)}{(0.494-0.477)} \times (0.482-0.477).$$

$$y = \frac{M_U}{bd^2} = 1.565 \ N/mm^2.$$

$$M_U = 1.565 \times 200 \times (417)^2 = 81.64 \times 10^6 \ N\text{-}mm.$$

$$L_{d_1} \leq 1.3 \times \frac{M_U}{V_U} + L_d.$$

$$753 \leq \frac{1.3 \times 81.64 \times 10^6}{200 \times 10^3} + L_d.$$

$$222.34 \leq L_d.$$

$$L_d = 223 \ mm.$$

(ii) Minimum length of bar to be extended beyond the centre of support

$$= \frac{L_{d_1}}{3} - \frac{\text{Width of support}}{2}$$

$$= \frac{753}{3} - \frac{300}{2} = 101 \ mm < 223 \ mm.$$

Provide anchorage length, $L_d = 225$ mm.

rectangular beam section of 300mm width and 450mm (1) effective depth is reinforced with 4 bars 20mm ϕ. Determine the shear reinforcement required to resist shear force of 40 kN. Consider concrete of grade M_{20} and steel of grade Fe-415.

Given Data :-

$b = 300mm$.

$d = 450mm$.

$A_{st} = 4 - 20mm\,\phi$

$= 1256mm^2$.

$V_u = 40\,kN$

Factored shear force, $V_u = 1.5 \times 40 \times 10^3$

$= 60 \times 10^3 N$.

$f_{ck} = 20\,N/mm^2$, $f_y = 415\,N/mm^2$.

Solution :-

Shear resistade by concrete, $V_{uc} = \tau_{uc} \times b \times d$.

% steel provided, $P = \frac{100 \times A_{st}}{b \times d}$

$= \frac{1000 \times 1256}{300 \times 450}$

$= 0.93\,\%$

p τ_c (M-20)

$x_1 = 0.75$ $y_1 = 0.56$

$x_2 = 1.00$ $y_2 = 0.62$

$x_3 = 0.93$ $y = ?$

$$y = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$$= 0.56 + \frac{(0.62 - 0.56)}{(1 - 0.75)}(0.93 - 0.75)$$

$\tau_c = 0.6032\ N/mm^2$.

$V_{uc} = 0.6032 \times 300 \times 450$

$= 81.432\ kN > V_u$.

Hence minimum shear reinforcement shall be provided.

Provide 2 legged - 8mm ϕ stirrups.

Spacing b/n shear rft can be calulated by,

(i) Spacing; $S_v = 2.175 \times 2 \times 50 \times \frac{415}{300} = 300.875mm$.

(ii) $S_v = 0.75 \times 450 = 337.5\ mm$.

(iii) $S_v = 300mm$.

2 legged - 8mm ϕ @ 300mm c/c stirrups.

A rectangular beam section of 300mm width and 450 mm effective depth is reinforcement with 4 bars of 20mm ϕ. Determine the shear reinforcement rqd to resist shear force of 140 kN. Consider concrete of grade M_{20} and steel of grade Fe-415. (7)

Given Data :-

$b = 300mm$.

$d = 450mm$.

$A_{st} = 4 \# 20mm \phi$.

$V = 140 kN$.

$V_U = 210 kN$

$f_{ck} = 20 N/mm^2$.

$f_y = 415 N/mm^2$.

Solution :-

Shear resisted by concrete, $V_{uc} = \tau_{uc} \times bd$

$$p_t = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 1256}{300 \times 450}$$

$$= 0.93 \%$$

P	$\tau_c (M_{20})$
$x_1 = 0.75$	$y_1 = 0.56$
$x_2 = 1$	$y_2 = 0.62$
$x_3 = 0.93$	$y = ?$

$\tau_c = 0.6032\ N/mm^2$.

$V_{uc} = 0.6032 \times 300 \times 450$

$= 81.432\ kN$.

Hence shear reinforcement is required.

Shear to be resisted by Steel $V_{us} = V_u - V_{uc}$

$= 210 - 81.432$

$V_{us} = 128.568\ kN$.

Provide 2 legged - 8mm ϕ stirrups.

(i) $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 415 \times 2 \times 50 \times 450}{128.568 \times 10^3}$

$= 126.37\ mm$.

(ii) $S_v = 2.175\ A_{sv} f_y / b$

$= 2.175 \times 2 \times 50 \times \frac{415}{300}$

$= 300.875\ mm$.

(iii) $S_v = 0.75d = 337.5\ mm$.

(iv) $S_v = 300\ mm$.

Provide 2 legged - 8mm ϕ stirrups @ 125mm c/c.

(8)

Design shear reinforcements for the RC beam of effective c/s of 300 x 450 mm with the following load conditions. Consider concrete of grade M_{20} and steel grade of Fe-415. Span of beam = 4m, U.D.L. over the beam = 80kN/m. (including S.W). Reinforcement details are,

A_{st} (support) = 2 # 25 mm ϕ

Ast (mid-span) = 2 # 25 mmϕ + 2 # 20mm ϕ.

Given Data :-

Eff. size of beam = 300 x 450 mm.

U.D.L = 80 kN/m.

Ast (support) = 2 # 25 mm ϕ.

Ast (mid-span) = 2 #25mmϕ + 2 # 20mm ϕ.

f_{ck} = 20 N/mm², f_y = 415 N/mm².

Solution :-

(i) End segment = 0.25 l

= 0.25 x 4.

= 1m from each support of a beam.

(ii) Interior segment = 0.25l to 0.75l from the support.

= (0.25x4) + (0.75x4)

= 1 to 3m from the support.

(i) End segment shear reinforcement design.

Gen. shear failure occurs @ d from face of the support.

Assume width of support = 280mm.

Hence end segments are designed for shear force at a distance of

$$= \frac{\text{bearing width}}{2} + d$$

$$= \frac{280}{2} + 450$$

= 565 mm from the centre of support.

S.F @ 0.565m from the centre of support can be calculated by $\parallel^{lr}$ Δ^{le} principle of S.F. diagram.

$$\frac{160}{2} = \frac{y_1}{1.435}$$

$$V = y_1 = \frac{160 \times 1.435}{2} = 114.8 \text{ kN}$$

$$V_U = 1.5 \times 114.8$$

$$= 172.20 \text{ kN}$$

$$p = \frac{100 \times A_{st}\,(\text{support})}{bd}$$

A_{st} (support) = 2 # 25 mm ϕ

$= 2 \times 491 = 982 \text{ mm}^2$.

$$p = \frac{100 \times 982}{200 \times 450} = 0.78\%$$

(9)

P τ_c

$x_1 = 0.50$ $y_1 = 0.48$

$x_2 = 0.75$ $y_2 = 0.56$

$x = 0.73$ $y = ?$

$\tau_c = 0.55\ N/mm^2$.

$V_{uc} = \tau_{uc} \times b \times d$

$= 0.55 \times 300 \times 450$

$= 74.25\ kN$.

$V_{us} = V_u - V_{uc}$

$= 172.20 - 74.25$

$= 97.95\ kN$.

Provide 2 legged - 8mm ϕ stirrups /

(i) $S_v = 165.87\ mm$.

(ii) $S_v = 300.875\ mm$.

(iii) $S_v = 0.75\,d = 0.75 \times 450$
$= 337.5\ mm$.

(iv) $S_v = 300\ mm$.

Provide 2 legged 8mm ϕ stirrups @ 165mm c/c for end segments.

Interior segment

Max. S.F @ interior segment can be calulated by $\parallel^{lr}$ Δ^{le} principle of S.F diagram.

$$\frac{160}{2} = \frac{y_2}{1}$$

$$V = y_2 = 80 \text{ kN}.$$

$$V_U = 80 \times 1.5 = 120 \text{ kN}$$

$$A_{st\,(mid)} = 2 - 25mm\phi + 2 \# 20 mm\phi.$$

$$p = \frac{100 A_{st(mid)}}{b.d} = \frac{100 \times 1610}{300 \times 450}$$

$$= 1.19 \%$$

P τ_c

$x_1 = 1$ $y_1 = 0.62$

$x_2 = 1.25$ $y_2 = 0.67$

$x = 1.19$ $y = ?$

$$\tau_c = 0.658 \text{ N/mm}^2.$$

$$V_{uc} = \tau_c \times b \times d$$

$$= 0.658 \times 300 \times 450$$

$$= 88.83 \text{ kN}.$$

$$V_{us} = V_U - V_{uc} = 120 - 88.83$$

$$= 31.17 \text{ kN}.$$

Provide 2 legged - 8mm ϕ stirrups. (10)

(i) Spacing, $S_v = 2.175\, A_{sv} f_y / b$

$= 2.175 \times 2 \times 50 \times \frac{415}{300}$

$= 300.875\text{mm}$.

(ii) Spacing, $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$

$= \frac{0.87 \times 415 \times 2 \times 50 \times 450}{31.77 \times 10^3}$

$= 521.25\text{mm}$.

(iii) $S_v = 0.75d = 337.5\text{mm}$.

(iv) $S_v = 300\text{mm}$

Provide 2 legged - 8mm ϕ stirrups @ 230mm c/c for interior span.