

STRUCTURAL ANALYSIS - I

UNIT - I

Indeterminate Frames

Structural Analysis?

Structural analysis is the application of solid mechanics to predict the response [in terms of forces and displacement] of a given structure exposed subject to specified loads.

Statically Indeterminate Structure?

The structure for which the reaction at the supports and the internal force in the member cannot be found out by the condition of static equilibrium condition.

Types of indeterminacy?

- * Static indeterminacy [No of unknown force]
- * Kinematic indeterminacy [No. of available independent dof]

Degree of Static Indeterminacy?

The degree of static indeterminacy of the structure may be defined as the number of unknown force is excess of equations of statics.

It is also known as Degree of redundancy.

Degree of static indeterminacy = Total No. of unknown forces - No. of eqn of static available

$$D_s = D_{SE} + D_{SI}$$

The degree of indeterminacy of a beam is therefore equal to the external redundancy

$$D_s = D_{se}$$

as stated earlier

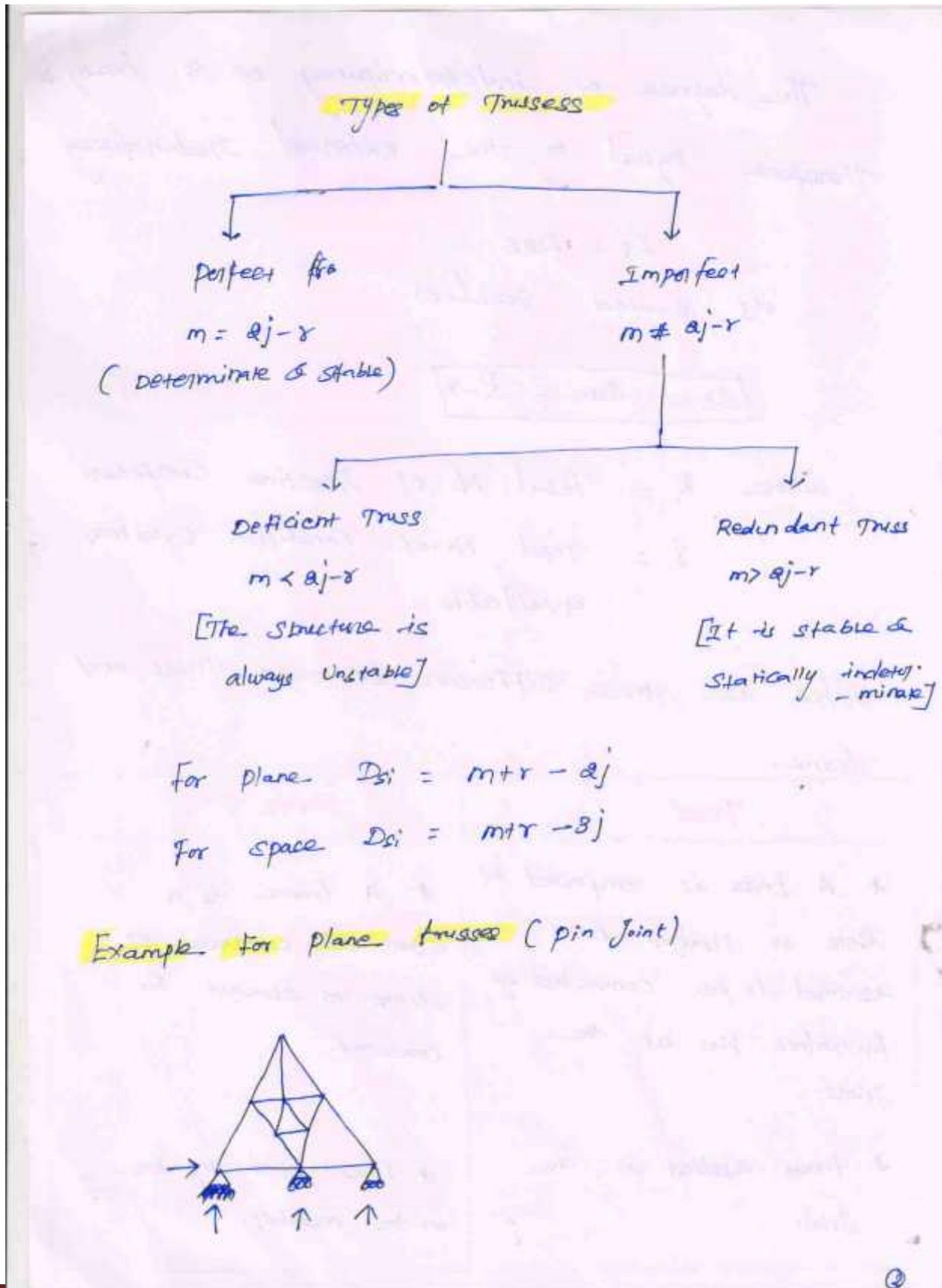
$$D_s = D_{se} = R - r$$

Where R = Total No. of Reaction Component

r = total No. of Condition Equation available.

What are the Difference between Truss and frame.

Truss	Frame
<ul style="list-style-type: none"> * A truss is composed of links or straight bars assumed to be connected by frictionless pins at the joints. * Forces applied at the joints 	<ul style="list-style-type: none"> * A frame is a structure composed of links or straight bars connected. * Forces acted anywhere on the member. 



$m = \text{member}$, $J = \text{Joint}$

$$m = 16, \quad J = 9$$

$$R = 4, \quad r = 3$$

$$\therefore D_{se} = R - r$$

$$= 4 - 3$$

$$= 1$$

\therefore Stable indeterminate First degree

$$D_{si} = (m+r) - 2j$$

$$= (16+3) - (2 \times 9)$$

$$= 1$$

\therefore Indeterminate First degree

$$D_s = D_{se} - D_{si}$$

$$= 1 + 1$$

$$= 2$$

\therefore indeterminate Second degree

\therefore Degree of Redundancy of plane frame

$$D_s = \underline{(3m+R)} - 3j$$

Degree of Redundancy of Space frame

$$D_s = \underline{6m + R} - 6j$$

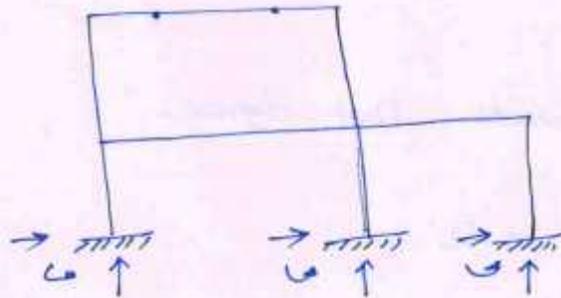
In case of hybrid structure

$$D_s = (3m - r_r) + R - 3(j + j')$$

r_r = No. of member connected to pin (or) hinge

j' = No. of hinges

Plane frame (Rigid Jointed)



$$m = 10, \quad j = 8$$

$$j' = 2, \quad R = 9$$

$$r_r = 3, \quad C = 1$$

$$D_s = (3m - r_r) + R - 3(j + j')$$

$$= (3 \times 10 - 3) + 9 - 3(8 + 2)$$

$$= 7$$

Principle of Superposition:-

Super position allows to separate the loads in any desired way. Analyse the structure for a separate set of loads

Types of Support

Hinged / pinned	—	
Roller	—	
Fixed	—	

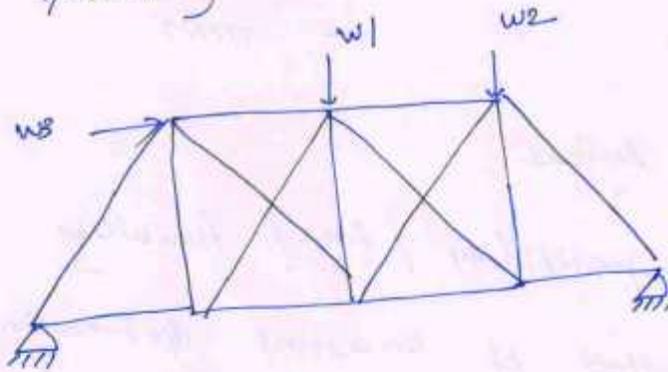
Method of Analysis:-

- ⇒ Compatibility / force / flexibility
- ⇒ method of consistent deformation.
- ⇒ Three moment theorem
- ⇒ Castiglione's theorem
- ⇒ Maxwell Mohr equation
- ⇒ Column analogy method
- ⇒ Elastic Centre method

- ⇒ Stiffness method
- ⇒ Slope Deflection
- ⇒ moment Distribution
- ⇒ minimum Potential Energy.

Examples

j) Find the static indeterminacy for the following structure. (Aug MAY/JUNE 2012)



$$\begin{aligned} D_{se} &= R - r \\ &= 4 - 3 \\ &= 1 \\ &= \end{aligned}$$

$$m = 15, \quad J = 8$$

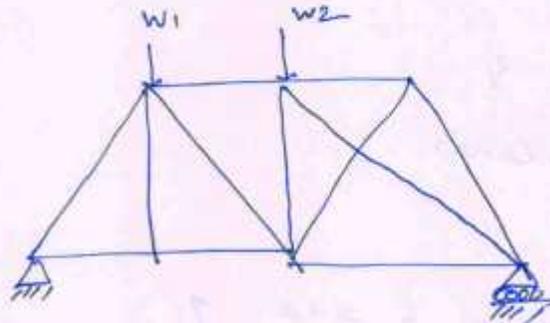
$$\begin{aligned} D_{si} &= m + r - 2j \\ &= 15 + 3 - 2 \times 8 \\ &= 2 \end{aligned}$$

$$D_s = D_{se} + D_{si}$$

$$= 1 + 2$$

$$D_s = 3$$

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$$D_{se} = R - r = 3 - 3 = 0$$

$$m = 12, \quad j = 7$$

$$D_{si} = (m + r) - 2j$$

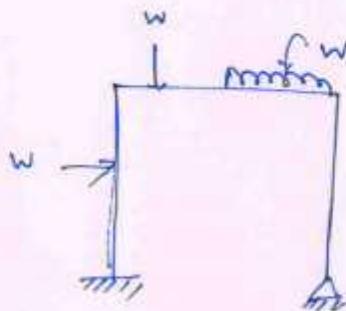
$$= (12 + 3) - (2 \times 7)$$

$$= 1$$

$$D_{se} = D_{se} + D_{si}$$

$$= 0 + 1 = 1$$

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$$D_{se} = R - r$$

$$= 5 - 3$$

$$= 2$$

$$m = 3, \quad j = 2$$

$$D_{si} = m + r - 2j$$

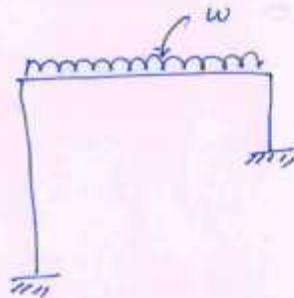
$$= 3 + 3 - (2 \times 2)$$

$$= 2$$

$$D_s = D_{se} + D_{si} = 2 + 2 = 4$$



4)



$$D_{se} = R - r$$

$$= 6 - 3$$

$$= 3$$

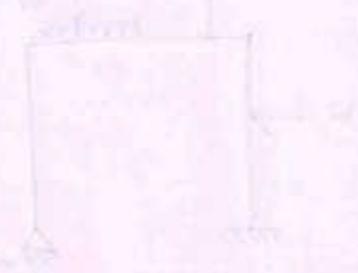
$$m = 3, \quad j = 2$$

$$D_{si} = m + r - 2j$$

$$= 6 - 2(2)$$

$$= 2$$

$$D_s = 3 + 2 = 5$$



Energy Method:-

- * Castigliano's Second Theorem
- * Principle of Virtual Work
- * Theorem of Least Work / minimum potential Energy.

The principle of least work / minimum strain energy theorem:-

$$\frac{\partial U}{\partial R_j} = 0 \quad j = 1, 2, \dots, n$$

$$\frac{\partial U}{\partial R_1} = 0 \quad \frac{\partial U}{\partial R_2} = 0$$

For a given pin jointed structure...

$$U = \frac{\sum PL^2}{2AE}$$

$$\frac{\partial \left(\frac{\sum PL^2}{2AE} \right)}{\partial R_i} = 0$$

$$\sum \frac{2PL}{2AE} \frac{\partial P}{\partial R_j} = 0$$

$$\sum \frac{PL}{AE} \frac{\partial P}{\partial R_j} = 0$$

frames and Beams

$$U = \int \frac{m^2 \cdot dx}{2EI}$$

$$\frac{\partial U}{\partial R_j} = 0 \Rightarrow \int \frac{2mx}{2EI} \frac{\partial mx}{\partial R_j} = 0$$

$$\frac{\partial U_{AB}}{\partial R_v} = \frac{1}{EI} \int_0^L (R_v \cdot x - \frac{wx^2}{2}) x \cdot dx$$

$$= \frac{1}{EI} \left(\frac{R_v x^3}{3} - \frac{wx^4}{8} \right)_0^L$$

$$\frac{\partial U_{AB}}{\partial R_v} = \frac{1}{EI} \left(\frac{R_v L^3}{3} - \frac{wL^4}{8} \right)$$

For span BC

$$\frac{\partial U_{BC}}{\partial R_4} = \frac{1}{EI} \int_0^L (R_4 x + R_v L - \frac{wx^2}{2}) x \cdot dx$$

$$= \frac{1}{EI} \left[R_4 \frac{x^3}{3} + R_v L \frac{x^2}{2} - \frac{wx^3}{4} \right]$$

$$\frac{\partial U_{BC}}{\partial R_H} = \frac{1}{EI} \left[\frac{R_H L^3}{3} + \frac{R_V L^3}{2} - \frac{w L^4}{4} \right]$$

$$= \frac{1}{EI} \left[R_H \frac{x^2}{2} + R_V L^2 x - \frac{w L x^3}{2} \right]$$

$$\frac{\partial U_{BC}}{\partial R_V} = \frac{1}{EI} \left[\frac{R_H L^3}{2} + R_V L^3 - \frac{w L^4}{2} \right]$$

$$\frac{\partial U}{\partial R_H} = 0$$

$$\frac{\partial U_{AB}}{\partial R_H} + \frac{\partial U_{BC}}{\partial R_H} = 0$$

$$0 + \frac{1}{EI} \left[\frac{R_H L^3}{3} + \frac{R_V L^3}{2} - \frac{w L^4}{4} \right] = 0$$

$$\frac{R_H L^3}{3} + \frac{R_V L^3}{2} - \frac{w L^4}{4} = 0$$

$$4 R_H L^3 + 6 R_V L^3 - 3 w L^4 = 0 \rightarrow \textcircled{1}$$

$$\frac{\partial U}{\partial R_V} = 0$$

$$\frac{\partial U_{AB}}{\partial R_V} + \frac{\partial U_{BC}}{\partial R_V} = 0$$

$$\frac{1}{EI} \left(\frac{R_V L^3}{3} - \frac{w L^4}{8} \right) + \frac{1}{EI} \left(\frac{R_H L^3}{2} + R_V L^2 - \frac{w L^4}{2} \right) = 0$$

$$\frac{1}{EI} \left[\frac{R_V L^3}{3} + R_V L^2 + \frac{R_H L^3}{2} - \frac{w L^4}{8} - \frac{w L^4}{2} \right] = 0$$

$$\frac{4 R_V L^3}{3} + \frac{R_H L^3}{2} - \frac{5 w L^4}{8} = 0$$

$$32 R_V L^3 + 12 R_H L^3 - 15 w L^4 = 0 \quad \text{--- (2)}$$

$$\textcircled{1} \Rightarrow 6 R_V L^3 + 4 R_H L^3 = 3 w L^4$$

$$\boxed{6 R_V + 4 R_H = 3 w L}$$

$$\textcircled{2} \Rightarrow 32 R_V L^3 + 12 R_H L^3 = 15 w L^4$$

$$\boxed{32 R_V + 12 R_H = 15 w L}$$

Solving $\textcircled{1}$ & $\textcircled{2}$

$$\textcircled{1} \times \textcircled{3} = 18 R_V + 12 R_H = 9 w L$$

$$\textcircled{2} = 32 R_V + 12 R_H = 15 w L$$

$$\begin{array}{r} 18 R_V + 12 R_H = 9 w L \\ \underline{32 R_V + 12 R_H = 15 w L} \\ 14 R_V - 6 w L \end{array}$$

$$R_V = \frac{3 w L}{7}$$

⑦

Sub in ①

$$6 \times \frac{3WL}{7} + 4RH = 3WL$$

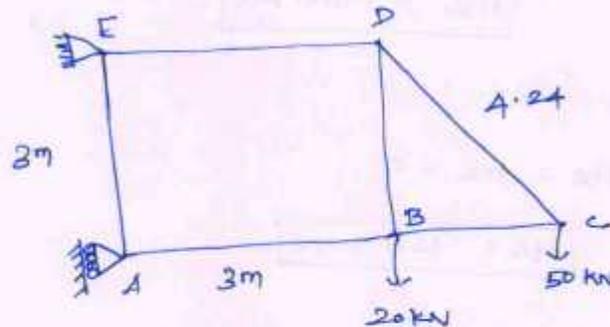
$$4RH = 3WL - \frac{18WL}{7}$$

$$4RH = \frac{3WL}{7}$$

$$RH = \frac{3WL}{28}$$



Find the axial force in the member AD as shown in fig. **AUG Nov/Dec - 2015**



All the members are of the same material and have the same cross sectional area.

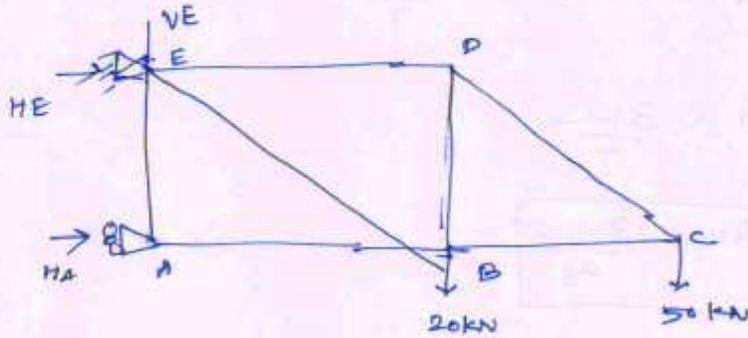
$$E \cdot I = R - \sigma = 3 - 3 = 0$$

$$I \cdot I = m + r - \omega$$

$$= 8+3 - 2(5)$$

$$= 11-10$$

$$= 1$$



$$\sum H_A = 0$$

$$H_E \times 3 + (20 \times 3) + (50 \times 6) = 0$$

$$3H_E = -360$$

$$H_E = -120 \text{ kN}$$

$$\sum H = 0$$

$$H_A + H_E = 0$$

$$H_A = 120 \text{ kN}$$

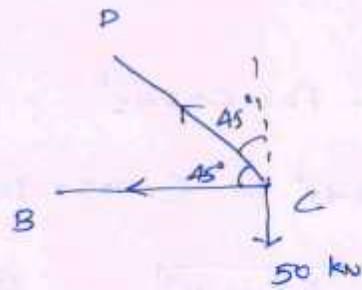
$$\sum V = 0$$

$$-V_E - 20 - 50 = 0$$

$$V_E = -70 \text{ kN}$$

Method of Joints

Joint C



$$\Sigma H = 0$$

$$-F_{BC} - F_{DC} \cos 45^\circ = 0$$

$$-F_{DC} \cos 45^\circ = +F_{BC}$$

$$F_{DC} \cos 45^\circ = -F_{BC}$$

$$\Sigma V = 0$$

$$-50 + F_{DC} \cos 45^\circ = 0$$

$$F_{DC} = \frac{50}{\cos 45^\circ}$$

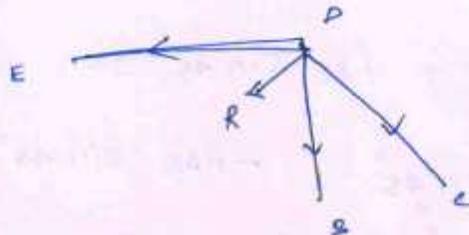
$$= \underline{70.71 \text{ kN T}}$$

$$-F_{BC} = -70.71 \times \cos 45^\circ$$

$$-F_{BC} = 50 \text{ kN}$$

$$F_{BC} = -50 \text{ kN (Comp)}$$

Joint D



$$\sum H = 0$$

$$-F_{ED} - R \cos 45^\circ + F_{DC} \cos 45^\circ$$

$$-F_{ED} = R \cos 45^\circ - (0.707 \times 70.71)$$

$$\boxed{F_{ED} = 50 - R \cos 45^\circ} \quad \text{--- (3)}$$

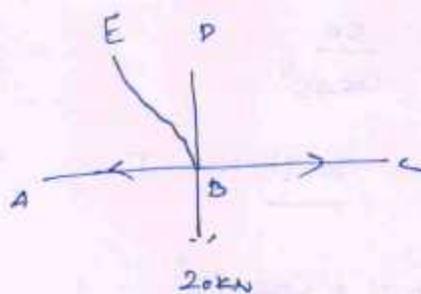
$$\sum V = 0$$

$$-F_{DB} - R \sin 45^\circ - F_{DC} \sin 45^\circ = 0$$

$$-F_{DB} = R \sin 45^\circ + 50$$

$$\boxed{F_{DB} = -R \sin 45^\circ - 50} \quad \text{--- (4)}$$

Joint B



$$\sum H = 0$$

$$-F_{BA} - F_{BE} \cos 45^\circ + F_{BC} = 0$$

$$-F_{BA} = 50 + F_{BE} \cos 45^\circ$$

$$\sum V = 0$$

$$-20 + F_{BD} + F_{BE} \sin 45^\circ = 0$$

$$-70 - R \sin 45^\circ = -F_{BE} \sin 45^\circ$$

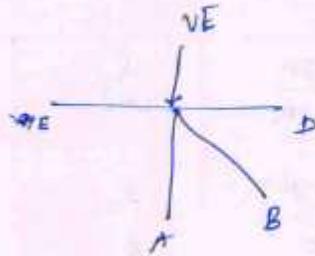
$$\boxed{F_{BE} = 98.99 + R} \quad \text{--- (5)}$$

$$F_{BA} = -50 - (98.99 + R) \cos 45^\circ$$

$$F_{BA} = -50 - 69.997 - R \cos 45^\circ$$

$$F_{BA} = -119.997 - R \cos 45^\circ \quad \text{--- (6)}$$

Joint E



$$\sum V = 0$$

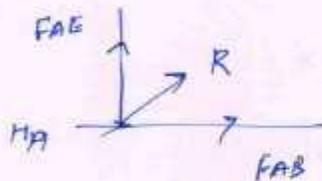
$$-V_E - F_{EA} - F_{EB} \cos 45^\circ = 0$$

$$70 - F_{EA} - 98.99 \cos 45^\circ - R \cos 45^\circ = 0$$

$$F_{EA} = 70 - 69.997 - R \cos 45^\circ$$

$$F_{EA} = -R \cos 45^\circ \quad \text{--- (7)}$$

Joint A



$$\sum V = 0$$

$$F_{AE} + R \cos 45^\circ = 0$$

$$F_{AE} = -R \cos 45^\circ$$

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$$\sum H = 0$$

$$H_A + F_{AB} + R \cos 45^\circ = 0$$

$$120 + F_{AB} + R \cos 45^\circ = 0$$

$$F_{AB} = -120 - R \cos 45^\circ$$

member	P (kN)	$\frac{\partial P}{\partial R}$	l (m)	P
AB	$-120 - 0.707R$	-0.707	3	$254.52 + 1.5R$
BC	-50	0	3	0
DC	70.71	0	4.24	0
DE	$50 - 0.707R$	-0.707	3	$-106.05 + 1.5R$
AE	$-0.707R$	-0.707	3	1.5R
BE	$98.99 + R$	1	4.24	$419.72 + 4.24R$
BD	$-50 - 0.707R$	-0.707	3	$106.05 + 1.5R$
AD	R	1	4.24	4.24R

$$674.24 + 14.48R$$

$$\sum = \frac{dP}{dR} = 0$$

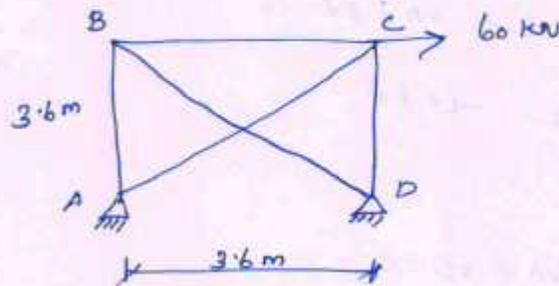
$$674.24 + 14.48R = 0$$

$$14.48R = -674.24$$

$$R = -46.56 \text{ kN}$$

$$R = 46.56 \text{ kN (Comp)}$$

Determine the forces in the member of the truss shown in fig. Axial rigidity is constant for all the members. (Aug Nov/Dec 2012)



$$D_{si} = m + r - 2j$$

$$= 5 + 3 - 2(4)$$

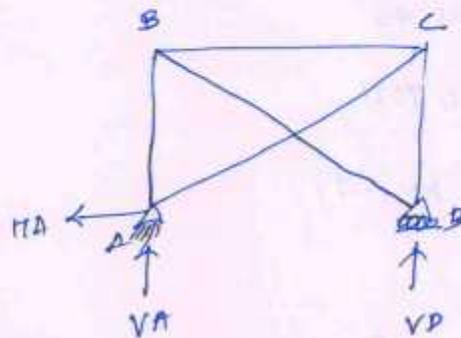
$$= 8 - 8$$

$$D_{si} = \underline{0}$$

$$D_{se} = R - r$$

$$= 4 - 3$$

$$D_{se} = \underline{1}$$



$$\Sigma H = 0$$

$$-H_A + 60 = 0$$

$$H_A = 60 \text{ KN}$$

$$\Sigma M_D = 0$$

$$60 \times 3.6 + V_A \times 3.6 = 0$$

$$V_A = -60 \text{ KN}$$

$$\Sigma V = 0$$

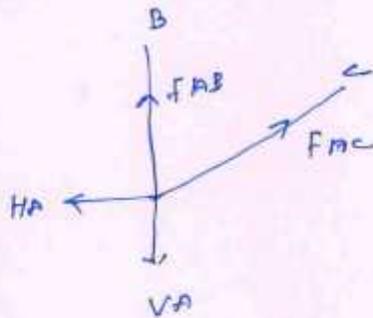
$$V_A + V_D = 0$$

$$V_A = -V_D$$

$$V_D = 60 \text{ KN}$$



Joint A



$$\Sigma H = 0$$

$$H_A = F_{AC} \cos 45^\circ$$

$$60 = F_{AC} \times 0.7071$$

$$F_{AC} = 84.85 \text{ KN (T)}$$

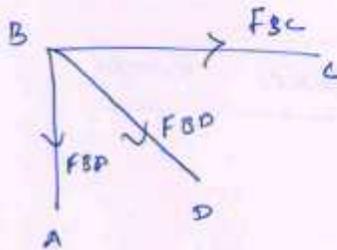
$$\sum V = 0$$

$$F_{AB} + F_{AC} \cos 45^\circ = V_A$$

$$F_{AB} + (84.85 \times 0.7071) = 60$$

$$F_{AB} = 0.002565 \text{ kN (T)}$$

Joint B



$$\sum V = 0$$

$$-F_{BA} - F_{BD} \cos 45^\circ = 0$$

$$F_{BD} \cos 45^\circ = -0.0026$$

$$F_{BD} = -0.0037 \text{ kN}$$

$$F_{BD} = 0.0037 \text{ kN (Comp)}$$

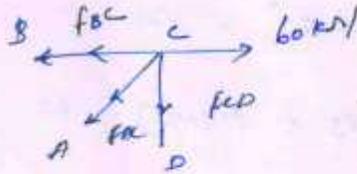
$$\sum H = 0$$

$$F_{BC} + F_{BD} \cos 45^\circ = 0$$

$$F_{BC} + 0 = 0$$

$$F_{BC} = 0 \text{ kN.}$$

Joint C



$$\sum V = 0$$

$$F_{CD} + F_{AC} \cos 45^\circ = 0$$

$$F_{CD} = -59.99 \text{ kN}$$

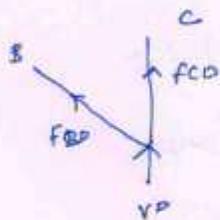
$$F_{CD} = 60 \text{ kN (Comp)}$$

$$\sum H = 0$$

$$60 - F_{BC} - F_{AC} \cos 45^\circ = 0$$

$$F_{BC} = 0 \text{ kN}$$

Joint D



$$\sum H = 0$$

$$F_{BD} \cos 45^\circ = 0$$

$$F_{BD} = 0$$

$$\sum V = 0$$

$$F_{CD} + F_{BD} \cos 45^\circ + V_D = 0$$

$$F_{CD} = -60 \text{ kN}$$

$$F_{CD} = 60 \text{ kN (Comp)}$$

$$\sum H = 0$$

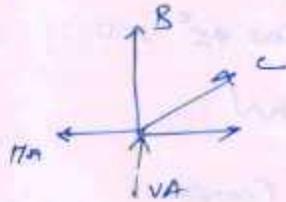
$$H_A = 1 \text{ kN}$$

$$\sum V = 0$$

$$V_A + V_D = 0$$

$$V_D = 0$$

Joint A



$$\sum H = 0$$

$$H_A = K_{AC} \cos 45^\circ$$

$$K_{AC} = 1.414 \text{ kN (T)}$$

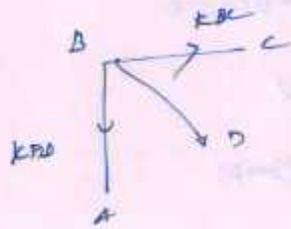
$$\sum V = 0$$

$$V_A + K_{AB} + K_{AC} \cos 45^\circ = 0$$

$$K_{AB} = -1.414 \times 0.707$$

$$K_{AB} = -1 \text{ kN} \quad \therefore K_{AB} = 1 \text{ kN (Comp)}$$

Joint B



$$\sum V = 0$$

$$K_{AB} + K_{BD} \cos 45^\circ = 0$$

$$K_{BD} = \frac{1}{\cos 45^\circ}$$

$$K_{BD} = 1.414 \text{ (KNCT)}$$

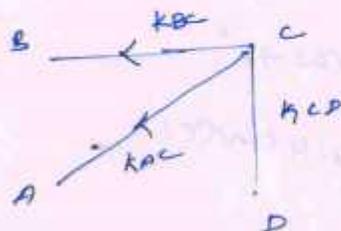
$$\sum H = 0$$

$$K_{BC} + K_{BD} \cos 45^\circ = 0$$

$$K_{BC} = -1 \text{ KN}$$

$$K_{BC} = 1 \text{ KN (Comp)}$$

Joint C



$$\sum V = 0 - K_{CD} + K_{AC} \cos 45^\circ = 0$$

$$K_{CD} = -1 \text{ KN}$$

$$K_{CD} = 1 \text{ KN (Comp)}$$

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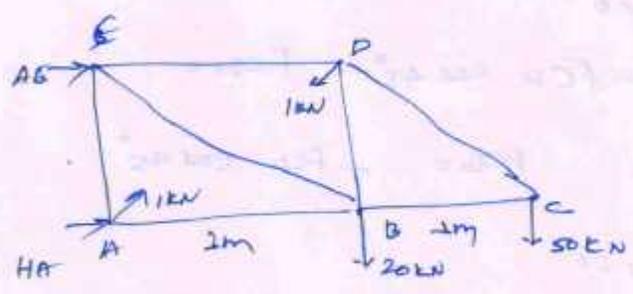
member	F (kN)	K (kN)	l (m)	Fkl	K^2l	$\frac{Fkl}{K^2l}$
AB	0	-1	3.6	0	3.6	26.53
BC	0	-1	3.6	0	3.6	26.53
CD	-60	-1	3.6	716	3.6	-53.47
AC	84.85	1.414	5.09	610.69	10.18	47.54
BD	0	1.414	5.09	0	10.18	-37.51
				<u>826.69</u>		<u>31.14</u>

$$R = -\frac{\sum Fkl}{\sum \frac{K^2l}{AE}}$$

$$R = \frac{-826.69}{31.14}$$

$$R = -26.52$$

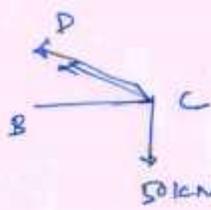
2)



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$\sum H = 0$
 $H_A + H_E = 0$
 $H_A = -H_E$
 $\sum V = 0$
 $V_E + 20 + 50 = 0$
 $V_E = -70 \text{ kN}$
 $\sum M_A = 0$
 $H_E \times 3 + (20 \times 3) + (50 \times 6) = 0$
 $H_E = -120 \text{ kN}$
 $H_A = 120 \text{ kN}$

Joint C

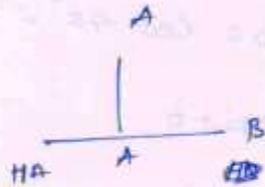


$\sum H = 0$
 $-F_{CD} \cos 45^\circ - F_{CB} = 0$
 $F_{CB} = -F_{CD} \cos 45^\circ$
 $\sum V = 0$
 $-50 + F_{CD} \cos 45^\circ = 0$
 $F_{CD} = 70.71 \text{ kN (C)}$

$$F_{CB} = -50 \text{ kN}$$

$$F_{CB} = 50 \text{ kN (Comp)}$$

Joint A



$$\sum V = 0$$

$$F_{AE} = 0$$

$$\sum H = 0$$

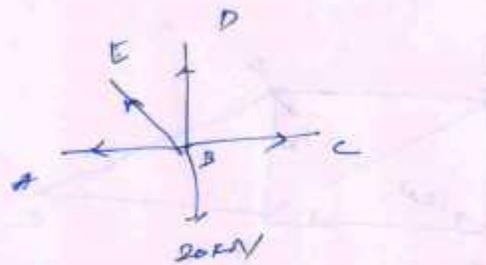
$$H_A + F_{AB} = 0$$

$$F_{AB} = -H_A$$

$$F_{AB} = -120 \text{ kN}$$

$$F_{AB} = 120 \text{ k (Comp)}$$

Joint B



$$\sum H = 0$$

$$F_{BC} - F_{AB} - F_{BE} \cos 45^\circ = 0$$

$$-50 + 120 - F_{BE} \cos 45^\circ = 0$$

$$-F_{BE} \cos 45^\circ = 70$$

$$F_{BE} = 99 \text{ kN (C)}$$

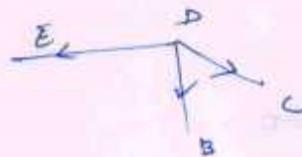
$$\sum V = 0$$

$$-20 + F_{BD} + F_{DE} \cos 45^\circ = 0$$

$$-89.99 + F_{BD} = 0$$

$$F_{BD} = 70 \text{ kN (T)}$$

Joint D

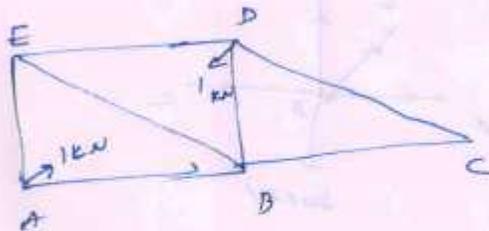


$$\sum H = 0$$

$$-F_{DE} + F_{DC} \cos 45^\circ = 0$$

$$F_{DE} = 70.71 \cos 45^\circ$$

$$F_{DE} = 60 \text{ kN (T)}$$



$$\sum V = 0$$

$$\sum H = 0$$

$$V_E = 0$$

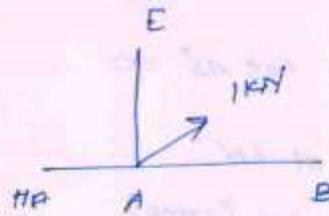
$$H_E = 0$$

$$\sum H = 0$$

$$H_C = 0$$

$$H_A = 0$$

Joint A



$$\sum V = 0$$

$$K_{AE} + 1 \sin 45^\circ = 0$$

$$K_{AE} = -0.7071 \text{ kN}$$

$$K_{AE} = 0.707 \text{ kN (Comp)}$$

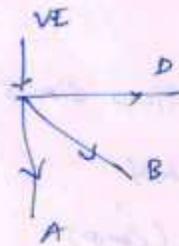
$$\sum H = 0$$

$$H_A + K_{AB} + 1 \cos 45^\circ = 0$$

$$K_{AB} = -0.707 \text{ kN}$$

$$K_{AB} = 0.707 \text{ kN (Comp)}$$

Joint E



$$\sum V = 0$$

$$V_E + K_{EA} + K_{EB} \cos 45^\circ = 0$$

$$0 - 0.707 + K_{EB} \cos 45^\circ = 0$$

$$K_{EB} = 1 \text{ kN (T)}$$

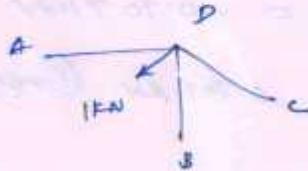
$$\sum H = 0$$

$$H_E + K_{DE} + K_{BE} \cos 45^\circ = 0$$

$$K_{DE} = -0.707 \text{ kN}$$

$$K_{DE} = 0.707 \text{ kN (Comp)}$$

Joint D



$$\sum H = 0$$

$$K_{CD} \cos 45^\circ + 1 \cos 45^\circ - K_{DE} = 0$$

$$K_{CD} \cos 45^\circ - 0.707 + 0.707 = 0$$

$$K_{CD} = 0$$

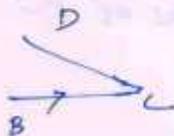
$$\sum V = 0$$

$$-K_{BD} - 1 \sin 45^\circ - K_{CD} \sin 45^\circ = 0$$

$$K_{BD} = -0.707 \text{ kN}$$

$$K_{BD} = 0.707 \text{ kN (Comp)}$$

Joint C

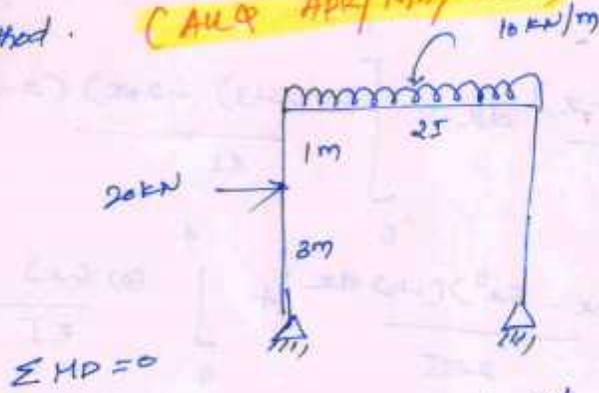


$$\sum H = 0$$

$$-K_{BC} - K_{DC} \cos 45^\circ = 0$$

$$K_{BC} = 0$$

Analyse the frame shown in fig by method of consistent deformation force method. @ flexibility method. (ALLQ APR/MAY 2011)



$$\sum MD = 0$$

$$V_A \times 6 + 20 \times 3 - 10 \times 6 \times \frac{6}{2} = 0$$

$$6V_A + 60 - 180 = 0$$

$$V_A = \frac{120}{6}$$

$$V_A = 20 \text{ kN.}$$

$$\sum MA = 0$$

$$V_D \times 6 = 0$$

$$V_D = 0$$

$$\sum V = 0$$

$$V_A + V_D = 0$$

$$V_A = 0$$

$$\sum H = 0$$

$$H_A + H = 0$$

$$H_A = -1 \text{ kN}$$

$$\Delta = \Delta_{10} + \Delta_{11} R_1 = 0$$

$$\Delta_{10} = \int_0^2 \frac{M_m}{EI} dx + \int_0^3 \frac{(20x)(-x)}{EI} dx + \int_0^1 \frac{20(x+3) - 20x}{EI} (x-1) dx + \int_0^6 \frac{(40x - 5x^2)(-4)}{2EI} dx + \int_0^4 \frac{(0)(-x)}{EI} dx$$

$$= \frac{1}{EI} \left[\int_0^3 (-20x^2) dx + \int_0^1 (-60x - 180) dx + \int_0^6 (-80x + 10x^2) dx \right]$$

$$= \frac{1}{EI} \left[\left(-\frac{20x^3}{3} \right)_0^3 + \left(-\frac{60x^2}{2} - 180x \right)_0^1 + \left(-80\frac{x^2}{2} + \frac{10x^3}{3} \right)_0^6 \right]$$

$$= \frac{1}{EI} \left[-180 + (-30 - 180) + (-1440 + 720) \right]$$

$$\Delta_{10} = \frac{-1110}{EI}$$

$$\Delta_{11} = \int_0^2 \frac{M_1^2}{EI} dx$$

$$\begin{aligned}
 &= \int_0^3 \frac{(-x)^2}{EI} dx + \int_0^1 \frac{(-x-3)^2}{EI} dx + \int_0^6 \frac{(-4)^2}{2EI} dx + \\
 &\quad \int_0^4 \frac{(-x)^2}{EI} dx \\
 &= \frac{1}{EI} \left[\left(\frac{x^3}{3} \right)_0^3 + \int_0^1 (x^2 + 9 + 6x) dx + \left(\frac{16x}{2} \right)_0^6 + \right. \\
 &\quad \left. \left(\frac{x^3}{3} \right)_0^4 \right] \\
 &= \frac{1}{EI} \left[9 + \left[\frac{x^3}{3} + 9x + \frac{6x^2}{2} \right]_0^1 + (8 \times 6) + \frac{64}{3} \right] \\
 &= \frac{1}{EI} \left[9 + \frac{1}{3} + 9 + 3 + 48 + \frac{64}{3} \right] \\
 &= \frac{1}{EI} \left[69 + \frac{65}{3} \right] \\
 \Delta_{11} &= \frac{90.667}{EI} \\
 \Delta_{10} + \Delta_{11} R_1 &= 0 \\
 -\frac{1110}{EI} + \frac{90.667}{EI} R_1 &= 0
 \end{aligned}$$

