SRI VIDYA COLLEGE OF ENGINEERING & TECHNOLOGY
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Year: II **Semester: IV**

Subject Code /Name: CE8402/Strength of Materials - II

Unit - III

COLUMNS AND CYLINDERS

Unit - III

Columns and cylinders.

Column and Strut

A member of structure or bar which carries an axial compressive load is called strut.

If the strut is vertical or inclined at 90° to the horizontal is known as column or pillar or stanchion.

Slenderness ratio :- K or λ

It is the ratio of unsupported length (or equivalent) of the column to minimum radius of gyration of the section.

$$K = \frac{\text{Equivalent length of column}}{\text{Minimum radius of gyration}}$$

units : No units.

Buckling Load :- (or) crippling Load :-

The maximum limiting load at which the column tends to have

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lateral displacement or tends to buckle is called buckling or crippling load.

Classifications of columns

Depends on the Slenderness ratio (or) l/d ratio, columns are divided into the 3 types.

1. Short columns
2. Medium columns.
   $\hookrightarrow l = 8d$ to $30d$
   $K = 32$ to 120
3. Long columns.

Short columns :-

Column which have length less than 8 times of respective diameters (or) Slenderness ratio less than 32 is called short columns.

$l <$ 8 times of diameter.

$K < 32$

long columns :-

The columns having their lengths more than 30 times their respective diameter

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or slenderness ratio more than 120 are called long columns.

$l < 30$ times diameter

$k > 120$

Effective length or Equivalent length:—

The distance between adjacent of inflexion is called equivalent length or effective length or simple column length.

End conditions of the columns

i) Both ends in pin jointed or hinged or rounded (or) free.

Eff. length (l_e) = Actual length.

2) one end fixed and other end free

$l_e = 2l$

3) one end fixed and other end pin jointed

$l_e = l/\sqrt{2}$

4) Both end fixed

$l_e = l/2$

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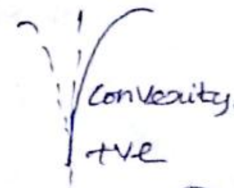
Euler's column theory:-

Assumptions:-

* column is intially straight and uniform lateral dimensions.
* The weight of the column is neglected.
* The column fails by buckling alone.
* compressive load is exactly axial and it passes through the centroid of the column section.

Sign convention:-

A moment which tends to bend the column with convexity towards it's intial central line as shown in figure is takes as positive



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A moment which tends to bend the column with concavity towards its initial central line as shown in figure is taken as negative

Euler's Formula:-

Euler's Formula is used for calculating the critical load for a column or strut.

$$P_{Euler} = \frac{\pi^2 EI}{le^2}$$

Where,
P = critical load
E = Modulus of Elasticity
I = least moment of Inertia of a section.
le = Equivalent length of column.

Limitations:-

* It is applicable to an ideal strut only.
* It takes no account of

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direct stress. It means that it may give a buckling load for structs for in excess of load which they can withstand under direct compression.

Critical loads for prismatic column with different end conditions

a) when both ends of the columns are hinged or pinned.

$$P = \frac{\pi^2 EI}{l^2}$$

b) when one end is fixed and other end is free.

$$P = \frac{\pi^2 EI}{4l^2}$$

c) when one end of the column is fixed and other end pinned or hinged.

$$P = \frac{2\pi^2 EI}{l^2}$$

d) when both ends of the column are fixed

$$P = \frac{4\pi^2 EI}{l^2}$$

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Rankine - Gordon formula

$$P_{Rankine} = \frac{\sigma_c A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

Where

a = Empirical constant = $\frac{\sigma_c}{\pi^2 E}$

σ_c = maximum Compressive stress

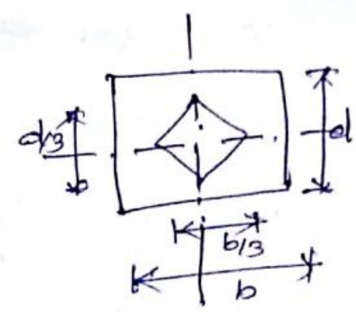
A = sectional area

l_e = Effective length.

Core of a section:-

The load acts on the any where in the section and there is no tension is known as core (or) Keen of the section.

In a rectangular section



Section modulus (Z)

$Z = \frac{1}{6} \times b \times d^2$

Area $A = b \times d$

No tension condition, $e \leq Z/A$

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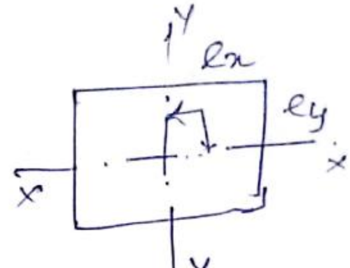
$$e \leq \frac{1/6\, bd^2}{bd}$$

$e \leq \frac{d}{b}$ for rectangular section.

Middle third rule:—

$$e_x \leq b/d$$

$$e_y \leq d/b$$



* Tensile stress should not occurs, Eccentric load acts at any of the geometrical axes. Let it be d/b it lies in the middle third ~~rule~~. The stress will be compressive

* The value of ~~eccentrically or~~ eccentricity of e_y on either side of x = axis does not exceed d/b.

Eccentrically loaded columns:—

Probelem:—

From the following data of a column of circular section, calculate

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extreme stresses on the column section, Also find maximum eccentricity in order that there may be no tension any where on the section.

External diameteres = 20 cm, Internal diameter = 16 cm, length of column = 4 m, load carried by column = 200 kN, Eccentricity of column load = 2.5 cm (from axis of column).

End conditions : Both ends are fixed.

$E = 94\, GN/m^2$

Solution :-

Area of column $A = \frac{\pi}{4} \times (20^2 - 16^2)$

$= 113.1 \times 10^{-4}\ m^2$

$I = \frac{\pi}{64}(20^4 - 16^4)$

$= 4637 \times 10^{-8}\ m^4$

$e = 2.5\ cm = 2.5 \times 10^{-2}\ m$

End condition : Both ends fixed

$l_e = \frac{l}{2} = \frac{4}{2} = 2m.$

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Max. B.M =

$$M_{max} = Pe \sec \frac{le}{2}\sqrt{P/EI}$$

$$M_{max} = 200\times10^3 \times (2.5\times10^{-2} \times \sec 2/2 \times \sqrt{\frac{200\times10^3}{94\times10^9\times4637\times10^{-8}}}$$

$$= 5.1\,KNm$$

Maximum compressive stress

$$\sigma_{max} = P/A + M/Z$$

$$= \frac{200\times10^3}{113.1\times10^{-4}} + \frac{5.1\times10^3}{463.7\times10^{-6}}$$

$$Z = I/y = \frac{4637\times10^{-8}}{10\times10^{-2}} = 463.7\times10^{-6}\,m^3$$

$$\sigma_{max} = 28.7\,MN/m^2$$

For no tension (corresponding to the maximum eccentricity).

$$P/A = M/Z$$

$$P/A = \frac{P.e.\sec le/2\sqrt{P/EI}}{Z}$$

$$\frac{200\times10^3}{113.1\times10^{-4}} = \frac{200\times10^3\times e\times\sec\frac{2}{2}\times\sqrt{\frac{200\times10^3}{94\times10^9\times4637\times10^{-8}}}}{463.7\times10^{-6}}$$

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$$\frac{200\times10^{3}}{113.1\times10^{-4}} = \frac{200\times10^{3}\times e\times1.02}{463.7\times10^{-6}}$$

$$e = \frac{463.7\times10^{-6}}{113.1\times10^{-4}\times1.02}$$

$$= 0.0402\,m$$

$$\boxed{e = 40.2\,mm}$$

—— x —— x —— x ——

Probelem:

A 1.5m long cast iron column has a circular cross section of 50mm diameter. One end of the column is fixed in direction and position and the other is free. Taking factor of safety as 3. Calculate the safe load using Rankine Gordon formula. Take yield as 560MPa and constant a = 1/1600 [May 2017]

Given:-

$a = 1/1600$ $\sigma_c = 560\times10^{6}\,N/m^{2}$

$l_e = 2L = 2\times1.5 = 3m$

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$A = 1.963 \times 10^{-3} m^2$

$$k^2 = \frac{I}{A} = \frac{\frac{\pi d^4}{64}}{\frac{\pi d^2}{4}} = \frac{\pi d^4}{64} \times \frac{4}{\pi d^2}$$

$$= \frac{0.05^2}{16}$$

$$= 1.5625 \times 10^{-4}$$

$$le^2 = 3^2 = 9 \; ; \; \frac{le^2}{k^2} = \frac{9}{1.5625 \times 10^{-4}} \times 57600$$

$$P_{Rankine} = \frac{\sigma_c \times A}{1 + a \times \frac{le^2}{k^2}} = \frac{560 \times 10^6 \times 1.96 \times 10^{-3}}{1 + \frac{1}{1600} \times 57600}$$

$$= 29.717 \times 10^3$$

$$\text{Safe load} = \frac{29.72}{F.O.S} = \frac{29.72}{3}$$

$$= 9.9 \text{ kN}$$

Probelem!-

Find Euler's crippling load for a hollow cylinderical steel column of 38mm external diameter and 2.5mm

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The length of the column is 2.3m and hinged at both ends. Take E = 205GPa. Also determine the crippling load by Rankine's formula using constant as 335 kN/mm² and 1/7500. [U.Q Dec. 2012]

Given

$a = \alpha = \frac{1}{7500}$ $\sigma_c = 335 \times 10^3 \ N/mm^2$

Solution:-

$$P_{Rankine} = \frac{\sigma_c \times A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

$$E = 205 \ GPa = 2.05 \times 10^5 \ N/mm^2$$

Support condition = hinged at both ends.

$$l_e = l = 2.3 \ m = 2300 \ mm$$

$$k = \sqrt{I/A}$$

$$= \sqrt{\frac{44140.11}{278.816}}$$

$$k = 12.58$$

$$\frac{l_e}{k} = \frac{2300}{12.58} = 182.83$$

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$$I = \frac{\pi D^4}{64} = \frac{\pi(38^4 - 33^4)}{64}$$

$$= 44140.11 \text{ mm}^4$$

$$A = \frac{\pi D^2}{4} = \frac{\pi(D^2 - d^2)}{4}$$

$$= \frac{\pi(38^2 - 33^2)}{4}$$

$$= 278.816 \text{ mm}^2$$

$$P_{(\text{rankine})} = \frac{335 \times 10^3 \times 278.816}{1 + \frac{1}{7500} \times (182.83)^2}$$

$$= 17116.5 \text{ KN}$$

$$\text{crippling load} = \frac{P \times F.O.S}{P_{(\text{rankine})}}.$$

$$\text{Crippling load} = P_{(\text{rankine})} \times F.O.S$$

$$= 17116.3 \times 3$$

$$= 51349 \text{ KN}$$

———— x ————

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Thin cylinderical and spherical Shells

If the thickness of the wall of the cylinderical vessel is less than 1/15 to 1/20 of its internal diameter, the cylinderical vessel is known as the thin cylinderical, ~~and~~ thin spherical shells.

Probelem

A cylinderical pipe of diameter 1.5m and thickness 1.5cm is subjected to an internal fluid pressure of 1.2 N/mm²

determine

(i) Longitudinal stress developed in the pipe

(ii) circumferential stress developed in the pipe.

Given! –

Internal fluid pressure $p = 1.2 N/mm^2$

$d = 1.5m$

$t = 1.5cm = 1.5 \times 10^{-2} m$

Solution! –

$$t/d = \frac{1.5 \times 10^{-2}}{1.5} = 1/100 = 0.01 < 1/20 = 0.05$$

It is Thin cylinder

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The longitudinal stress σ_2 :-

$$\sigma_2 = \frac{p \times d}{4t} = \frac{1.2 \times 1.5}{4 \times 1.5 \times 10^{-2}}$$

$$= 30 N/mm^2$$

The circumferential stress

$$\sigma_1 = \frac{pd}{2t}$$

$$= \frac{1.2 \times 1.5}{2 \times 1.5 \times 10^{-2}}$$

$$= 60 N/mm^2$$

Thick cylinder

Thick cylinder are the cylinderical vessels, containing fluid under pressure and whose wall thickness is not small

$$\boxed{t > d/20}$$

the following stress are exibit in the thick cylinder

* Radial pressure

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* Hoop stress or circumferential stress

* Longitudinal stress

These stresses are obtained using Lame's theory

1. Radial pressure

$$(\sigma_r) \text{ or } (P_x) = \frac{b}{r^2} - a$$

2. Hoop stress

$$(\sigma_c) \text{ or } (f_x) = \frac{b}{r^2} - a$$

sign convention.

Hoop stress ⇒ (+ve) at tensile

⇒ (−ve) at compression.

Radial stress ⇒ (+ve) at compressive

⇒ (−ve) at tensile.

3. Longitudinal stress

$$(\sigma_l) \text{ or } (P_0) = \left[\frac{r_1}{r_2^2 - r_1^2}\right] P$$

where

r = radius , r_1 = internal radius

σ_r = compressive stress

σ_c = Tensile "

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Change in dimension :-

$$e_v = \frac{\delta V}{V} \text{ or } \frac{\delta d}{d}$$

$$\boxed{e_v = 3e}$$

$$V = \frac{4}{3}\pi r^3 \text{ or } \frac{\pi d^3}{6}$$

$$\delta d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$$

Compound cylinder :-

A compound cylinder is defined as one cylinder that is shrunk onto the top of another cylinder. The inner cylinder will than be subjected to compressive hoop stresses due to outer cylinder will be in tension due to shrinkage.

Shrinking on stresses :-

When the working pressure is now applied to the inner cylinder, the stress in the inner cylinder will be

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the algebric sum of the stress due to the Shrinkage and the stress due to internal pressure.

Probelem:-

A pipe of 200mm internal diameters and 50mm thickness carries a fluid at a pressure of $10 MN/m^2$. calculate the maximum and minimum intensities of circumferential stress across the section.

Solution:-

internal $r_1 = \frac{200}{2} = 100mm = 0.1m$

Extend $r_2 = \frac{(200 + 2\times 50)}{2} = 150mm = 0.15m$

$P_1 = 10 MN/m^2$

Lame's eqn.

$$\sigma_r = b/r^2 - a \rightarrow (1) \; ; \; \sigma_c = \frac{b}{r^2} + a \rightarrow (2)$$

at $r = 0.1m$ $\sigma_r = 10 MN/m^2$

When $r = 0.15m$ $\sigma_r = 0$

Substituting in (i) we get

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$$10 = \frac{b}{0.1^2} - a$$

$$0 = \frac{b}{0.15^2} - a$$

solving the above eqn. we get.

$b = 0.18$ and $a = 8$

at $r = 0.1m$ $\sigma_r = \frac{0.18}{0.1^2} - 8 = 10 \, MN/m^2$

$$\sigma_c = \left[\frac{0.18}{0.1^2}\right] + 8 = 26 \, MN/m^2$$

$r = 0.15m$

$$\sigma_c = \left[\frac{0.18}{0.15^2}\right] + 8 = 16 \, MN/m^2$$

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