Unit - IV

Initial Value Problem for ordinary differential Equation

Method - 1

Taylor Series:

The taylor Series formula is

$$y = y_0 + (x - x_0)\frac{y_0'}{1!} + (x - x_0)^2 \frac{y_0''}{2!} + (x - x_0)^3 \frac{y_0'''}{3!} + \cdots$$

Sri Vidya college of Engineering and Technology, virudhunagar

1. Use taylor series method to find $y(0.1)$ and $y(0.2)$. Given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$;

Soln: Given $\frac{dy}{dx} = y' = 3e^x + 2y$; $y(0) = 0$;

The taylor series formula is,

$$y = y_0 + (x - x_0)\frac{y_0'}{1!} + (x - x_0)^2 \frac{y_0''}{2!} + (x - x_0)^3 \frac{y_0'''}{3!} + (x - x_0)^4 \frac{y_0^{iv}}{4!}$$

x	0	x_0
y	0	y_0
$y' = 3e^x + 2y$	3	y_0'
$y'' = 3e^x + 2y'$	9	y_0''
$y''' = 3e^x + 2y''$	21	y_0'''
$y^{iv} = 3e^x + 2y'''$	45	y_0^{iv}

$$y = 0 + (x - 0)\cdot\frac{3}{1!} + (x - 0)^2 \cdot \frac{9}{2} + (x - 0)^3 \frac{21}{6} + (x - 0)^4 \cdot \frac{45}{24};$$

$$y = 3x + \frac{9}{2}x^2 + \frac{7}{2}x^3 + \frac{15}{8}x^4$$

$y(0.1) = 0.3487.$

$y(0.2) = 0.8110.$

Sri Vidya college of Engineering and Technology, virudhunagar

2. Use taylor series method, solve $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ at $x = 0.1, 0.2, 0.3$.

Soln:

The taylor series formula is,

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + (x-x_0)^4\frac{y_0^{iv}}{4!}$$

$y' = x^2 - y$; & $y(0) = 1$

x	0	x_0
y	1	y_0
$y' = x^2 - y$	-1	y_0'
$y'' = 2x - y'$	1	y_0''
$y''' = 2 - y''$	1	y_0'''
$y^{iv} = -y'''$	-1	y_0^{iv}

$$y = 1 + (x-0)\left(\frac{-1}{1!}\right) + (x-0)^2\frac{1}{2!} + (x-0)^3 \times \frac{1}{3!} + (x-0)^4\left(\frac{-1}{4!}\right)$$

$$y = 1 - x + \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24}.$$

$y(0.1) = \cancel{0.9100}$. 0.9052

Sri Vidya college of Engineering and Technology, virudhunagar

$$= \frac{7}{6}x^4 + \frac{4}{3}x^3$$

$$y = \frac{7}{6}x^4 + \frac{4}{3}x^3 + x^2 + x + 1$$

$y(0.1) = 1.1115$

$y(0.2) = 1.2525$

4. Obtain y by taylor series method given that $y' = xy + 1$; $y(0) = 1$; for $x = 0.1$; $x = 0.2$; Correct to four decimal places.

Soln: The formula is,

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + (x-x_0)^4\frac{y_0^{iv}}{4!} + \cdots$$

x	0	x_0
y	1	y_0
$y' = xy + 1$	1	y_0'
$y'' = y + xy'$	1	y_0''
$y''' = y' + y' + xy''$	2	y_0'''
$y^{iv} = y'' + y'' + y'' + xy'''$	3	y_0^{iv}

$$y = 1 + (x-0)\frac{1}{1} + (x-0)^2\frac{1}{2} + (x-0)^3\frac{2}{6} +$$

$$(x-0)^4\ 3$$

Sri Vidya college of Engineering and Technology, virudhunagar

$$y = 1+x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{8}x^4.$$

$y(0.1) = 1.1053$

$y(0.2) = 1.2229.$

5. G.T $y''+xy'+y=0$; $y(0)=1$; $y'(0)=0$ Obtain the value of y for $x=0.1$ & $x=0.2$; 0.3 by taylor series method.

Soln:

The taylor series formula is,

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + (x-x_0)^4\frac{y_0^{iv}}{4!} + \cdots$$

x	0	x_0
y	1	y_0
y'	0	y_0'
$y''=-xy'-y.$	-1	y_0''
$y'''=-xy''-y'-y'$	0	y_0'''
$y^{iv}=-xy'''-y''-y''-y''$	$+3$	y_0^{iv}

Sri Vidya college of Engineering and Technology, virudhunagar

$$y = 1 + (x-0)\frac{0}{1!} + (x-0)^2\frac{1}{2} + (x-0)^3\frac{0}{6} + (x-0)^4\frac{+3}{24}$$

$$y = 1 - x^2/2 + x^4/8$$

$y(0.1) = 0.9950$

$y(0.2) = 0.9802$

$y(0.3) = 0.9560$

Method-II : Euler's method :

Consider $\frac{dy}{dx} = f(x,y)$

The Euler's formula is,

$$y_{n+1} = y_n + hf(x_n, y_n) \text{ (or)}$$

$$y_{n+1} = y_n + hy_n'$$

1. Solve $y' = \frac{y-x}{y+x}$, $y(0) = 1$ at $x = 0.1$ by taking $h = 0.02$; by using by Euler's method.

Soln:

$$y' = \frac{y-x}{y+x} \; ; \; y(0) = 1$$

The Euler's formula is,

$$y_{n+1} = y_n + hf(x_n, y_n)$$

(or)

$$y_{n+1} = y_n + h \cdot y_n'$$

Sri Vidya college of Engineering and Technology, virudhunagar

x	0	0.02	0.04	0.06	0.08	0.
y	1	1.02	1.0392	1.0577	1.0756	1.
$y' = \frac{y-x}{y+x}$	1	0.9615	0.9259	0.8926	0.8615	0

$n=0$;

$y_1 = y_0 + hy_0' = 1 + 0.02 \times 1 = 1.02$.

$n=1$;

$y_2 = y_1 + hy_1' = 1.02 + 0.02 \times 0.9615 = 1.0392$.

$n=2$;

$y_3 = y_2 + h \cdot y_2' = 1.0392 + 0.02 \times 0.9259 = 1.0577$.

$n=3$;

$y_4 = y_3 + hy_3' = \cancel{0.9} \; 1.0577 + 0.02 \times 0.8926$

$y_4 = 1.0756$

$n=4$;

$y_5 = y_4 + hy_4' = 1.0756 + 0.02 \times 0.8615$

$y_5 = 1.0928$.

~~n=5;~~

~~$y_6 = y_5 + hy_5'$~~ $= 1.0928 + 0.$

2. Euler's method to find $y(0.4)$ Gi
using

$\frac{dy}{dx} = x+y$, $y(0) = 1$. Taking $h = 0.2$.

Sri Vidya college of Engineering and Technology, virudhunagar

Soln:

Given $\frac{dy}{dx} = x+y$, $y(0)=1$.

The Euler's formula is $y_{n+1} = y_n + h y_n'$

x	0	0.2	0.4
y	1	1.2	1.48
$y' = x+y$	1	1.4	1.88

$n=0 \Rightarrow y_1 = y_0 + h y_0' = 1 + (0.2 \times 1) = 1.2$

$n=1 \Rightarrow y_2 = y_1 + h y_1' = 1.2 + (0.2 \times 1.4) = 1.48$.

3. Using Euler's method find the solution of the initial value problem (IVP) $\frac{dy}{dx} = \log_e(x+y)$ by assuming $h = 0.2$. $y(0)=2$ at $x=0.6$

Soln:

Given $y' = \log_{10}(x+y)$; $y(0)=2$.

The Euler's formula is $y_{n+1} = y_n + h y_n'$

x	0	0.2	0.4	0.6
y	2	2.0602	2.1810	2.2117
$y' = \log_{10}(x+y)$	0.3010	0.3541	0.4033	0.4490

$n=0 \Rightarrow y_1 = y_0 + h y_0' = 2 + (0.2 \times 0.3010) = 2.0602$.

$n=1 \Rightarrow y_2 = y_1 + h y_1' = 2.0602 + (0.2 \times 0.3541) = 2.1810$.

$n=2 \Rightarrow y_3 = y_2 + h y_2' = 2.1810 + (0.2 \times 0.4033) = 2.2117$.

4. Using Euler's method, find $y(4.1)$ & $y(4.2)$ if $5x\frac{dy}{dx} + y^2 - 2 = 0$; $y(4) = 1$

Soln:

Given $5\frac{dy}{dx} + y^2 - 2 = 0$; $y(4) = 1$

$$\frac{dy}{dx} = \frac{-y^2+2}{5x}.$$

The Euler's formula is $y_{n+1} = y_n + h y_n'$

x	4	4.1	4.2
y	1	1.0050	1.0098
$y' = \frac{-y^2+2}{5x}$	0.05	0.0483	0.0467.

$n=0 \Rightarrow y_1 = y_0 + h y_0' = 1 + 0.1(0.05)$

$= 1.0050.$

$n=1 \Rightarrow y_2 = y_1 + h y_1' = 1.005 + 0.1(0.0483)$

$= 1.0098$ //.

5. find $y(0.2)$ for $y' = y + e^x$, $y(0) = 0$ by Euler's method. Take $h = 0.1$

Soln:

Given $y' = y + e^x$, $y(0) = 0$

The Euler's formula is $y_{n+1} = y_n + hy$

x	0	0.1	0.2
y	0	0.1	0.2205.
			1.4415

Sri Vidya college of Engineering and Technology, virudhunagar

$n=0 \Rightarrow$

$$y_1 = y_0 + hy_0' = 0 + 0.1(1) = 0.1$$

$n=1 \Rightarrow$

$$y_2 = y_1 + hy_1' = 0.1 + 0.1 \times (1.2052) = 0.2205.$$

Fourth order Range - kutta method.

Consider, $g(x, y, y') = 0$.

$$y' = f(x, y)$$

$$k_1 = hf(x, y)$$

$$k_2 = hf(x + h/2, y + k_1/2)$$

$$k_3 = hf(x + h/2, y + k_2/2)$$

$$k_4 = hf(x+h, y+k_3)$$

$$y = y_0 + 1/6(k_1 + 2k_2 + 2k_3 + k_4)$$

7. using Runge - kutta method of order 4; find y value when $x = 1.2$ in steps of 0.1 given that $y' = x^2 + y^2$, $y(1) = 1.5$.

Soln:

The Runge - kutta formula is

$$k_1 = h \cdot f(x, y)$$

$$k_2 = h \cdot f(x + h/2, y + k_1/2)$$

$$k_3 = h \cdot f(x + h/2, y + k_2/2)$$

Sri Vidya college of Engineering and Technology, virudhunagar

$k_4 = h \cdot f(x+h, y+k_3)$

Given $y' = x^2 + y^2$

here, $f(x,y) = x^2 + y^2$; $h = 0.1$

x	1	1.1	1.2
y	1.5	y_1 1.8955	y_2 2.5044

To find y_1

$x = 1$; $y = 1.5$.

$k_1 = h f(x,y) = 0.1 \times f(1, 1.5)$.

$= 0.1 \times 3.25 = 0.325$.

$k_2 = h \cdot f(x + h/2, y + k_1/2) = 0.1 \times f(1.05, 1.662)$

$= 0.1 \times 3.8664 = 0.3866$.

$k_3 = h \cdot f(x + h/2, y + k_2/2) = 0.1 f(1.05, 1.6933)$

$= 0.1 \times 3.9698 = 0.3970$.

$k_4 = h \cdot f(x+h, y+k_3) = 0.1 \times f(1.1, 1.8970)$

$= 0.4809$.

$y_1 = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$= 1.5 + \frac{1}{6}\left[0.325 + 2 \times 0.3866 + 2 \times 0.3970 + 0.4809\right]$

$y_1 = 1.8955$

$f(x,y) = x^2 + y^2$

$k_1 = h \cdot f(x,y) = 0.1 \times f(1.1, 1.8955)$

$= 0.1 \times 4.8029 = 0.4803$

$k_2 = h f(x + h/2, y + k_1/2) = \cancel{0.1 \times}$

$= 0.1 f(1.150, 2.1357)$

$= 0.1 \times 5.8837 = 0.5884$

$k_3 = h f(x + h/2, y + k_2/2)$

$= 0.1 \times f(1.150, 2.1897)$

$= 0.1 \times 6.1178 = 0.6117$

$k_4 = h f(x + h, y + k_3)$

$= 0.1 \times f(1.2, 2.5072)$

$= 0.7726$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1.8955 + \frac{1}{6}[0.4803 + 2 \times 0.5884 + 2 \times 0.6117 + 0.7726]$

2. Find $y(0.7)$ & $y(0.8)$ given that $y' = y - x^2$ $y(0.6) = 1.7379$ by using RK method of 4th order.

Soln:

$k_1 = h \cdot f(x, y)$

$k_2 = h \cdot f(x + h/2, y + k_1/2)$

$k_3 = h \cdot f(x + h/2, y + k_2/2)$

$k_4 = h \cdot f(x + h, y + k_3)$

given $y' = y - x^2$.

Here $f(x, y) = y - x^2$; $h = 0.1$

	x_0	x_1	x_2
x	0.6	0.7	0.8
y	1.7379	1.8763	2.0145.

To find y_1 :

$x = 0.6$; $y = 1.7379$.

$k_1 = h \cdot f(x, y) = 0.1 \times f(0.6, 1.7379)$

$= 0.1378$.

$k_2 = 0.1 \times f(0.6 + 0.1/2, 1.7379 + 0.1378/2)$

$k_2 = \cancel{0.0240}\ 0.1384$

$k_3 = 0.1 \times f\left[0.6 + \frac{0.1}{2}, 1.7879 + 0.1384/2\right]$

$= 0.1 \times f(0.65, 1.8071)$

$= 0.1385$

$k_4 = 0.1 \times f(0.7, 1.8764)$

$= 0.1386$

$y_1 = 1.7879 + 1/6\ (0.1378 + 0.1384 \times 2 + 0.1385 \times 2 + 0.1386)$

$= 1.8763$

To find y_2.

$x = 0.7\ ;\ y = 1.8763$

$k_1 = 0.1 \times f(0.7, 1.8763) = 0.1386$

$k_2 = 0.1 \times f(0.75, 1.9456) = 0.1383$

$k_3 = 0.1 \times f(0.75, 1.9455) = 0.1383$

$k_4 = 0.1 \times f(0.8, 2.0146) = 0.1375$

$y_2 = y_1 + 1/6\ [k_1 + 2k_2 + 2k_3 + k_4]$

$= 1.8763 + 1/6\ (0.1386 + 2 \times 0.1383 + 2 \times 0.1383 + \cancel{0.1402}\ 0.1375)$

Sri Vidya college of Engineering and Technology, virudhunagar

8. using R.K method to find $y(0.2)$, $y(0.4)$. Given to $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $y(0)=1$

Soln:

$$y' = \frac{y^2-x^2}{y^2+x^2}$$

Here, $f(x,y) = \frac{y^2-x^2}{y^2+x^2}$; $h=0.2$

x	0	0.2	0.4
y	1	1.1960	

To find y_1 :

$x=0$; $y=1$

$k_1 = h \cdot f(x,y) = 0.2 \times f(0,1)$

$= 0.2$

$k_2 = 0.2 \times f(0.1, 1.1) = 0.1967$.

$k_3 = 0.2 \times f(0.1, 1.0984) = 0.1967$.

$k_4 = 0.2 \times f(0.2, 1.1967) = 0.1891$

$y_1 = 1 + \frac{1}{6}(0.2 + 2 \times 0.1967 + 2 \times 0.1967 + 0.1891)$

$= 1.1960$

To find y_2 :-

$x = 0.2$; $y = 1.1960$.

$k_1 = 0.2 \times f(0.2, 1.1960) = 0.1891$

$k_2 = 0.2 \times f(0.3, 1.2906) = 0.1795$

$k_3 = 0.2 \times f(0.3, 1.2842) = 0.1798$.

$k_4 = 0.2 \times f(0.4, 1.3753) = 0.1688$

$$y_2 = 1.1960 + \frac{1}{6}(0.1891 + 2 \times 0.1763 + 0.1793 \times 2 + 0.1688)$$

$= 1.3753$.

Sri Vidya college of Engineering and Technology, virudhunagar

2

17/8/14. Using R.K method for solving simultaneous Equation :

consider,

$$\frac{dy}{dx} = f(x,y,z) \; ; \; \frac{dz}{dx} = g(x,y,z)$$

$f(x,y,z)$	$g(x,y,z)$
$k_1 = h \cdot f(x,y,z)$	$l_1 = h \cdot g(x,y,z)$
$k_2 = h \cdot f(x+h/2, y+k_1/2, z+l_1/2)$	$l_2 = h \cdot g(x+h/2, y+k_1/2, z+l_1/2)$
$k_3 = h \cdot f(x+h/2, y+k_2/2, z+l_2/2)$	$l_3 = h \cdot g(x+h/2, y+k_2/2, z+l_2/2)$
$k_4 = h \cdot f(x+h, y+k_3, z+l_3)$	$l_4 = h \cdot g(x+h, y+k_3, z+l_3)$

$$y_1 = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right].$$

$$z_1 = z_0 + 1/6\left[l_1 + 2l_2 + 2l_3 + l_4\right].$$

1. Solve for $y(0.1)$ and $z(0.1)$ from the simultaneous equation $\frac{dy}{dx} = 2y + z$; $\frac{dz}{dx} = y - 3z$ $y(0) = 0$; $z(0) = 0.5$; using R.K method of order 4 !

Soln :

Given, $\frac{dy}{dx} = y - 3z$; $g(x,y,z) = y - 3z$.

Sri Vidya college of Engineering and Technology, virudhunagar

x	0	0.1
y	0	0.0481
z	0.5	0.3726

$h = 0.1$

$f(x,y,z) = 2y+z$	$g(x,y,z) = y-3z$
$k_1 = h.f(x,y,z)$ $= 0.1 \times f(0,0,0.5)$ $k_1 = 0.05$	$l_1 = 0.1 \times g(0,0,0.5)$ $l_1 = -0.15$
$k_2 = h.f(x+h/2, y+k_1/2, z+l_1/2)$ $= 0.1 \times f(0.05, 0.025, 0.425)$ $k_2 = 0.0475$	$l_2 = 0.1 \times g(0.05, 0.025, 0.4$ $l_2 = -0.125$
$k_3 = h.f(x+h/2, y+k_2/2, z+l_2/2)$ $= 0.1 \times f(0.05, 0.0238, 0.4375)$ $k_3 = 0.0485$	$l_3 = 0.1 \times g(0.05 \times 0.238 \times 0.4375)$ $l_3 = -0.1289$
$k_4 = h.f(x+h, y+k_3, z+l_3)$ $= 0.1 \times f(0.1, 0.0485, 0.3711)$ $= 0.0468$	$l_4 = 0.1 \times g(0.1, 0.0485, 0.37$ $= -0.1065$

Sri Vidya college of Engineering and Technology, virudhunagar

$y_1 = y_0 + 1/6\ (k_1 + 2k_2 + 2k_3 + k_4)$

$= 0 + 1/6\ (0.05 + 2\times 0.0475 + 2\times 0.0485 + 0.0460)$

$= 0.0481$

$z_1 = 0.5 + 1/6\ (-0.15 - 2\times 0.125 - 2\times 0.1289 - 0.1065)$

$= 0.3726$

R.K method for solving second order equation.

Consider, $\varphi(x, y, y', y'') = 0$ —— ①

take $y' = z$ —— ②

By using ② in ①, we get

$z' = g(x, y, z)$

1. Given $y'' + xy' + y = 0$; $y(0) = 1$; $y'(0) = 0$; find the value of $y(0.1)$ by using R.K method

Soln:

Given, $y'' + xy' + y = 0$. —— ①

Take $y' = z$;

$z' + xz + y = 0$.

Sri Vidya college of Engineering and Technology, virudhunagar

$z' = -xz - y.$

x	0	0.1
y	1	0.9950
$z = y'$	0	-0.0995

$h = 0.1$

$f(x,y,z) = z$	$g(x,y,z) = -xz - y$
$k_1 = h \cdot f(x,y,z)$ $= 0.1 \times f(0,1,0)$ $k_1 = 0.$	$l_1 = 0.1 \times g(0,1,0)$ $= -0.1$
$k_2 = h \cdot f(x + h/2, y + k_1/2, z + l_1/2)$ $= 0.1 \times f(0.05, 1, -0.05)$ $= -0.005$	$l_2 = 0.1 \times g(0.05, 1, -0.05)$ $= -0.0998$
$k_3 = h \cdot f(x + h/2, y + k_2/2, z + l_2/2)$ $= 0.1 \times f(0.05, 0.9975, -0.0499)$ $= -0.005$	$l_3 = 0.1 \times g(0.05, 0.9975, -0.0499)$ $= -0.0995$
$k_4 = h \cdot f(x+h, y+k_3, z+l_3)$ $= 0.1 \times f(0.1, 0.9950, -0.0995)$ $= -0.0100$	$l_4 = 0.1 \times g(0.1, 0.9950, -0.09$ $= \cancel{0.1015} = -0.0985$

Sri Vidya college of Engineering and Technology, virudhunagar

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1 + \frac{1}{6}(0 - 2\times 0.005 - 2\times 0.005 - 0.01)$$

$$= 0.9950 \,//.$$

$$z_1 = 0 + \frac{1}{6}(-0.1 - 2\times 0.0998 - 2\times 0.0997 - 0.0985)$$

$$= -0.0995 \,//.$$

2. Consider the 2nd order initial value pbm: $y'' - 2y' + 2y = e^{2t}\sin t$; $y(0) = -0.4$; $y'(0) = -0.6$ using 4th order R.K method find $y(0.2) = ?$

Soln:

Given $y'' - 2y' + 2y = e^{2x}\sin x$

Take $y' = z$.

$$f(x, y, z) = z.$$

$$z' - 2z + 2y = e^{2x}\sin x.$$

$$z' = e^{2x}\sin x - 2y + 2z.$$

$$g(x, y, z) = e^{2x}\sin x - 2y + 2z.$$

Sri Vidya college of Engineering and Technology, virudhunagar

x	0	0.2
y	-0.4	
$z = y'$	-0.6	

$h = 0.2$

$f(x,y,z) = z$	$g(x,y,z) = e^{2x}\sin x - 2y + 2z$
$k_1 = h\cdot f(x,y,z)$ $= 0.2 \times f(0,-0.4,-0.6)$ $= -0.12$	$l_1 = 0.2 \times g(0,-0.4,-0.6)$ $l_1 = -0.08$
$k_2 = h\cdot f(x+h/2,\ y+k_1/2,\ z+l_1/2)$ $= 0.2 \times f(0.1,-0.46,-0.64)$ $= -0.1280$	$l_2 = 0.2 \times g(0.1,-0.46,-0.64)$ $= \cancel{-0.0592}\ -0.0476$
$k_3 = h\cdot f(x+h/2,\ y+k_2/2,\ z+l_2/2)$ $= 0.2 \times f(0.1,\ \cancel{-0.4360}\ -0.4640,\ \cancel{-0.6288}\ -0.6238)$ $= \cancel{-0.1257}\ -0.1248$	$l_3 = 0.2 \times g(0.1,-0.4640,-0.6286)$ $= \cancel{-0.0588}\ -0.0395$
$k_4 = h\cdot f(x+h,\ y+k_3,\ z+l_3)$ $= 0.2 \times f(0.2,\ \cancel{-0.5257}\ -0.5248,\ \cancel{-0.6511}\ -0.6395)$ $= \cancel{-0.1302}\ -0.1279$	$l_4 = 0.2 \times g(0.2,\ \cancel{-0.5257}\ -0.5248,\ \cancel{-0.6511}\ -0.6395)$ $= \cancel{+0.0086}\ -0.0134$

Sri Vidya college of Engineering and Technology, virudhunagar

$y_1 = y_0 + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$

$= -0.47\ 1/6\ (-0.12 - 2\times0.1280 - 2\times \cancel{0.1257}^{0.1248} - \cancel{0.1302}^{-0.1279})$

$= \cancel{-0.5263} \quad -0.5256$

$z_1 = -0.6 + 1/6\,(-0.08 - 2\times\cancel{0.0572}^{0.0476} - 2\times\cancel{0.0511}^{0.0395} + \cancel{0.0086}\ -0.0134)$

$= -0.6480$

$= -0.6401$

13/3/14.

Milne's Predictor - corrector Method.

Consider $\dfrac{dy}{dx} = f(x,y)$

$$P: \quad y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$C: \quad y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

1. By using Milne's predictor - corrector formula to find $y(0.4)$ & $y(0.5)$; G.T $\dfrac{dy}{dx} = \dfrac{(1+x^2)y^2}{2}$, $y(0) = 1$; $y(0.1) = 1.06$; $y(0.2) = 1.12$; $y(0.3) = 1.21$

Sri Vidya college of Engineering and Technology, virudhunagar

Soln: The Milne's predictor - corrector formula is,

$$P: \; y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right] \quad \text{--- ①}$$

$$C: \; y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right] \quad \text{--- ②}$$

x	0 x_0	0.1 x_1	0.2 x_2	0.3 x_3	0.4 x_4		[illegible] x_5
y	1 y_0	1.06 y_1	1.12 y_2	1.21 y_3	1.2771 y_4	1.2797	$1.$[illegible]
$y' = \frac{(1+x^2)y^2}{2}$	y_0' 0.5	y_1' 0.5674	y_2' 0.6523	y_3' 0.7979	0.9460	0.9478	$1.$[illegible]

Put n=3 in ①.

$$P: \; y_4 = y_0 + \frac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right)$$

$$= 1 + \frac{4 \times 0.1}{3}\left(2 \times 0.5674 - 0.6523 + 2 \times 0.7979\right)$$

$$P: \; y_4 = 1.2771.$$

put n=3 in eqn ②.

$$C: \; y_4 = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right)$$

$$= 1.12 + \frac{0.1}{3}\left(0.6523 + 4 \times 0.7979 + 0.9460\right]$$

$$C: \; y_4 = 1.2797.$$

put $n=4$ in ①,

$$P: y_5 = y_1 + \frac{4h}{3}\left[2y_2' - y_3' + 2y_4'\right]$$

$$= 1.06 + \frac{4\times0.1}{3}\left[2\times0.6523 - 0.7979 + 2\times0.9498\right]$$

$$P: y_5 = 1.3808.$$

put $n=4$ in ②,

$$C: y_5 = y_3 + \frac{h}{3}\left(y_3' + 4y_4' + y_5'\right)$$

$$= 1.21 + \frac{0.1}{3}\left(0.7979 + 4\times0.9498 + 1.1916.\right)$$

$$y_5 = 1.4030.$$

3808	1.4030
1916	1.2303

② Given $y' = \frac{1}{x+y}$; $y(0) = 2$; $y(0.2) = 2.0933$; $y(0.4) = 2.1755$, $y(0.6) = 2.2493$. Find $y(0.8)$ by using Milne's method.

Soln: The Milne's formula is,

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y_{n-2}' - y_{n-1}' + 2y_n'\right] \quad — ①$$

$$C: y_{n+1} = y_{n-1} + \frac{h}{3}\left[y_{n-1}' + 4y_n' + y_{n+1}'\right] \quad — ②$$

	x_0	x_1	x_2	x_3	x_4
x	0	0.2	0.4	0.6	0.8
y	y_0 2	y_1 2.0933	y_2 2.1755	y_3 2.2493	2.3162
$y' = \frac{1}{x+y}$	y_0' 0.5	y_1' 0.4861	y_2' 0.3883	y_3' 0.3510	0.3209

put $n=0$ in ①

$$P:\ y_4 = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right].$$

$$= 2 + \frac{4\times 0.2}{3}\left[2\times 0.4861 - 0.3883 + 2\times 0.3510\right]$$

$$P: y_4 = 2.3162.$$

put $n=3$ in ②

$$C:\ y_4 = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 2.1755 + \frac{0.2}{3}\left[0.3883 + 4\times 0.3510 + 0.3209\right]$$

$$C: y_4 = 2.3164 \,//.$$

19/3/14.

3. Given $y' = xy + y^2$, $y(0) = 1$; $y(0.1) = 1.1169$; $y(0.2) = 1.2774$. using R.K method of 4th order, find $y(0.3)$. Continue the solution at $x = 0.4$ using milne's method.

Sri Vidya college of Engineering and Technology, virudhunagar

soln:

x	0	0.1	0.2	0.3
y	1	1.1169	1.2774	1.5042

Here, $h = 0.1$;

$y' = xy + y^2$

$f(x,y) = xy + y^2$

To find y_3;

$x = 0.2$; $y = 1.2774$.

$k_1 = h.f(x,y) = 0.1 \times f(0.2, 1.2774) = 0.1887$.

$k_2 = h \cdot f\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right) = 0.1 \times f(0.25, 1.3718)$

$= 0.2225$.

$k_3 = h \cdot f\left(x + h/2, y + \frac{k_2}{2}\right) = 0.1 \times f(0.25, 1.3887)$

$= 0.2276$

$k_4 = h \cdot f(x+h, y+k_3) = 0.1 \times f(0.3, 1.5050) = 0.2717$.

$y_3 = y_2 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$= 1.2774 + \frac{1}{6}\left[0.1887 + 2 \times 0.2225 + 2 \times 0.2276 + 0.2717\right]$

$= 1.5042$.

Sri Vidya college of Engineering and Technology, virudhunagar

Milne's formula is,

$$P: \quad y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$C: \quad y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

	x_0	x_1	x_2	x_3	x_4	
x	0	0.1	0.2	0.3	0.4	
y	y_0 1	y_1 1.1169	y_2 1.2774	y_3 1.5042	1.8345	1.8
$y' = xy + y^2$	y'_0 1	y'_1 1.3592	y'_2 1.8872	y'_3 2.7139	4.0992	4.1

Put $n = 3$ in (1)

$$P: \quad y_4 = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$= 1 + \frac{4 \times 0.1}{3}\left[2 \times 1.3592 - 1.8872 + 2 \times 2.7139\right]$$

$$= 1.8345.$$

Put $n = 3$ in (2)

$$C: \quad y_4 = y_2 + \frac{0.1}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$$= 1.2774 + \frac{0.1}{3}\left[1.8872 + 4 \times 2.7139 + 4.099\right.$$

$$= 1.8388$$

4. Given that $y'' + xy' + y = 0$, $y(0) = 1$; $y'(0) = 0$ obtain y for $x = 0.1, 0.2$ and 0.3 by Taylor series method and find the soln for $y(0.4)$ by Milne's method.

soln:

The Taylor series is,

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + (x-x_0)^4\frac{y_0^{iv}}{4!} + \cdots$$

$y'' + xy' + y = 0$.

$y'' = -xy' - y$.

x	0	x_0
y	1	y_0
y'	0	y_0'
$y'' = -xy' - y$	-1	y_0''
$y''' = -xy'' - y' - y'$	0	y_0'''
$y^{iv} = -xy''' - y''' - y'' - y''$	3	y_0^{iv}

$$y = 1 + (x-0)\,0/1 + (x-0)^2 \times -1/2 + (x-0)^3\,0/6 +$$

$$y = 1 - \frac{x^2}{2} + \frac{x^4}{8}$$

$$y' = -\frac{2x}{2} + \frac{4x^3}{8} \Rightarrow y' = -x + \frac{x^3}{2}$$

$y(0.1) = 0.9950$

$y(0.2) = 0.9802$

$y(0.3) = 0.9560$

The Milne's formula is,

$$P: \quad y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_{n}\right]$$

$$C: \quad y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_{n} + y'_{n+1}\right]$$

x	x_0 0	x_1 0.1	x_2 0.2	x_3 0.3	x_4 0.4	
y	y_0 1	y_1 0.9950	y_2 0.9802	y_3 0.9560	0.9232	0.9232
$y' = -x + \frac{x^3}{2}$	y'_0 0	y'_1 −0.0995	y'_2 −0.1960	y'_3 −0.2865	−0.3680	−0.3680

put $n = 3$;

$$P: \quad y_4 = y_0 + \frac{4 \times 0.1}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$= 1 + \frac{0.4}{3}\left[2 \times (-0.0995) + 0.1960 + 2 \times (-0.2\ldots\right.$$

$$= 0.9232.$$

put $n = 3$;

$$C: \quad y_4 = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$$= 0.9802 + \frac{0.1}{3}\left[-0.1960 - 4\times 0.2865 \cancel{+0.9232} - 0.3680\right]$$

$C: y_4 = 0.9232$

Adam's Bashforth predictor - corrector formula:

$$P: y_{n+1} = y_n + \frac{h}{24}\left[55 y_n' - 59 y_{n-1}' + 37 y_{n-2}' - 9 y_{n-3}'\right]$$

$$C: y_{n+1} = y_n + \frac{h}{24}\left[19 y_n' - 5 y_{n-1}' + y_{n-2}' + 9 y_{n+1}'\right].$$

1. Using Adam's method find $y(1.4)$
given $y' = x^2(1+y)$, $y(1) = 1$; $y(1.1) = 1.233$;
$y(1.2) = 1.548$ & $y(1.3) = 1.979$.

Soln:

The Adam's formula is,

$$P: y_{n+1} = y_n + \frac{h}{24}\left[55 y_n' - 59 y_{n-1}' + 37 y_{n-2}' - 9 y_{n-3}'\right]$$

$$C: y_{n+1} = y_n + \frac{h}{24}\left[19 y_n' - 5 y_{n-1}' + y_{n-2}' + 9 y_{n+1}'\right]$$

x	x_0 1	x_1 1.1	x_2 1.2	x_3 1.3	x_4 1.4
y	y_0 1	y_1 1.233	y_2 1.548	y_3 1.979	y_4 2.5723 2.5749
$y' = x^2(1+y)$	y_0' 2	y_1' 2.7019	y_2' 3.6691	y_3' 5.0345	y_4' 7.0017 7.0068

Sri Vidya college of Engineering and Technology, virudhunagar

put $n=3$;

$$P: y_4 = y_3 + \frac{0.1}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 1.979 + \frac{0.1}{24}\left[55\times 5.0345 - 59\times 3.6691 + 37\times 2.7019 - 9\times 2\right]$$

$$P: y_4 = 2.5723.$$

put $n=3$ in ①

$$C: y_4 = y_3 + \frac{h}{24}\left[19y_3' - 5y_2' + y_1' + 9y_4'\right]$$

$$= 1.979 + \frac{0.1}{24}\left[19\times 5.0345 - 5\times 3.6691 + 2.70 + 9\times 7.0017\right]$$

$$C: y_4 = 2.5749.$$

2. Use Adam's method to find $y(2)$ if

$y' = \frac{x+y}{2}$, $y(0)=2$; $y(0.5)=2.636$; $y(1)=$ and $y(1.5)=4.968$.

Soln:

The Adam's formula is,

$$P: y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - \right.$$

$$C: y_{n+1} = y_n + \frac{h}{24}\left[19y_n' - 5y_{n-1}' + y_{n-2}' + 9y_{n+1}'\right]$$

Sri Vidya college of Engineering and Technology, virudhunagar

	x_0	x_1	x_2	x_3	x_4	
x	0	0.5	1	1.5	2	
y	y_0 2	y_1 2.636	y_2 3.595	y_3 4.968	6.8708	6.8731
$y' = \frac{x+y}{2}$	y_0' 1	y_1' 1.5680	y_2' 2.2975	y_3' 3.2340	4.4354	4.4366

put $n=3$ in ①

$$P: \quad y_4 = y_3 + \frac{0.5}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 4.968 + \frac{0.5}{24}\left[55 \times 3.2340 - 59 \times 2.2975 + 37 \times 1.5680 - 9 \times 1\right]$$

$$= 6.8708$$

$$C: \quad y_4 = y_3 + \frac{h}{24}\left[19y_3' - 5y_2' + y_1' + 9y_4'\right]$$

$$= 4.968 + \frac{0.5}{24}\left[19 \times 3.2340 - 5 \times 2.2975 + 1.5680 + 9 \times 4.4354\right]$$

$$= 6.8731.$$

21/3/14.

3. Using Adam's method find $y(0.4)$ given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$; $y(0.1) = 1.1169$; $y(0.2) = 1.2774$; and $y(0.3) = 1.5041$

Soln:

The Adam's formula is,

$$P: \quad y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

Sri Vidya college of Engineering and Technology, virudhunagar

$$c : y_{n+1} = y_n + \frac{h}{24}\left[19y_n' - 5y_{n-1}' + y_{n-2}' + 9y_{n+1}'\right.$$

x	x_0 0	x_1 0.1	x_2 0.2	x_3 0.3	x_4
y	y_0 1	y_1 1.1169	y_2 1.2774	y_3 1.5041	1.8841
$\frac{dy}{dx} = xy+y^2$	y_0' 1	y_1' 1.3592 0.0895	y_2' 1.8872 0.1566	y_3' 2.7185	4.0976

put n = 3 in

$$P : y_4 = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9\right.$$

$$= 1.5041 + \frac{0.1}{24}\left[55\times2.7185 - 59\times1.8872 + 37\times1.3592 - 9\times1\right]$$

$$P : y_4 = 1.8841$$

put n = 3 in

$$c : y_4 = y_3 + \frac{h}{24}\left[19y_3' - 5y_2' + y_1' + 9y_4'\right]$$

$$= 1.5041 + \frac{0.1}{24}\left[19\times2.7185 - 5\times1.8872 + 1.3592 + 9\times4.0976\right]$$

$$= 1.8889.$$