SRI VIDYA COLLEGE OF ENGINEERING & TECHNOLOGY
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DEPARTMENT OF CIVILENGINEERING

Year: II **Semester: IV**

Subject Code /Name: CE8402/Strength of Materials - II

Unit - II

INDETERMINATE BEAMS

Unit – II

Indeterminate Beams

Concept of analysis :-

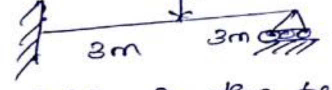
Any structure which cannot be solved by means of the three static equilibrium conditions, then structure is called indeterminate or redundant structures.

Propped Cantilever beam

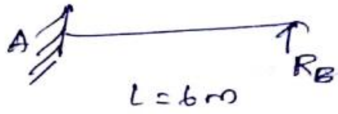
A cantilever beam supported at free end using prop (or) support is known as propped cantilever beam.

Probelem:-

Draw SFD and BMD for the propped Cantilever beam loaded AB shown in figure.



Draw BMD for the prop.



Solution:-

Deflection due to prop.

$$y_B' = \frac{A_1 \bar{x}_1}{EI} = \frac{18R_B \times 4}{EI} \qquad x_1 = 2/3 \times l = 4m$$

(1)

$$= \frac{72 R_B}{EI}$$

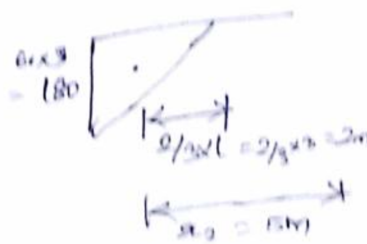
$$A_1 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 6R_B \times 6$$

$$A_1 = 18 R_B$$

Draw a BMD for cantilever beam

$$A_2 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 180 \times 3$$

$$= 270 \, kNm^2$$



$$\bar{x}_2 = 5m$$

Downward deflection due to load

$$y_B = \frac{A_2 \bar{x}_2}{EI} = \frac{270 \times 5}{EI} = \frac{1350}{EI}$$

$$y_B' = y_B$$

$$\frac{72 R_B}{EI} = \frac{1350}{EI}$$

$$R_B = \frac{1350}{72} = 18.75 \, kN$$

$$R_A + R_B = 60$$

$$R_A = 60 - 18.75 = 41.25 \, kN$$

$$M_A = 18.75 \times 6 - 60 \times 3 = -67.5 \, kNm.$$

$$M_B = 0 \, kNm$$

$$M_C = R_B \times 3 = 18.75 \times 3 = 56.25 \, kNm.$$

(2)

S.F.D

Start from free end B

consider left of B = 0

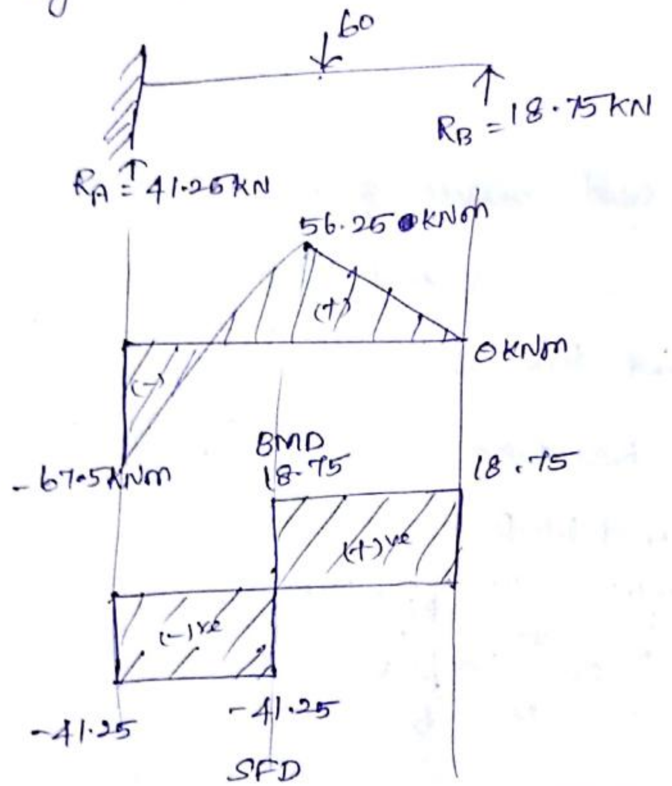
Right of B = 18.75 KN

Left of C = 18.75 KN

Right of C = 18.75 − 60 = −41.25 KN

Left of A = −41.25 KN

Right of A = −41.25 + 41.25 = 0 KN



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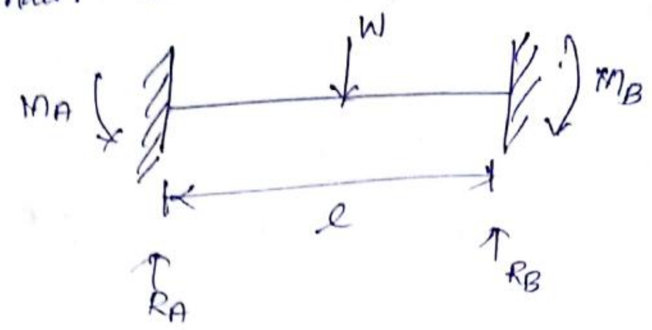
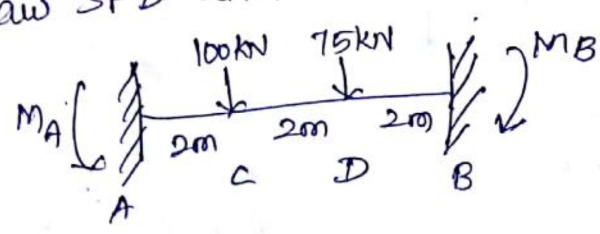
Fixed beams :-

A fixed beam is a beam the ends of which are constrained (or) built in to remain in horizontal position.



Probelem :-

A fixed beam of 6m span carries point loads of 100kN and 75kN as shown in figure. Find the following i) Fixing moments at ends. ii) Reactions at the supports. Draw SFD and BMD



Solution :-

$A_1 = 1/2 \times 6R_B \times 6 = 18R_B$

$\bar{x}_1 = 4m$

Consider moment $A_2 = 6M_B$ $x_2 = 3m$

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Consider 75KN load

$A_3 = \frac{1}{2} \times 300 \times 4 = 600 kNm^2$

$\bar{x} = 4.667$

consider 100KN load

$A_4 = \frac{1}{2} \times 200 \times 2 = 200 kNm^2$

$\bar{x} = 5.333$

$\Sigma A = 0$

$18R_B - 6M_B - 600 - 200 = 0$

$18R_B - 6M_B = 800$

$\Sigma Ax = 0$

$18R_B \times 4 - 6M_B \times 3 - 600 \times 4.667$

$- 200 \times 5.333 = 0$

$72R_B - 18M_B = 8866.66$

$R_B = 81.48 KN$

$M_B = 111.11 KNm$

$R_A = 100 + 75 - R_B$

$= 93.52 KN$

$81.48 \times 6 - 75 \times 4 - 100 \times 2 - 111.11 = 0$

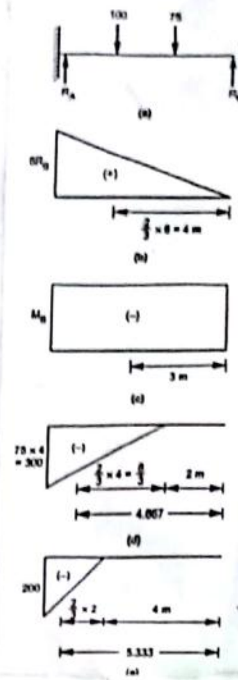
$M_B = 122.23 KNm$

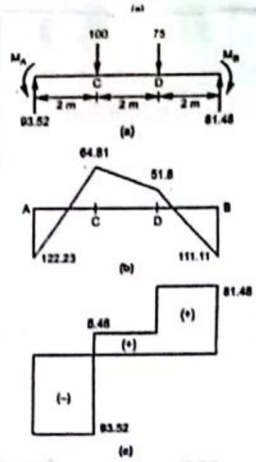
$M_C = 81.48 \times 4 - 75 \times 2 - 111.11$

$= 64.81 KNm$

$M_D = 81.48 \times 2 - 111.11$

$= 51.85 KNm$





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Consider point B:

Left of B = 0

Right of B = 81.48

Left of B = 81.48

Right of D = 81.48 - 75

= 6.48 kNm

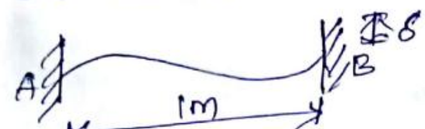
Left of C = 6.48 kNm

Right of C = -93.52 kNm

Left of A = -93.52 kNm

Right of A = 0 kNm

Effect of sinking of support: & Rotation of Supports.



The above figure shows a beam AB of span 1m and the support B settles down by an amount δ with respect A.

The difference in level causes the fixed end moments M_A and M_B respectively.

The additional moment caused by settlement are to be added in the fixed end moments due to the applied loads.

The beam is split into two halves with the load W acting at centre.

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Support reaction to be $\frac{W}{2}$

For each half the deflection is $(\frac{\delta}{2})$

Hence fixed end moments is $(W \times l/2)$ at each end.

$$\frac{\delta}{2} = \frac{Wl^3}{3EI} \qquad l = l/2$$

$$\delta/2 = \frac{W(l/2)^3}{3EI} \qquad W = \frac{12EI\delta}{2}$$

$$M = W(l/2) = \frac{12EI\delta}{l^2} \times (\delta/2)$$

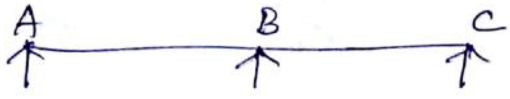
$$= \frac{6EI\delta}{l^2}$$

If support B lower by $\delta M_A = M_B = \frac{-6EI\delta}{l^2}$

If support B lower by $\delta M_A = M_B = \frac{6EI\delta}{l^2}$

If 'B' lower $R_A = \frac{12EI\delta}{l^3}$, $R_B = \frac{-12EI\delta}{l^2}$

Theorem of three moments:-



$$\frac{M_A l_1}{E_1 I_1} + 2M_B\left[\frac{l_1}{E_1 I_1} + \frac{l_2}{E_2 I_2}\right] + \frac{M_C l_2}{E_2 I_2} + \frac{6a_1\bar{x}_1}{l_1 E_1 I_1} + \frac{6a_2\bar{x}_2}{l_2 E_2 I_2} = \frac{6\delta_1}{l_1} + \frac{6\delta_2}{l_2} \quad (7)$$

l_1 = length of span AB

l_2 = length of span BC

$a_1 \bar{x}_1$ = First moment of BMD for span AB considering the origin at A.

$a_2 \bar{x}_2$ = First moment of BMD for span BC, considering the origin at C.

$E_1 I_1$ = Flexural rigidity for the span AB

$E_2 I_2$ = Flexural rigidity for the span BC.

δ_1 = sinking of the support A w.r.t to support B.

δ_2 = " " " C w.r.t to support B.

case (i) $\delta_1 = \delta_2 = 0$

$$\frac{M_A l_1}{E_1 I_1} + 2M_B\left[\frac{l_1}{E_1 I_1} + \frac{l_2}{E_2 I_2}\right] + M_C \frac{l_2}{E_2 I_2} + \frac{6a_1\bar{x}_1}{l_1 E_1 I_1} + \frac{6a_2\bar{x}_2}{l_2 E_2 I_2} = 0$$

case (ii) Flexural rigidity same.

$E_1 I_1 = E_2 I_2$; $\delta_1 = \delta_2 = 0$

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 + \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2} = 0$$

— x — x —

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Continuous Beams :-

A beam is generally supported on a hinge at one end and a roller bearing at the other end.

The reactions are determined by using static equilibrium equations. such as known as statically determined structure.

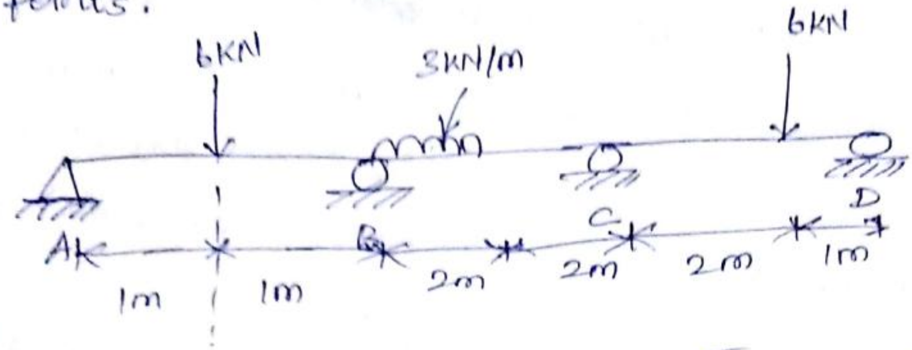
If the ends of the beam are restrained (clamped / encastered fixed then the moments make the structural element to be statically indeterminate structure.

A continuous beam is one being more than the one span & it is carried by several supports. (minimum three).

(9)

Problem

A continuous beam ABCD for Figure. Draw SFD and BMD indicating the salient points.



EI constant through out. [U.Q Dec. 2011]

Solution:-

We know that simply supported moments are

Span AB

$$\frac{Wl}{4} = \frac{6\times 2}{4} = 3$$

Span BC

$$\frac{Wl^2}{8} = \frac{3\times 4^2}{8} = 6$$

Span CD

$$\frac{Wab}{l} = \frac{6\times 2\times 1}{3} = 4$$

Consider Span AB and BC

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$$M_A l_1 + 2 M_B (l_1 + l_2) + M_C l_2 = -6\left[\frac{a_1 x_1}{l_1} + \frac{a_2 x_2}{l_2}\right]$$

$$a_1 = \tfrac{1}{2} \times b \times h = \tfrac{1}{2} \times 2 \times 3 = 3$$

$$a_2 = \tfrac{2}{3} \times 4 \times 6 = 16$$

$x_1 = 1m$, $x_2 = 2m$

$l_1 = 2m$ $l_2 = 4m$

$$\frac{a_1 x_1}{l_1} = \frac{3 \times 1}{2m} = 1.5$$

$$\frac{a_2 x_2}{l_2} = \frac{16 \times 2}{4} = 8$$

$$2M_B (2+4) + (M_C \times 4) = -6(1.5+8)$$

$$12 M_B + 4 M_C = -57$$

Consider the span BC and CB.

$$M_B l_1 + 2M_C (l_1 + l_2) + \cancel{M_D}^{\,0} l_2 = -6\left[\frac{a_1 x_1}{l_1} + \frac{a_2 x_2}{l_2}\right]$$

$$4M_B + 2M_C (4+3) = -6\left[\frac{a_1 x_1}{l_1} + \frac{a_2 x_2}{l_2}\right]$$

$$a_2 = \tfrac{1}{2} \times 3 \times 4 = 6$$

$$\frac{a_1 x_1}{l_1} = 8$$

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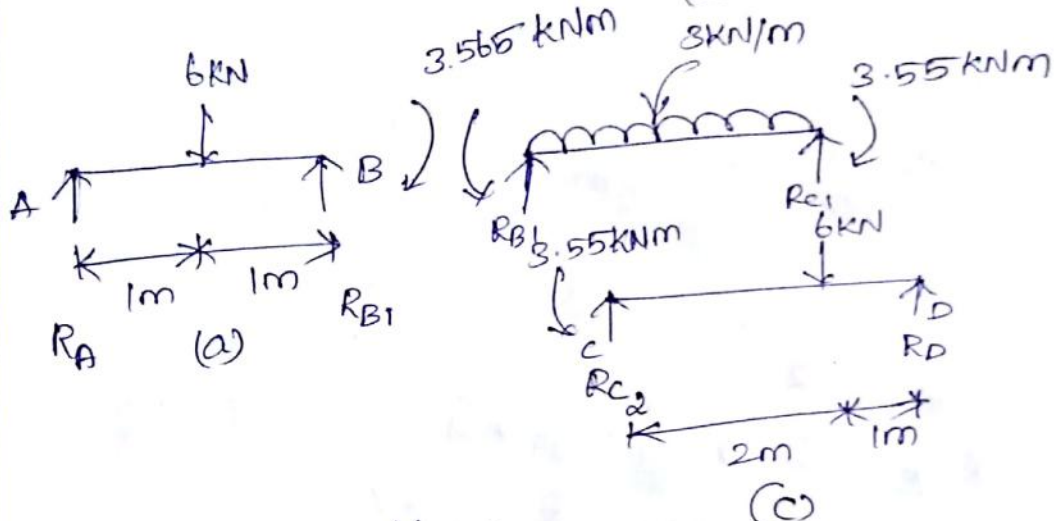
$$\frac{a_2 x_2}{l_2} = \frac{6 \times 1.33}{3} = 2.667$$

$$4M_B + 14M_C = -6(8 + 2.667)$$

$$4M_B + 14M_C = -64$$

Solve (1) and (2)

$M_B = -3.565$ $M_C = -3.55$ KNm



a) $R_{B1} \times 2 - 6 \times 1 - 3.566 = 0$

$R_{B1} = 4.78$ kN

$R_A = 1.22$ kN

b) $3.566 = R_{C1} \times 4 - \frac{3 \times 4^2}{2} - 3.55$

$6\text{kN} = R_{C1}$

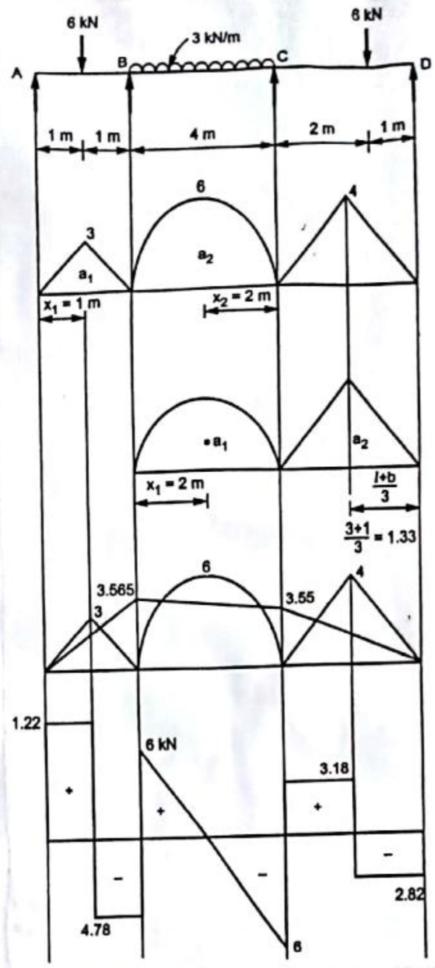
$R_{B1} = 6\text{kN}$

$R_D \times 3 - 6 \times 2 = 0$

$R_D = 4\text{kN}$

$R_{C2} = 6 - 4 = 2\text{kN}$

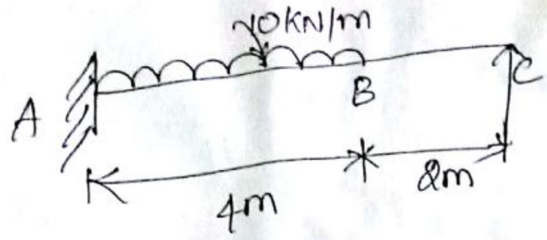
(12)



Probelems :-

A propped cantilever beam ABC of span 6m fixed at A and propped at C is loaded with an UDL of 10kN/m for a length of 4m from the fixed end. Find the prop reactions. Draw shear force and bending moment diagrams. Find the maximum sagging B.M and point of Contra flexure (DEC. 2012 U.Q)

(13)



$\delta_{cp} = \delta_c$

Deflection due to prop $= \frac{A_1 \bar{x}}{EI}$

$A_1 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6 \times 6 R_c$

$\bar{x} = 4m$

$= \frac{18 R_c \times 4}{EI}$

$\delta_{cp} = \frac{72 R_c}{EI}$

Deflection due to load : $\frac{A\bar{x}}{EI}$

$A_2 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \times 80$

$= 106.667 m^2$

$\frac{72 R_c}{EI} = \frac{533.33}{EI}$

$72 R_c = 533.33$

$R_c = 7.40 kN$

R_A = Total load $- R_c$

$= (10 \times 4) - 7.4$

(14)

$= 32.6 kN$

S.F.D

S.F at left of A = 0

" at right of B = $32.6 - (40 \times 4)$

$= -7.4 kN$

S.F at right of B = $-7.4 kN$

" Left of C = $-7.4 kN$

Right of C = 0

B.M.D

B.M at C = 0 kNm

B.M at B = $7.4 \times 2 = 14.8 kNm$

B.M at A = $7.4 \times 6 - 10 \times \frac{4^2}{2}$

$= -35.6 kNm$

To find the max. B.M (S.F = 0)

S.F at E = 0

x distance from A

Find shear force at E = $32.6 - 10\frac{x^2}{2}$

$-5x^2 = -32.6$

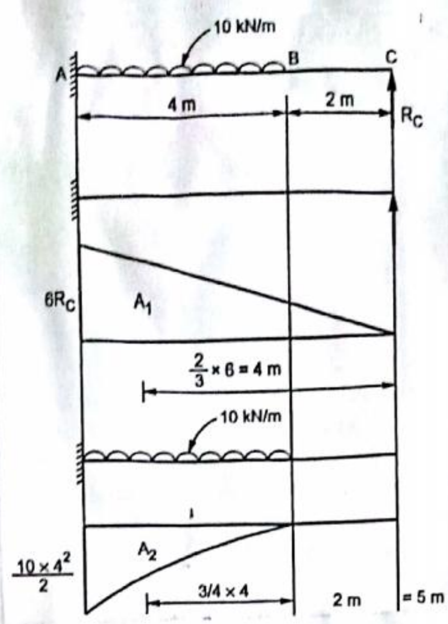
$x^2 = 6.52$; $x = 2.55 m$

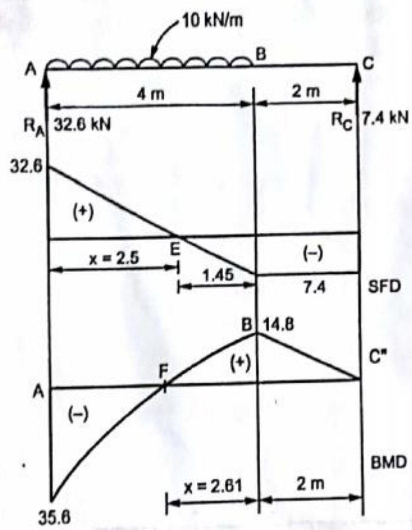
moment at E = $7.4 \times (2 + 1.45) - \frac{10 \times 1.45^2}{2}$

Consider right side

$= 25.53 - 10.512 = 15.0 kNm$

(15)





Point of contraflexure $M_F = 0$

consider 'x' from Right end from B

$M_F = 7.4 \times (2 + x) - \frac{10x^2}{2} = 0$

$14.8 + 7.4x - 5x^2 = 0$

$y = 2.61$ m from B

(16)