UNIT – II
MOVING LOADS & INFLUENCE DIAGRAM

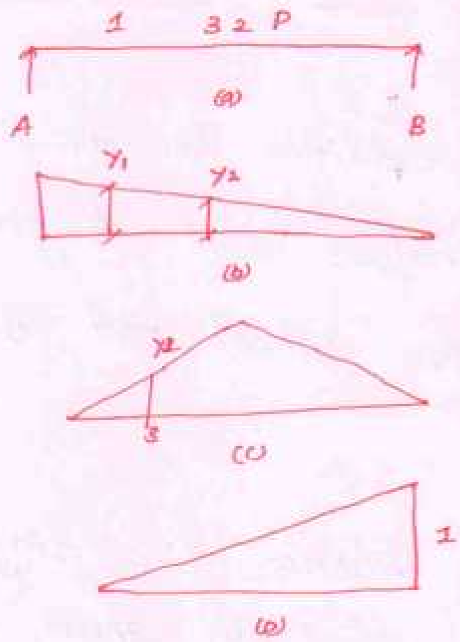
Introduction :-

In this addition you are working in a design office analyses and designing bridge griders for rolling wheel load will be done.

In an influence line diagram the ordinates show the BM, shear, reaction etc.. at chosen point in the structure say P and the abscissa (x co-ordinate).

Influence Line : ?

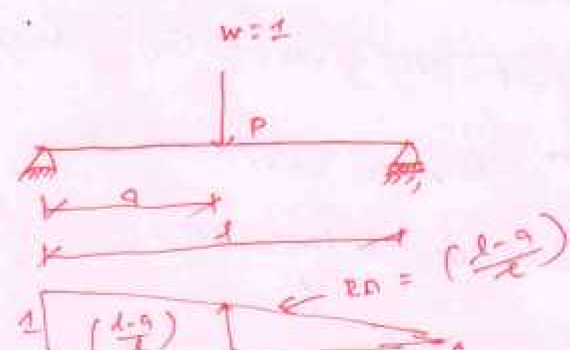
An influence line is a graph showing, for any given beam, frame or truss, the variation of any force (or displacement quantity (such as shear force, bending moment, tension & Deflection).



Influence lines are very useful in quickly determining the force component at any given point due to set of moving loads.

Getting the Influence Lines:-

External force like reactions are the easiest force.



①

Let us try to get the IL for RA for the beam AB in fig. Let a unit load act at P, distance a from A. Then

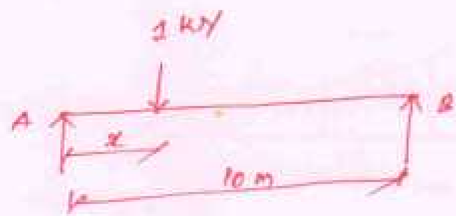
$$R_A = \left(\frac{l-a}{l}\right)$$

When we plot equation, that is the influence line for RA.

When will extend this reasoning to RB and load at $R_B = \frac{a}{l}$

Examples

Draw the influence line diagram for the reactions RA & RB for 2m, 4m, 6m.



Solution

$$\Sigma M_A = 0$$

$$(R_B \times 10) = x$$

$$R_B = \frac{x}{10}$$

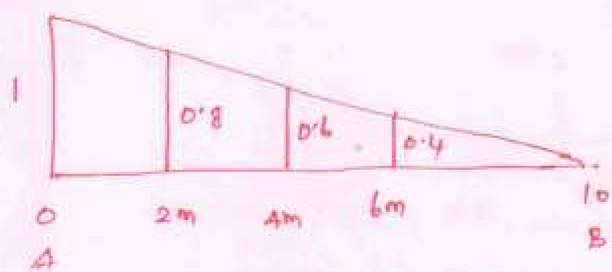
$\Sigma V = 0$

$R_A + R_B = 1$

$R_A = 1 - \frac{x}{10}$

$R_A = \frac{10-x}{10}$

x	RA	RB
0m	1	0
2m	0.8	0.2
4m	0.6	0.4
6m	0.4	0.6
8m		1
10m	0	



ILD For RA

②

Let us try to get the IL for RA for the beam AB in fig. Let a unit load act at P, distance a from A. Then

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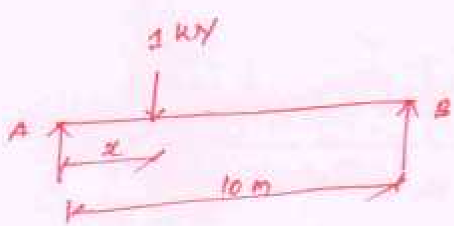
When we plot equation, that is the influence line for RA.

When will extend this reasoning to R_B and load at $R_B = \frac{a}{l}$

Examples

Draw the influence line diagram for the reactions RA & RB for 2m, 4m, 6m.



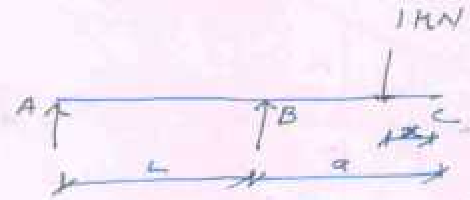
Solution

$\Sigma M_A = 0$

$(R_B \times 10) = x$

$R_B = \frac{x}{10}$

Case (ii)



$\Sigma M_A = 0$

$R_B \times L = 1 \times (L + a - x)$

$R_B = \frac{L+a-x}{L}$

$\Sigma V = 0$

$R_A + R_B = 1$

$R_A = 1 - \frac{L+a-x}{L}$

$R_A = \frac{x-a}{L}$

@ $x = 0$, $R_A = \frac{-a}{L}$

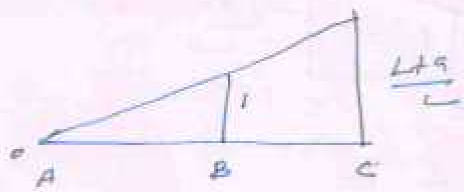
@ $x = a$, $R_A = 0$

@ $x = 0$, $R_B = L + a/L$

@ $x = a$, $R_B = 1$

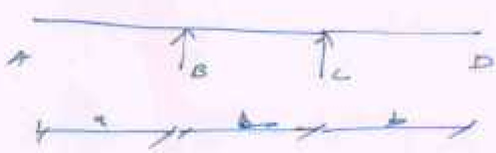
ILD for R_A

③



ILD for RB

3]



$\Sigma M_B = 0$

$1(a-x) + R_C \times L = 0$

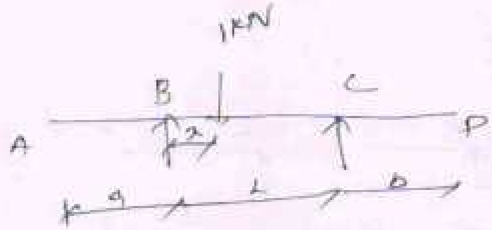
$R_C = \dfrac{-a+x}{L}$

$\Sigma V = 0$

$R_B + R_C = 1$

$R_B = \dfrac{L+a-x}{L}$

ILD for R_B



$\Sigma M_B = 0$

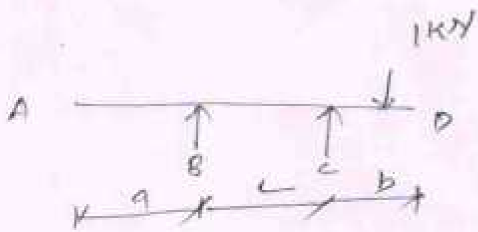
$R_C \times L = x$

$R_C = \frac{x}{L}$

$\Sigma V = 0$

$R_B + R_C = 1$

$R_B = \frac{L-x}{L}$



$\Sigma M_B = 0$

$R_C \times L = 1(L+b-x)$

$R_C = \frac{L+b-x}{L}$

(4)

$\Sigma V = 0$

$R_B + R_C = 1$

$$R_B = \frac{L - L - b + x}{L}$$

$$R_B = \frac{x - b}{L}$$

Span AB

@ $x = 0$, $R_C = \frac{-a}{L}$

$$R_B = \frac{L + a}{L}$$

@ $x = a$, $R_C = 0$

$R_B = 1$

Span BC

@ $x = 0$, $R_C = 0$

$R_B = 1$

@ $x = L$, $R_C = 1$

$R_B = 0$

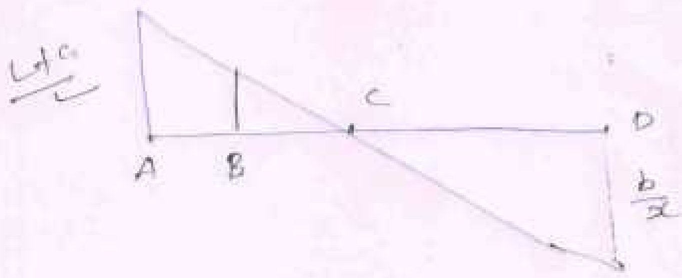
Span CD

$x = 0$, $R_C = \frac{L + b}{L}$

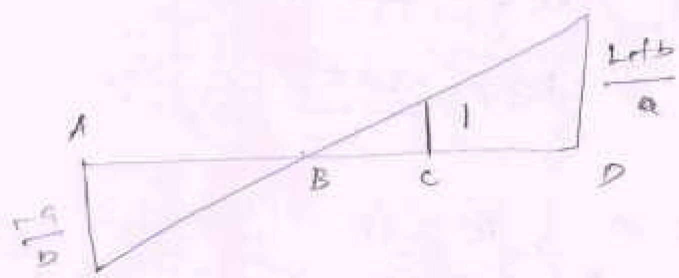
$$R_B = \frac{-b}{L}$$

@ $x = b$, $R_C = 1$

$R_B = 0$



ILD for RA



ILD for RC.

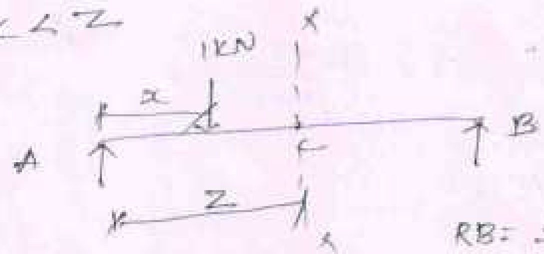
Draw the ILD for the following Condition.

* UDL longer than the span
* UDL Shorter than the span
* Train of loads rolls over the beam.

6

Singly Concentrated Load

i) $x < z$



$RB = \frac{x}{L}$

$RA = \frac{L-x}{L}$

$VC = RA - 1 = \frac{L-x}{L} - 1$

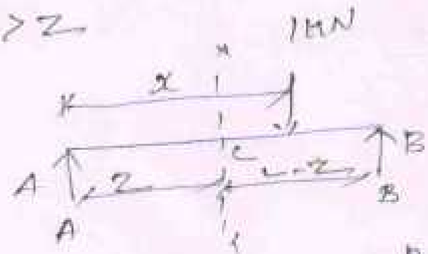
$VC = -\frac{x}{L} = -RB$

When $x < z$

when $x = 0$, $VC = 0$

$x = z$, $VC = -z/L$

ii) $x > z$



$RB = \frac{x}{L}$

$RA = \frac{L-x}{L}$

$VC = -RB + 1$

$= -\frac{x}{L} + 1$

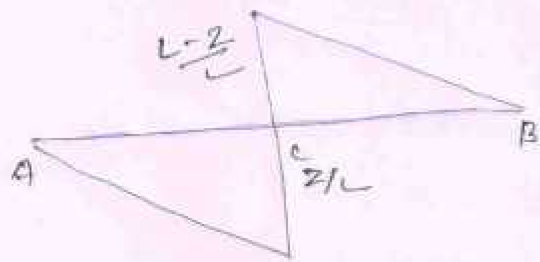
$Vc = \frac{L-x}{L} = RA$

$x > z$

When $x = z$, $Vc = \frac{L-z}{L}$

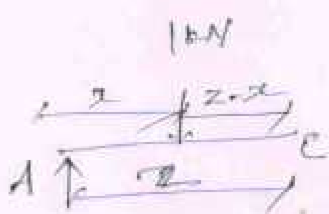
$x = L$, $Vc = 0$

$x = L1$, $Vc = 0$



ILD for VC.

Influence line Diagram for the SF @ point c.



$RA = \frac{L-x}{L}$

$Mc = RA\,z - 1(z-x)$

$= \left(\frac{L-x}{L}\right)z - (z-x)$

6

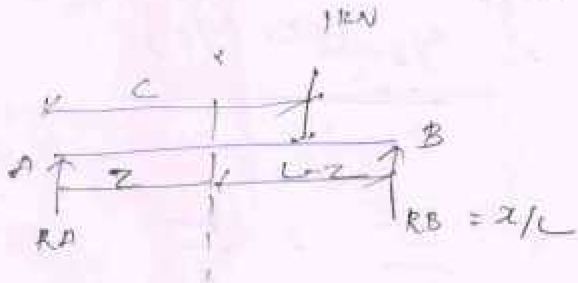
$$= \frac{Lz - zx - Lz + Lx}{L}$$

$$Mc = \frac{x(L-z)}{L}$$

$x < z$

When $x = 0$, $\quad Mc = 0$

$x = z$ $\quad Mc = \frac{z(L-z)}{L}$



$$Mc = RB(L-z) - 1(x-z)$$

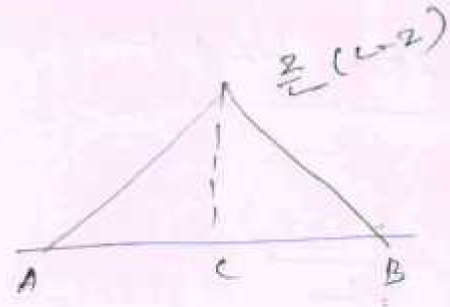
$$= \frac{x}{L}(L-z) - x + z$$

$$= \frac{xL - xz - Lx + Lz}{L}$$

$$Mc = \frac{z(L-x)}{L}$$

When $x = z$, $\quad Mc = \frac{z(L-z)}{L}$

$x = L$, $\quad Mc = 0$



@ $z = L/2$

$$M_C = \frac{z(L-z)}{L}$$

$$= \frac{L/2\,(L-L/2)}{L}$$

$$= \frac{L^2}{4L} = \frac{L}{4}$$

$$M_C = \frac{L}{4}$$

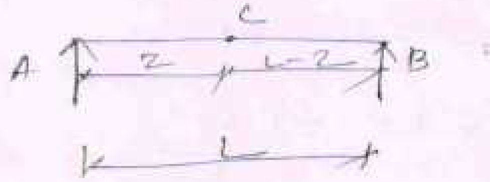
UDL for longer than the span:-

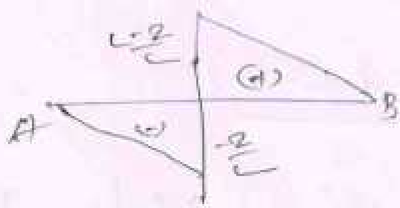
w - intensity of load move from left to right of the span.

Stress Resultant = Load intensity × Area of ILD ordinate over the loaded region.

(7)

Shear force





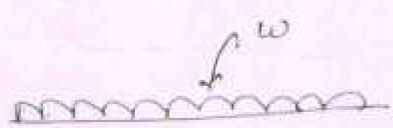
Maximum Negative SF $= w \times -\frac{z}{L} \times z \times \frac{1}{2}$

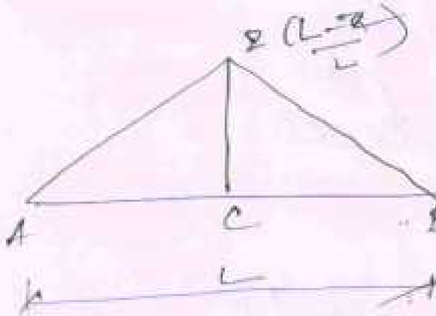
$$= -\frac{wz^2}{2L}$$

Maximum Positive SF $= w \times \frac{1}{2}(L-z) \cdot \left(\frac{L-z}{L}\right)$

$$= \frac{w(L-z)^2}{2L}$$

Bending moment





$$M_C = w \times \frac{1}{2} \times L \times \frac{z(L-z)}{L}$$

$$= \frac{wz(L-z)}{2}$$

Absolute maximum Value For Negative

SF @ $z = L$

$$= -\frac{wz^2}{2L} = -\frac{wL^2}{2L}$$

$$= -\frac{wL}{2}$$

Absolute Bending @ $z = \frac{L}{2}$

$$= \frac{w\frac{L}{2}\left(L-\frac{L}{2}\right)}{2}$$

$$= \frac{wL}{2} \times \frac{L}{2} \times \frac{1}{2}$$

$$= \frac{wL^2}{8}$$

Absolute max. Value for positive SF

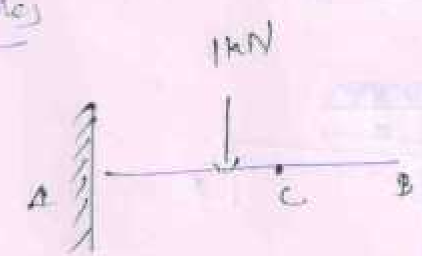
@ $z = 0$

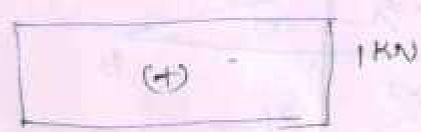
$$= \frac{w(L-0)^2}{2L}$$

$$= \frac{wL}{2}$$

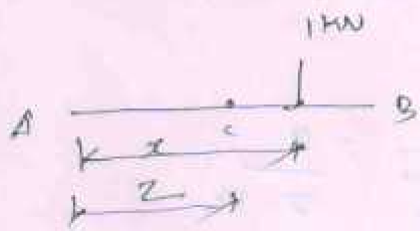
Examples

i)

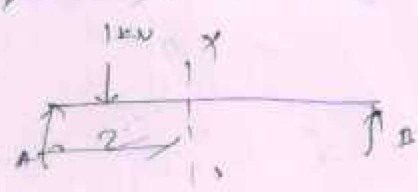




ILD for RA



$V_C = 1$ KN

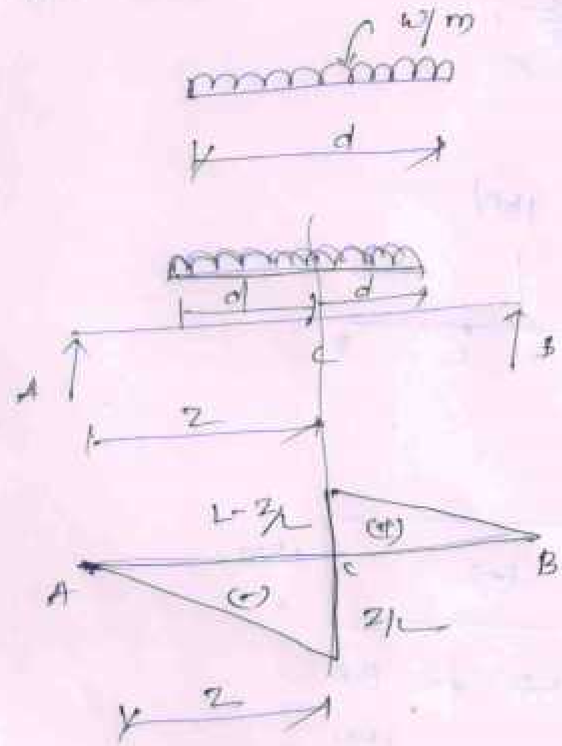


$VC = RA - 1$

$= 1 - 1$

$VC = 0$,,

2] UDL shorter than the span



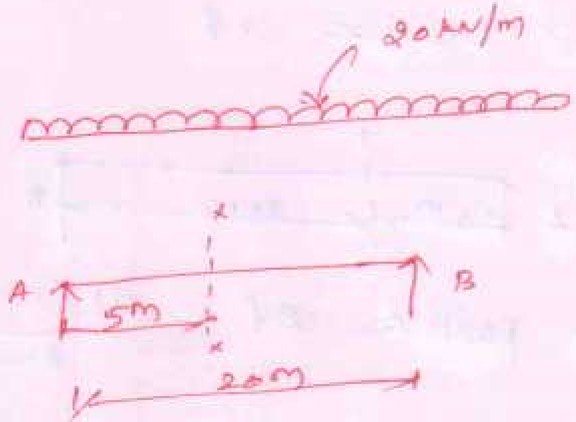
$$\frac{z/L}{z} = \frac{x}{z-D}$$

(9)

problem

In a simply supported girder of span 20m. Determine the maximum BM & SF at a section 5m from A, due to the passage of uniformly distributed load of intensity 20 kN/m longer than the span.

Solution

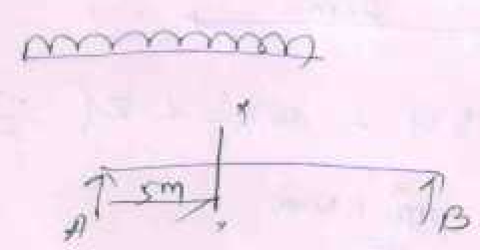


To find

(i) Max BM

(ii) Max S.F (Negative & Positive)

i) max. Negative S.F



Maximum Negative SF $= -\frac{wx^2}{2L}$

$$= -\frac{20\times5^2}{2\times20}$$

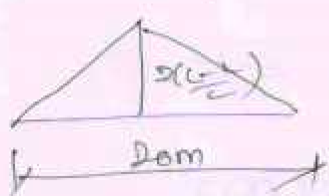
Maximum Negative S.F $= -12.5$ kN

Maximum positive SF

maximum positive SF $= \frac{w(L-x)^2}{2L}$

$$= \frac{20\,(20-5)^2}{2(20)}$$

Max. Positive SF $=$ 112.5 kN.



maximum BM $= w\times\frac{1}{2}\times2\left(\frac{L-x}{L}\right)\times L$

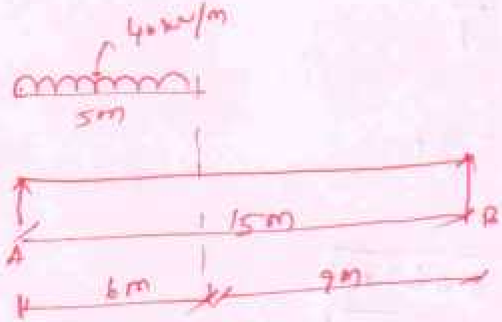
$= 750$ kN.m

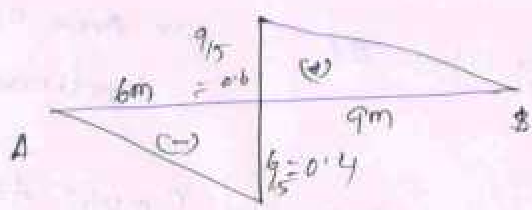
(18)

Problem

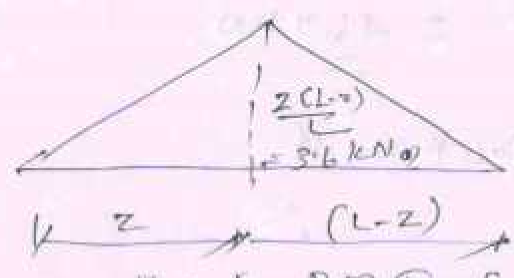
A simply supported beam has a span of 15 m uniformly distributed load of 40 kN/m and 5m long crosses the girder from left to right draw the ILD for S.F and B.M at a section 6m from left end use this diagram to calculate the max S.F and this diagram to calculate the max B.m at this section.
S.F and B.m at this section.

Sol





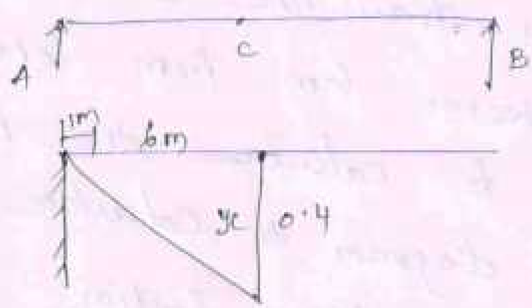
ILD for S.F @ C



ILD for BM @ C

(i) Maximum SF

Max. Negative SF



$$\frac{y_c}{2} = \frac{y_1}{1}$$

$$\frac{0 \cdot 4}{6} = y_1$$

$$\boxed{y_1 = 0 \cdot 067\ m}$$

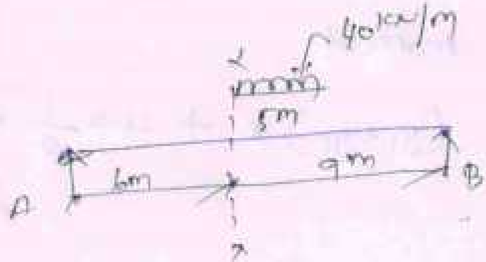
Max. Negative SF = w × Area of ILD ordinate

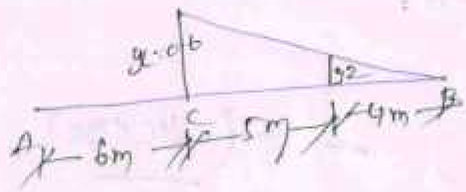
$$= 40 \times \frac{1}{2} \times (0 \cdot 067 + 0 \cdot 4) \times 5$$

= 46.7 kN

Max. Positive SF

(11)





$$\frac{y_c}{9} = \frac{y_2}{4}$$

$$\frac{4 \times 0.6}{9} = y_2$$

$$\boxed{y_2 = 0.267\,m}$$

Maximum positive M = w × Area of ILD ordinate

$$= 40 \times \frac{1}{2}\,(0.6 + 0.267) \times 5$$

$$= 86.67\ kN$$

Maximum Bending moment

$$Mc = w \times \frac{1}{2} (y_1 + y_c) x + w \times \frac{1}{2} \times (y_c + y_2)(d - x)$$

$$\frac{dM_c}{dx} = 0$$

$$w\left(\frac{y_1 + y_c}{2}\right) - w\left(\frac{y_c + y_2}{2}\right) = 0$$

$$y_1 + y_2 = y_c + y_2$$

$$\boxed{y_1 = y_2}$$

$$\frac{y_1}{z - x} = \frac{y_c}{z}$$

$$y_1 = \frac{y_c (z - x)}{z}$$

$$\frac{y_2}{(L - z) - (d - x)} = \frac{y_c}{L - z}$$

$$y_2 = y_c \frac{(L - z - d + x)}{L - z}$$

$$(L - z)(z - x) = Lz - z^2 - dz + xz$$

②

$$\cancel{L/2} - \cancel{z/2} + \cancel{z/2} - Lx = \cancel{L/2} - \cancel{z/2} - dz + \cancel{x/2}$$

$$\cancel{-}Lx = \cancel{-}dz$$

$$Lx = dz$$

$$\boxed{x = \frac{dz}{L}}$$

$$= \frac{5 \times 6}{15}$$

$$\boxed{x = 2\,m}$$

$$\frac{3.6}{6} = \frac{y_1}{4}$$

$$y_1 = \frac{3.6 \times 4}{6}$$

$$\boxed{y_1 = 2.4\,m}$$

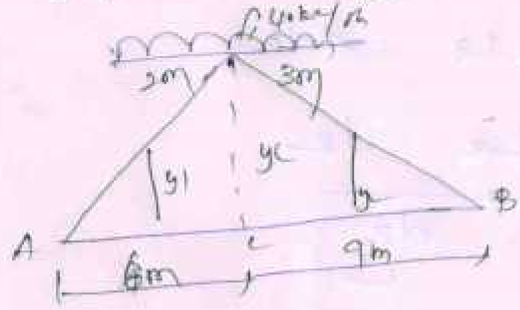
$$\frac{dy_c}{dz} = 0 \qquad y_c = \frac{z(L-z)}{L}$$

$$\frac{d}{dz}\left(\frac{z(L-z)}{L}\right) = 0$$

$$\frac{L - 2z}{L} = 0$$

$$z = L/2$$

Maximum B.M @ $z = 4_2$



$$\text{Max. BM} = 40 \times \left[\frac{1}{2} (2.4 + 3.6) \times 2 + \frac{1}{2} \times 3 (2.4 + 3.6) \right]$$

$$= 40 [15]$$

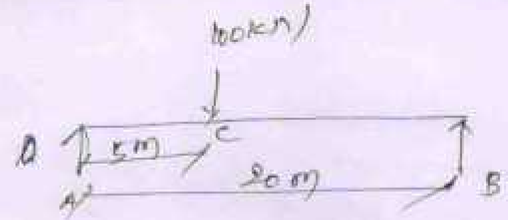
$$= 600 \text{ kN.m}$$

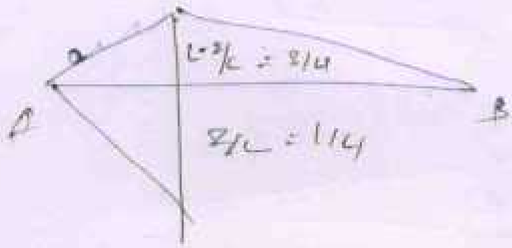
Problem

A single Load of 100 kN moves along a girder of 20 m span draw the diagram of max B.m and S.f at 5 m from the left support. What will be the absolute max positive Sf, max Negative S f and B.M.

Solution

(13)





Maximum Negative S.F = $\frac{Wz}{L} = \frac{100 \times 5}{20}$

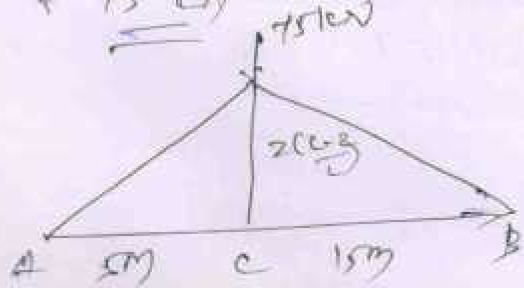
$= 25$ kN

Maximum positive S.F value at a section z = 5 m

$= \frac{W \times (L-z)}{L}$

$= \frac{100 \times (20-5)}{20}$

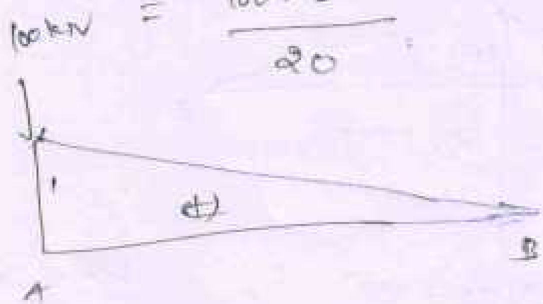
$= 75$ kN



Max. BM at a section C

$$= \frac{w \times z(L-z)}{L}$$

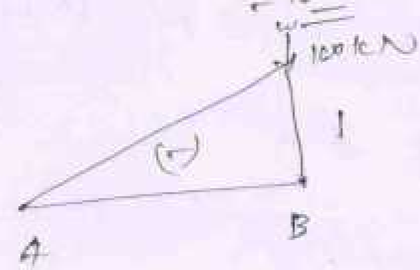
$$= \frac{100 \times 5 \times 15}{20}$$



@ z = 0 Absolute max positive SF

w = 1

= 100 kN



Absolute max BM @ z = L/2

$= 100 \times 20/4$

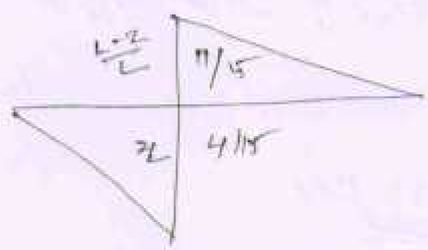
= 500 kN·m

$\boxed{B.M = 500}$

Problem

A system of concentrated loads shown in the fig rolls from left to right on the girder of span 15m. 90kN load leading, for a section 4m from the left support, Determine i) max BM ii) max S.F. What will be the absolute max S.F and B.M, when a set of this loads rolls from left to right.

Solution



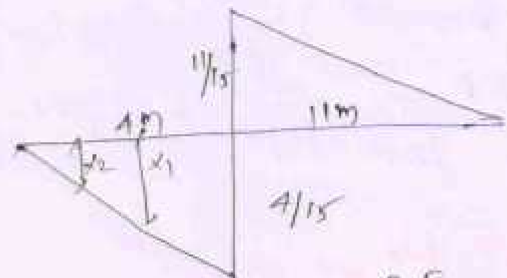
$$\frac{x_1}{3} = \frac{4}{15\times 4} \Rightarrow x_1 = \frac{3}{15}$$

$$\frac{x_2}{1} = \frac{4}{15\times 4} \Rightarrow x_2 = \frac{1}{15}$$

Max. Negative S.F as 40 kN at point C.

$= 40 \times \frac{4}{15} + 50 \times \frac{3}{15} + 60 \times \frac{1}{15}$

$= 24.67$ kN



$x_1 = 2/15$, $x_2 = \frac{0.5}{15}$, $x_3 = \frac{10 \times 11}{15 \times 11}$

$x_3 = 10/15$

Maximum Negative S.F as 50 kN at point C

$= 50 \times 4/15 + 60 \times 2/15 + 60 \times 0.5/15 -$

$40 \times 10/15$

$= -3.33$ kN

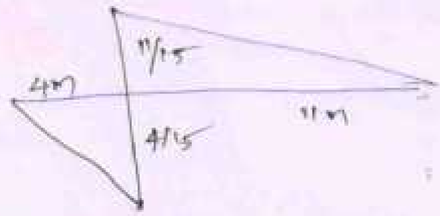
Maximum positive SF as 20 kN @ point C

$= \left(20 \times \frac{11}{15}\right) + \left(60 \times \frac{9.5}{15}\right) + \left(60 \times \frac{8}{15}\right) + \left(50 \times \frac{6}{15}\right)$

$+ \left(40 \times \frac{5}{15}\right)$

$= 118$ kN

(15)



Maximum positive SF at 60 kN @ point C

$$= \left(60 \times \frac{11}{15}\right) + \left(60 \times \frac{9.5}{15}\right) + \left(50 \times \frac{7.5}{15}\right) + \left(40 \times \frac{6.5}{15}\right) - \left(20 \times \frac{9.5}{15}\right)$$

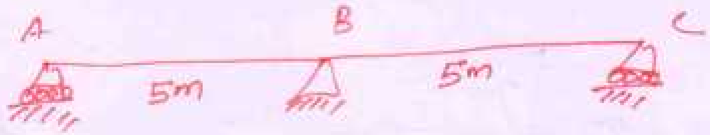
$= 121\ KN$

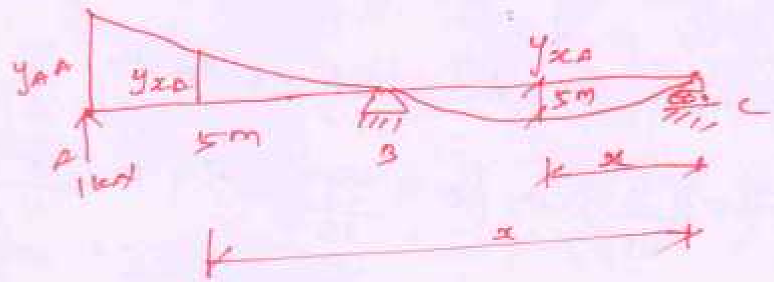
Muller Breslau's principle

⇒ 1886 → statically determinate Indeterminate structure Analysis

Problem:-

Draw the influence line for reaction at A for the continuous beam show in fig. Compute the influence line ordinates at 1m interval. (AUQ Nov/Dec 2014)





$\Sigma M_C = 0$

$(1 \times 10) + (R_B \times 5) = 0$

$5R_B = -10$

$R_B = -2\,kN$

$\Sigma V = 0$

$1 + R_B + R_C = 0$

$R_C = -1 + 2$

$R_C = 1\,kN$

$M_x = -EI \dfrac{d^2y}{dx^2}$

$M_x = 1 \cdot x \mid -2(x-5)$

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~~Mx~~ -

$$EI \cdot \frac{d^2y}{dx^2} = -x + 2(x-5)$$

$$\frac{dy}{dx} = \frac{1}{EI}\left[-\frac{x^2}{2} + C_1 + 2\left(\frac{x-5}{2}\right)^2\right]$$

$$y = \frac{1}{EI}\left[-\frac{x^3}{6} + C_1x + C_2 \;\Big|\; + \frac{2(x-5)^3}{3}\right]$$

Bcs at point C

$x = 0, \quad y = 0$

At point B

$x = 5m, \quad y = 0$

$x = 0, \; y = 0$

$$0 = \frac{1}{EI}\left[C_2 + \frac{(-5)^3}{3}\right]$$

~~$x=0$~~, $C_2 = 41.67$

$$0 = \frac{1}{EI}[C_2] \Rightarrow C_2 = 0$$

$x = 5, \quad y = 0$

$$0 = \frac{1}{EI}\left[-\frac{5^3}{6} + 5C_1\right]$$

$$5C_1 = \frac{5^3}{6}$$

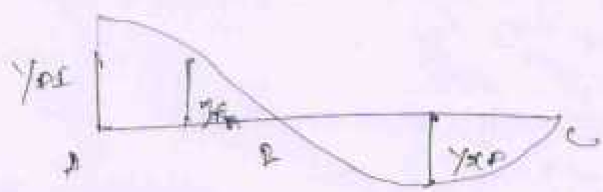
$C_1 = 4.167$

$$y = \frac{1}{EI}\left[-\frac{x^3}{6} + 4.167x + \frac{(x-5)^3}{3}\right]$$

$$\underset{(x=10m)}{y_{AD}} = \frac{1}{EI}\left[-\frac{10^3}{6} + (4.167\times10) + \frac{(10-5)^3}{3}\right]$$

$$= \frac{-83.33}{EI}$$

To get ILD ordinates at x



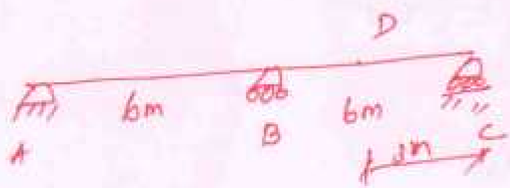
$$\frac{y_{xA}}{y_{AD}} = \frac{\frac{1}{EI}\left(-\frac{x^3}{6} + 4.167x + \frac{(x-5)^3}{3}\right)}{\frac{1}{EI}(83.33)}$$

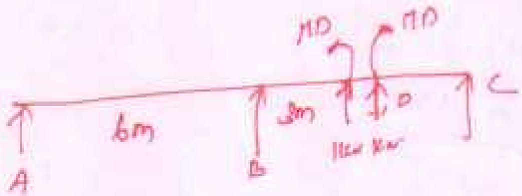
x	0	1	2	3	4	5	
R_A	0	-0.048	-0.08	-0.096	-0.072	0	
x	6	7	8	9	10		
						0	

(17)

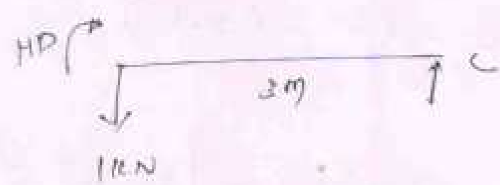
Problem

Determine the influence line for M at point D as mid point of span BC of a continuous beam shown in fig. Compute the influence line ordinate at every 1.5m interval.





Consider DC



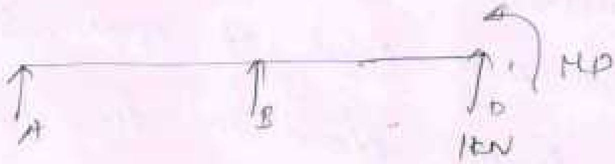
$\Sigma V = 0$

$R_C = 1\,kN$

$M_D = (1 \times 3)$

$M_D = 3\,kN \cdot m$

Consider ABD



$\Sigma M_A = 0$

$(R_B \times 6) + M_D + (1 \times 9) = 0$

$6 R_A = -12$

$R_B = -2\,kN$

$\Sigma V = 0$

$R_A + R_B + 1 = 0$

$R_A = -1 + 2$

$\boxed{R_A = 1\,kN}$

$M_x = (1 \times x) \mid -2(x-6) = -EI\left(\frac{d^2y}{dx^2}\right)$

$EI\frac{d^2y}{dx^2} = -x + 2(x-6)$

$EI\frac{dy}{dx} = \frac{-x^2}{2} + C_1 \mid + \frac{(x-6)^2}{2}$

$EI = \frac{-x^3}{6} + C_1x + C_2 \mid + \frac{(x-6)^3}{3}$

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Bcs,

At $x=0,\ y=0$

At $x=6,\ y=0$

At $x=0,\ y=0$

$$0=\frac{1}{EI}[C_2]$$

$$\boxed{C_2=0}$$

At $x=6,\ y=0$

$$0=\frac{1}{EI}\left[-\frac{6^3}{6}+6C_1\right]$$

$$6C_1=36$$

$$\boxed{C_1=6}$$

$$y_{xA}=\frac{1}{EI}\left[-\frac{x^3}{6}+6x \;\middle|\; +\frac{(x-6)^3}{3}\right]$$

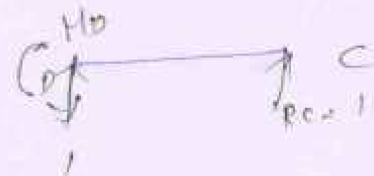
$$y_{(D)\,x=9m}=\frac{1}{EI}\left[-\frac{9^3}{6}+(6\times 9)+\frac{(9-6)^3}{3}\right]$$

$$=\frac{-58.5}{EI}$$

$$\theta_{xA}=\frac{1}{EI}\left[-\frac{x^2}{2}+6+(9-6)^2\right]$$

θ_{Dn} $x = 9m$ $= \frac{1}{EI}\left[-\frac{9^2}{2} + 6 \times 9 - 6^2\right]$

$= \frac{-25.5}{EI}$



$M_x = 3 - x = -EI\frac{d^2y}{dx^2}$

$EI\frac{d^2y}{dx^2} = -3 + x$

$EI\frac{dy}{dx} = -3x + \frac{x^2}{2} + C_3$

$EIy = \frac{-3x^2}{2} + \frac{x^3}{6} + C_3x + C_4$

Boundary Condition :

$x = 0$ $(\theta_{Dn}) = \theta_x$

$x = 3$ $y = 0$

$x = 0$

$$\theta_{DC} = \frac{1}{EI}\left[-3(0) + 0 + C_3\right]$$

$$= \frac{-25.5}{EI} = \frac{C_3}{EI}$$

$$\boxed{C_3 = -25.5}$$

$x = 3,\ y = 0$

$$0 = \frac{1}{EI}\left[\frac{-3 \times 3^2}{2} + \frac{3^3}{6} - (25.5 \times 3) + C_4\right]$$

$$C_4 = 13.5 - 4.5 + 76.5$$

$$\boxed{C_4 = 85.5}$$