Sri Vidya college of Engineering and Technology,virudhunagar

UNIT - V

BOUNDARY VALUE PROBLEM IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATION.

Finite difference Method :

Replace x by x_k

y by y_k

y' by $\frac{y_{k+1} - y_k}{h}$

y'' by $\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2}$

where, $h = \frac{b-a}{n}$

1. Solve $y'' = x+y$ with the boundary condition $y(0) = y(1) = 0$.

Soln :

x	0	0.25	0.5	0.75	1
y	0	-0.0349	-0.0564	-0.05	0

$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$.

$y'' = x+y$.

$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} = x_k + y_k$.

Sri Vidya college of Engineering and Technology,virudhunagar

$$y_{k-1} - 2y_k + y_{k+1} = h^2 x_k + h^2 y_k$$

$$y_{k-1} - 2y_k + y_{k+1} - h^2 y_k = h^2 x_k$$

$$y_{k-1} + y_k(-2-h^2) + y_{k+1} = h^2 x_k$$

$$y_{k-1} - 2.0625\, y_k + y_{k+1} = 0.0625\, x_k$$

$k=1$;

$$y_0 - 2.0625\, y_1 + y_2 = 0.0625\, x_1$$

$$-2.0625 y_1 + y_2 = 0.0156 \quad \text{—} (1)$$

$k=2$;

$$y_1 - 2.0625 y_2 + y_3 = 0.0625 x_2$$

$$y_1 - 2.0625 y_2 + y_3 = 0.0313 \quad \text{—} (2)$$

$k=3$;

$$y_2 - 2.0625 y_3 + y_4 = 0.0625 x_3$$

$$y_2 - 2.0625 y_3 = 0.0469 \quad \text{—} (3)$$

Solve (1), (2) & (3)

$$y_1 = -0.0349; \quad y_2 = -0.0564; \quad y_3 = -0.0501;$$

Sri Vidya college of Engineering and Technology,virudhunagar

2. using a finite difference method compute

24/3/14. $y(0.5)$. Given $y'' - 64y + 10 = 0$; $y(0) = y(1) = 0$.

Sub dividing the interval into i) 4 Equal parts.

ii) 2 equal parts.

soln:

Given $y'' - 64y + 10 = 0$

$$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} - 64y_k + 10 = 0$$

$$\frac{y_{k-1} - 2y_k + y_{k+1} - 64y_k h^2 + 10h^2}{h^2} = 0$$

$$y_{k-1} + y_k(-2 - 64h^2) + y_{k+1} = -10h^2 \quad \text{--- (1)}$$

i) subdiving into 4parts.

$$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$$

	x_0	x_1	x_2	x_3	x_4
x	0	0.25	0.5	0.75	1
y	y_0 0	y_1 0.1287	y_2 0.1471	y_3 0.1287	y_4 0

for $h = 0.25$, (1) becomes,

$$y_{k-1} - 6y_k + y_{k+1} = -0.625 \quad \text{--- (2)}$$

put $k = 1$.

$$y_0 - 6y_1 + y_2 = -0.625$$

$$-6y_1 + y_2 = -0.625 \quad \text{--- (3)}$$

Sri Vidya college of Engineering and Technology,virudhunagar

put $k=2$;

$$y_1 - 6y_2 + y_3 = -0.625 \quad \text{--- (4)}$$

put $k=3$;

$$y_2 - 6y_3 + y_4 = -0.625$$

$$y_2 - 6y_3 = -0.625 \quad \text{--- (5)}$$

Solving by (3), (4) & (5)

$y_1 = 0.1287$; $\boxed{y_2 = 0.1471}$; $y_3 = 0.1287$.

ii) Sub dividing into 2 parts :

$$h = \frac{b-a}{n} = \frac{1-0}{2} = 0.5.$$

	x_1	x_2	x_3
x	0	0.5	1
	y_0	y_1	y_2
y	0	0.1389.	0.

for $h=0.5$. Eqn (1) becomes

$$\cancel{y_{k-1} + y_k}$$

$$y_{k-1} - 18y_k + y_{k+1} = -2.5 \quad \text{--- (1)}.$$

$k=1$.

$$y_0 - 18y_1 + y_2 = -2.5.$$

$$\cancel{-}18y_1 = \cancel{-}2.5$$

$$\boxed{y_1 = 0.1389.}$$

Sri Vidya college of Engineering and Technology,virudhunagar

5. Solve by finite difference method, the BVP $y'' - y = 0$ where $y(0) = 0; y(1) = 1$; take $\Delta x = 0.25$

Soln:

Given $y'' - y = 0$

$$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} - y_k = 0.$$

$$\frac{y_{k-1} - 2y_k + y_{k+1} - y_k h^2}{h^2} = 0.$$

$$y_{k-1} + y_k(-2 - h^2) + y_{k+1} = 0 \quad \text{——} (1)$$

for $h = 0.25$ eqn (1) becomes,

$$\cancel{y_{k-1} + y_k} \quad y_{k-1} - 2.0625\, y_k + y_{k+1} = 0. \quad \text{——} (2)$$

x	x_0 0	x_1 0.25	x_2 0.5	x_3 0.75	x_4 1
y	y_0 0	0.2151	0.4437	0.7	y_4 1

$k = 1$;

$$y_0 - 2.0625\, y_1 + y_2 = 0.$$

$$-2.0625\, y_1 + y_2 = 0 \quad \text{——} (3)$$

$k = 2$;

$$y_1 - 2.0625\, y_2 + y_3 = 0 \quad \text{——} (4)$$

Sri Vidya college of Engineering and Technology,virudhunagar

$k=3$;

$$y_2 - 2.0625\,y_3 + y_4 = 0$$

$$y_2 - 2.0625\,y_3 + 1 = 0$$

$$y_2 - 2.0625\,y_3 = -1 \quad \text{—(5)}$$

Solve by (3), (4) & (5)

$$y_1 = 0.8151\ ;\ y_2 = 0.4487\ ;\ y_3 = 0.7000$$

24/3/14

Classification of partial differential Equation

consider,

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = 0$$

$B^2 - 4ac < 0$ The P.D.E is elliptic

$B^2 - 4ac = 0$ The P.D.E is parapolic

$B^2 - 4ac > 0$ The P.D.E is hyperbolic.

One dimensional heat equation :

The One dimensional heat eqn is

$$\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t} \quad (\text{or}) \quad U_{xx} = a\,U_t$$

$$\frac{\partial^2 u}{\partial x^2} - a\frac{\partial u}{\partial t} = 0.$$

$A=1$; $B=0$; $C=0$

Sri Vidya college of Engineering and Technology,virudhunagar

$B^2 - 4AC = 0 - 4 \times 1 \times 0$

$= 0$

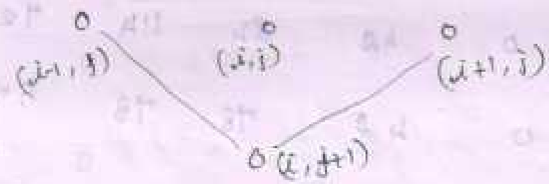
The one dimensional heat eqn is parabolic

There are two methods to solve one dimensional heat equations

i) Bender - Schmidt formula (Explicit)

ii) Crank - Nicolson method (Implicit)

Bender - Schmidt formula:



$$u_{i,j+1} = \frac{u_{i-1\,j} + u_{i+1\,j}}{2}$$

Here, $k = \frac{ah^2}{2}$

1. Solve $u_t = u_{xx}$ in $0 < x < 5$, $t > 0$ given that $u(0,t) = 0$, $u(5,t) = 0$, $u(x,0) = x^2(25 - x^2)$

Compute u upto 3sec with $\Delta x = 1$ by using Bender Schmidt formula.

Sri Vidya college of Engineering and Technology,virudhunagar

Soln:

Given $u_t = u_{xx} \Rightarrow a = 1$

$h = \Delta x = 1$

$k = \frac{ah^2}{2} = \frac{1 \times 1}{2} = 0.5$.

$$u_{i,j+1} = \frac{u_{i-1,j} + u_{i+1,j}}{2}$$

t \ x	0	1	2	3	4	5
0	0	24	84	144	144	0
0.5	0	42	84	114	72	0
1	0	42	78	78	57	0
1.5	0	39	60	67.5	39	0
2.	0	30	53.25	49.5	33.75	0
2.5	0	26.625	39.75	43.5	24.75	0
3	0	19.875	35.0625	32.25	21.75	0

2. Solve $u_{xx} = 32u_t$, $h = 0.25$ for $t \geq 0$, $0 < x < 1$, with $u(0,t) = 0$, $u(x,0) = 0$; $u(1,t) = t$.

soln:

$u_{xx} = 32 u_t \qquad a = 32.$

$h = 0.25.$

$$k = \frac{ah^2}{2} = \frac{32 \times 0.25}{2} = 1$$

$$u_{i,j+1} = \frac{u_{i-1,j} + u_{i+1,j}}{2}$$

$t \backslash x$	0	0.25	0.5	0.75	1
0	0	0	0	0	0
1	0	0	0	0	1
2	0	0	0	0.5	2
3	0	0	0.25	1	3
4	0	0.125	0.5	1.625	4
5	0	0.25	0.875	2.25	5.

2. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subjected to $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin(\pi x)$ using Bender schmidt method.

Sri Vidya college of Engineering and Technology,virudhunagar

Soln:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

$$u_{xx} = u_t \qquad a=1$$

$$h = \frac{B-A}{n} = \frac{1-0}{5} = 0.2$$

$$k = \frac{ah^2}{2} = \frac{1 \times 0.2^2}{2} = 0.02$$

Bender schmidth formula is,

$$u_{i,j+1} = \frac{u_{i-1,j} + u_{i+1,j}}{2}$$

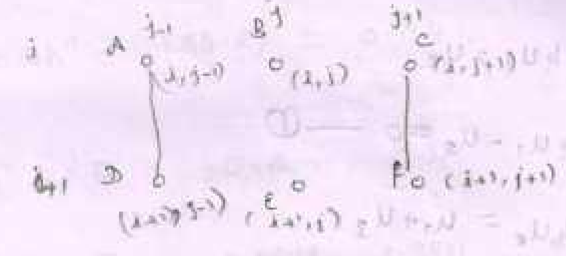
t \ x	0	0.2	0.4	0.6	0.8
0	0	0.5878	0.9511	0.9510	0.5878
0.02	0	0.4756	0.7695	0.7695	0.4756
0.04	0	0.3848	0.6226	0.6226	0.3848
0.06	0	0.3113	0.5037	0.5037	0.3113
0.08	0	0.2519	0.4075	0.4075	0.2519
1	0	0.2028	0.3297	0.3297	0.2038

Sri Vidya college of Engineering and Technology,virudhunagar

25/3/14. Crank – Nicolson's Method (Implicit method):

Consider, $\frac{\partial^2 u}{\partial x^2} = a \frac{\partial u}{\partial t}$ (one dimensional heat eqn).

$k = ah^2$



$4u_E = u_A + u_C + u_D + u_F$

1. Using Crank – Nicolson's scheme solve

$16 \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$.

Subjected to $u(x,0) = 0$; $u(0,t) = 0$; $u(1,t) = 100t$. Compute u for one step in t-direction. taking $h = 1/4$

Soln:

$16 \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

$a = 16$.

$h = 0.25$;

$k = ah^2 = 16 \times (0.25)^2 = 1$

t/x	0	0.25	0.5	0.75	1
0	0	0	0	0	0
1	0	u_1	u_2	u_3	100

$4u_1 = u_2$

$4u_1 - u_2 = 0$ —— (1)

$4u_2 = u_1 + u_3$

$u_1 - 4u_2 + u_3 = 0$ —— (2)

$4u_3 = u_2 + 100$

$-u_2 + 4u_3 = 100$ —— (3)

Solve (1), (2) & (3)

$u_1 = 1.7857$

$u_2 = 7.1429$

$u_3 = 26.7857$

2. find $u(x,t)$ for one time step the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given $u(x,0) = \sin(\pi x)$; $u(0,t) = u(1,t) = 0$ Take $h = 0.2$ using implicit method.

Sri Vidya college of Engineering and Technology,virudhunagar

Soln:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

$a = 1$

$h = 0.2$

$k = ah^2 = 1 \times (0.2)^2 = 0.04$.

t/x	0	0.2	0.4	0.6	0.8	1
0	0	0.5878	0.9511	0.9511	0.5878	0
0.04	0	u_1	u_2	u_3	u_4	0

$4u_1 = u_2 + 0.9511$

$4u_1 - u_2 = 0.9511$ — (1)

$4u_2 = u_1 + u_3 + 1.5389$.

$\cancel{-u_1 + 4u_2}$ $u_1 - 4u_2 + u_3 = -1.5389$ — (2)

$4u_3 = u_2 + u_4 + 1.5389$.

$u_2 - 4u_3 + u_4 = -1.5389$ — (3)

$4u_4 = 0.9511 + u_3$.

$\cancel{-u_3 + 4u_4 = 0.9511}$ — (4).

$u_4 = \frac{u_3}{4} + 0.2378$ — (4).

Sub (4) in (3)

$$u_2 - 4u_3 + u_4 = -1.5389.$$

$$u_2 - 4u_3 + \frac{u_3}{4} + 0.2378 = -1.5389.$$

$$u_2 - \frac{15}{4}u_3 = -1.7767.$$

$$u_2 - 3.75u_3 = -1.7767 \quad \text{—— (5)}$$

Solve ~~eqn~~ (1), (2), (5).

$u_1 = 0.3993$

$u_2 = 0.6461$

$u_3 = 0.6461$

$$(4) \Rightarrow \quad u_4 = \frac{0.6461}{4} + 0.2378 = 0.3993.$$

$$u_4 = 0.3993.$$

27/3/14.

3. Solve by crank nicolson's method, eqn $u_{xx} = u_t$ subjected to $u(x,0) = 0$; $u(0,t) = 0$; $u(1,t) = t$ for two time step.

Sri Vidya college of Engineering and Technology,virudhunagar

soln :

$u_{xx} = u_t$.

$a = 1$

$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$.

$k = ah^2 = 1 \times 0.25^2 = 0.0625$.

$t \backslash x$	0	0.25	0.5	0.75	1
0	0	0	0	0	0
0.0625	0	0.0011	0.0045	0.0167	0.0625
0.125	0	0.0059	0.0191	0.0528	0.125

$4u_1 = u_2$.

$4u_1 - u_2 = 0$ — (1)

$4u_2 = u_1 + u_3$

$u_1 - 4u_2 + u_3 = 0$ — (2)

$4u_3 = u_2 + 0.0625$.

$-u_2 + 4u_3 = 0.0625$ — (3)

solve by (1), (2), (3)

$u_1 = 0.0011$; $u_2 = 0.0045$; $u_3 = 0.0167$.

$4u_4 = u_5 + 0.0045$.

$4u_4 - u_5 = 0.0045$ — (4)

Sri Vidya college of Engineering and Technology,virudhunagar

$4u_5 = u_4 + u_6 + 0.0178$

$~~u_4+~~\; u_4 - 4u_5 + u_6 = -0.0178$ —— (5)

$4u_6 = u_5 + 0.1920$

$-u_5 + 4u_6 = 0.1920$ —— (6)

solve by (4), (5), & (6)

$u_4 = 0.0059 \quad u_5 = 0.0191 \quad u_6 = 0.0528$

One dimentional wave Equation:

The One dimentional wave Equation is, $\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial t^2}$; $k = ah$

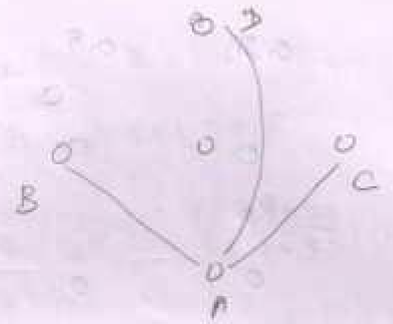
$U_{xx} = a^2 U_{tt}$

$U_{xx} - a^2 U_{tt} = 0$

$A = 1$; $B = 0$; $C = -a^2$

$B^2 - 4AC = 0 + 4a^2 = 4a^2 > 0$

The P.D.E is hyperbolic.



The formula is,

$$U_A = U_B + U_C - U_D$$

1. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0<x<1$, $t>0$

Given $u(x,0)=0$; $\frac{\partial u}{\partial t}(x,0)=0$; $u(0,t)=0$; $u(1,t)=100 \sin(\pi t)$. compute $u(x,t)$ for 4 times steps with $h=0.25$

soln:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

$a^2 = 1$

$a = 1$.

$h = 0.25$.

$k = ah = 1 \times 0.25 = 0.25$.

x: 0, 0.25, 0.5, 0.75, 1

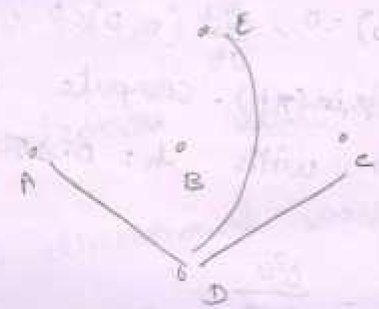
t: 0, 0.25, 0.5, 0.75

$u_1 = \frac{0+0}{2} + k.A$

$u_1 = \frac{\text{add side two values}}{2} + k.A$

22/5/14.

$t \backslash x$	0	0.25	0.5	0.75	
0	0	0	0	0	
0.25	0	$\frac{0+0+0.0}{2}$ u_1 0	u_2 0	u_3 0	70
0.5	0	0	0	70.7107	10
0.75	0	0	70.7107	100	70.
1	0	70.7107	100	70.7107	0



$$U_D = U_A + U_C - U_E$$

2. Solve the Eqn. $\frac{1}{4}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with $u(0,t)=0$; $u(4,t)=0$; $u(x,0) = x(4-x)$; $\frac{\partial u}{\partial t}(x,0) = 0$; by taking $h=1$; upto

soln:

$$\frac{1}{4}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

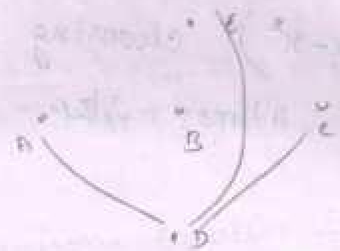
$$a^2 = 1/4$$

$$a = 1/2.$$

Sri Vidya college of Engineering and Technology,virudhunagar

$h = 1$

$k = ah = 0.5 \times 1 = 0.5.$



$$u_D = u_A + u_C - u_E$$

t \ x	0	1	2	3	4
0	0	3	4	3	0
0.5	0	2	3	2	0
1	0	0	0	0	0
1.5	0	-2	-3	-2	0
2	0	-3	-4	-3	0
2.5	0	-2	-3	-2	0
3	0	0	0	0	0
3.5	0	+2	3	2	0
4.	0	3	4	3.	0

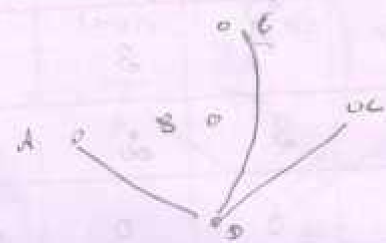
8. Solve $u_{tt} = u_{xx}$. $0 < x < 2$; $t > 0$. subject to $u(x,0) = 0$; $u(0,t) = 0$; $u(2,t) = 0$; $u_t(x,0) = 100(2x - x^2)$ choosing $h = 1/2$ compute 'u' for 4 times step.

soln:

$$u_{tt} = u_{xx}.$$

$$a^2 = 1 \Rightarrow a = 1 \; ; \; h = 0.5.$$

$$k = ah = 1 \times 0.5 = 0.5.$$



$$u_D = u_A + u_C - u_E$$

$\frac{0+0}{2} + k \times 100(2x - x^2)$

t \ x	0	0.5	1	1.5	2
0	0	0	0	0	0
0.5	0	37.5	50	37.5	0
1	0	50	75	50	0
1.5	0	37.5	50	37.5	0
2	0	0	0	0	0

Sri Vidya college of Engineering and Technology,virudhunagar

1/4/14. Laplace and poisson Equation.

The laplace Equation is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

$U_{xx} + U_{yy} = 0$ (or) $\nabla^2 u = 0$.

The poisons Equation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$$

(or)

$$U_{xx} + U_{yy} = f(x,y)$$

(or)

$$\nabla^2 u = f(x,y)$$

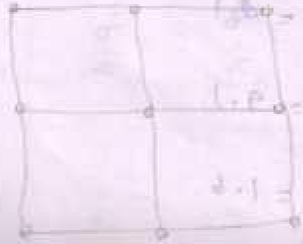
Here $A=1$; $B=0$; $C=1$

$B^2 - 4AC = 0 - 4 \times 1 \times 1$

$= -4 < 0$.

Hence, Laplace and poisson equation are elliptic

Standard Diagonal five point formula,



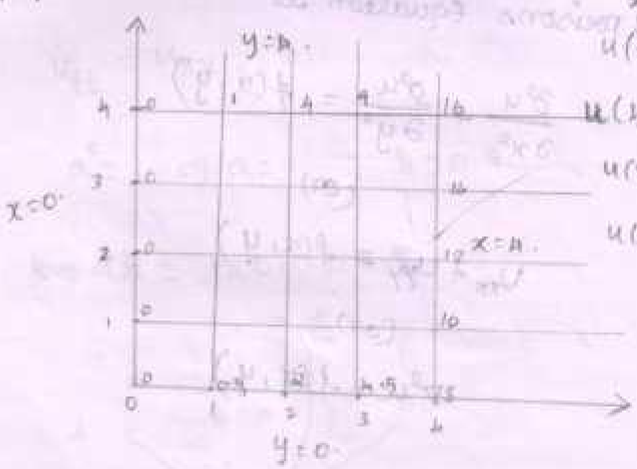
(+) SFPF : $U_E = \frac{U_B + U_D + U_F + U_H}{4}$.

(x) DFPF : $U_E = \frac{U_A + U_C + U_G + U_I}{4}$

Sri Vidya college of Engineering and Technology,virudhunagar

1. By Liebmann iteration method solve $u_{xx}+u_{yy}$ over the ~~surface~~ square region of side 4 satisfying $u(0,y)=0$ $0 \le y \le 4$; $u(4,y)=8+2y$; $u(x,0)=x^2/2$ $0 \le x \le 4$; $u(x,4)=x^2$, $0 \le x \le 4$. Com the values at the interior points with $h=k=$

Soln:



$u(0,y)=0$

$u(4,y)=8+$

$u(x,0)=x^2/2$

$u(x,4)=x^2$

Rough values:

SFPF : $u_5 = \frac{0+4+12+2}{4} = 4.5$.

DFPF : $u_1 = \frac{0+4+0+u_5}{4} = 2.1$

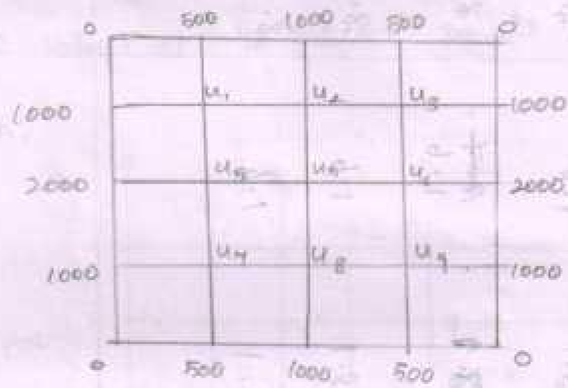
DFPF : $u_3 = \frac{4+16+12+u_5}{4} = 9.1$

DFPF : $u_7 = \frac{0+u_5+0+2}{4} = 1.6$

DFPF : $u_9 = \frac{u_5+12+2+8}{} = 6.6$

$u_1 = \frac{1+u_2+u_4}{4}$	$u_2 = \frac{4+u_1+u_3+u_5}{4}$	$u_3 = \frac{8+u_2+u_6}{4}$	$u_4 = \frac{u_1+u_5+u_7}{4}$	$u_5 = \frac{u_2+u_4+u_6+u_8}{4}$	$u_6 = \frac{13+u_5+u_3+u_9}{4}$	$u_7 = \frac{0.5+u_4+u_8}{4}$	$u_8 = \frac{2+u_5+u_7+u_9}{4}$	$u_9 = \frac{4.5+u_6+u_8}{4}$
2.1	4.9.	9.1	2.1	4.5	8.1	1.6	3.7.	6.6
2	4.9.	9.	2	~~2.8~~ 4.7.	8.1.	~~2.7~~. 1.6	3.7.	6.6.
2.	4.9.	9	2.1	4.7.	8.1	1.6	3.7.	6.6.
2	4.9	9	2.1	4.7	8.1	1.6	3.7	6.6

2. Solve the Elliptic Eqn $U_{xx}+U_{yy}=0$ following square mesh with the bounda[ry] values are shown below



Soln :

By symmetry

$u_1 = u_3 \qquad u_1 = u_7$

$u_4 = u_6 \quad \& \quad u_2 = u_8$

$u_7 = u_9 \qquad u_3 = u_9$

Hence,

$u_1 = u_3 = u_7 = u_9$

$u_2 = u_8$

$u_4 = u_6$

Now, we find only u_1, u_2, u_4, u_5

Sri Vidya college of Engineering and Technology,virudhunagar

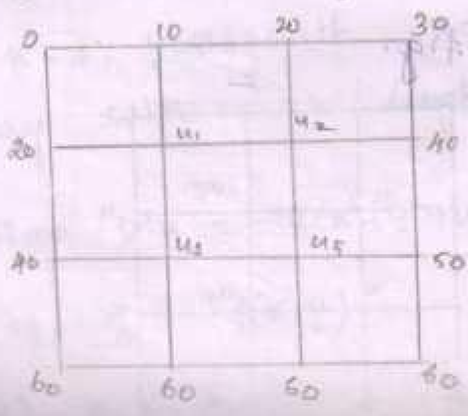
$u_1 = \frac{1500+u_2+u_4}{4}$	$u_2 = \frac{1000+2u_1+u_5}{4}$	$u_4 = \frac{2000+2u_1+u_5}{4}$	$u_5 = \frac{2u_2+2u_4}{4}$
1125	1187.5	1437.5	1500
~~1068.8~~ 1017	~~1190.9~~	~~1380.9~~	~~1234.4~~
1031.3	1140.7	1390.7	1265.4
1007.9	~~1089.0~~ 1070.3	1320.9	~~1254~~ 1195.4
972.7	1035.2	1285.2	1160.2
955.1	1017.6	1267.6	1142.6
946.3	1008.8	1258.8	1133.8
941.9	1004.4	1254.4	1129.4
939.7	1002.2	1252.2	1127.2
938.6	1001.1	1251.1	1126.1
938.1	1000.8	1250.6	1125.7
937.9	1000.4	1250.4	1125.4
937.9	1000.3	1250.3	1125.3

Rough value :

SPPF : $u_5 = \frac{1000+2000+2000+1000}{4} = \frac{6000}{4} = 1500$

DFPF : $u_1 = \frac{0+1000+1500+2000}{4} = 1125$.

3. Solve $\nabla^2 u = 0$ over the square region given by the boundary condition as in the fig. below.



Sri Vidya college of Engineering and Technology,virudhunagar

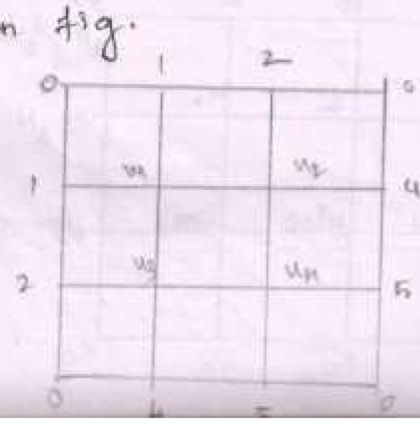
$u_1 = \frac{30+u_2+u_3}{4}$	$u_2 = \frac{60+u_1+u_4}{4}$	$u_3 = \frac{100+u_1+u_4}{4}$	$u_4 = \frac{110+}{}$
4	18.8	28.8	0
19.4	19.9	29.9.	40
20	30	40	45
25	32.5	42.5	46.3
26.3	33.2	43.2	46.6
26.6	33.3	43.3	46.7
26.7.	33.4	43.4	46.7
26.7.	33.4	43.4	46.7.

~~soln:~~ Rough :

$u_4 = 0$.

DFPF: $u_1 = \frac{0+20+40+0}{4} = 15$.

4. Solve $u_{xx} + u_{yy} = 0$ over the squa[re] region given by the boundary condi[tions] shown in fig.



Sri Vidya college of Engineering and Technology,virudhunagar

Soln:

By symmetry, $u_3 = u_2$.

Rough; Assume, $R_4 = 0$

DFPF : $u_1 = \frac{2+2+0+0}{4} = 1$

$u_1 = \frac{2+2u_2}{4}$	$u_2 = \frac{6+u_1+u_4}{4}$	$u_4 = \frac{10+2u_2}{4}$
1	1.6	0
1.4	1.9	3.5
1.5	2.8	3.9
1.9	3	4
2	3	4
2	3	4.

3/4/14. Poisson equation problems $u_{xx} + u_{yy} = -f(x,y)$

1. Solve the equation $\nabla^2 u = -10(x^2+y^2+10)$ over the square mesh with sides $x=0$; $y=0$; $x=3$; $y=3$, with $u=0$ on the boundary with mesh length 1 unit.

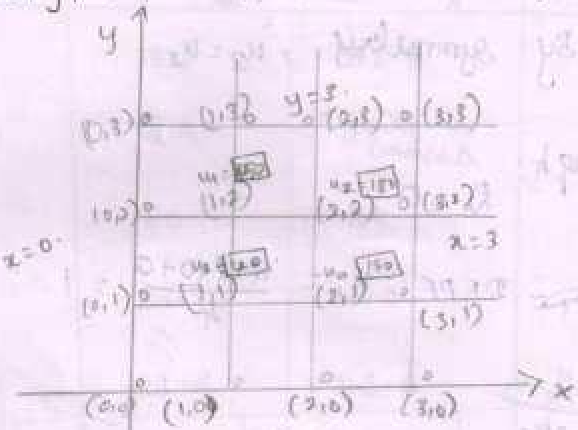
Soln:

given $\nabla^2 u = -10(x^2+y^2+10)$

$u_{xx} + u_{yy} = -f(x,y)$

$f(x,y) = 10(x^2+y^2+10)$

$h^2 f(x,y) = f(x,y) = 10(x^2+y^2+10)$ $\therefore h =$



By symmetry,

$u_1 = u_4$.

$u_1 = \frac{u_2+u_3+150}{4}$	$u_2 = \frac{u_1+u_4+180}{3} = \frac{2u_1+180}{4}$	$u_3 = \frac{u_1+u_4 \ldots}{4} = \frac{2u_1+\ldots}{4}$
0	0	0
37.5	63.8	48.8
65.7	77.9	62.9
72.7	81.4	66.4
74.5	82.3	67.3
74.9	82.5	67.5
75	82.5	67.5
75	82.5	67.5

Sri Vidya college of Engineering and Technology,virudhunagar

2. Solve $\nabla^2 u = 8x^2y^2$ over the square bounded by the lines $x=-2$; $x=2$, $y=-2$, $y=2$ with $u=0$ on the boundary and mesh length $=1$

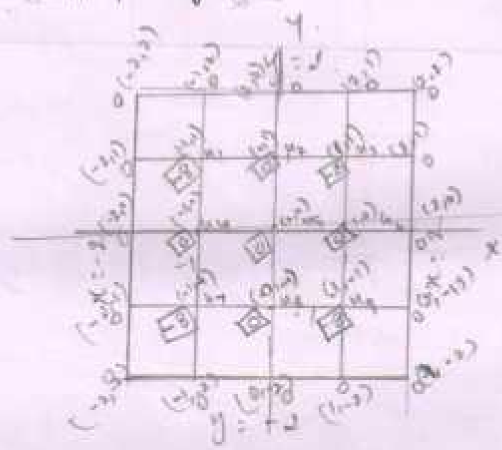
Soln :

Given $\nabla^2 u = 8x^2y^2$

w.k.t $\nabla^2 u = -f(x,y)$

$f(x,y) = -8x^2y^2$

$h^2 f(x,y) = -8x^2y^2$; $(\because h=1)$



By symmetry :

~~$u_1 = u_7$~~	~~$u_1 = u_3$~~	$u_2 = u_4$	~~$u_1 = u_9$~~
$u_2 = u_8$	$u_4 = u_6$	~~$u_3 = u_7$~~	$u_4 = u_8$
~~$u_3 = u_9$~~	~~$u_7 = u_9$~~	$u_6 = u_8$	$u_2 = u_6$

$u_1 = u_7 = u_3 = u_9$

$u_2 = u_8 = u_6 = u_4$.

$u_1 = \frac{u_2+u_4-8}{4} = \frac{2u_2-8}{4}$	$u_2 = \frac{u_1+u_3+u_5}{4} = \frac{2u_1+u_5}{4}$	$u_5 = \frac{u_2+u_4+u_6+\ldots}{4}$, $u_5 = u_2$
0	0	0
-2	-1	-1
-2.5	-1.5	-1.5
-2.8	-1.8	-1.8
-2.9	-1.9	-1.9
-3	-2	-2
-3	-2	-2