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UNIT - 3

Numerical Differentiation and Integration

Numerical differentiation :

It is the Process of finding the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ & $\frac{d^3y}{dx^3}$, ... for some particular value of x.

① find the first derivatives of $f(x)$ at $x=2$ for the data $f(-1)=-21$, $f(1)=15$, $f(2)=12$, $f(3)=3$. using Newtons divided difference formula.

Soln

x	-1	1	2	3
y	-21	15	12	3

The Newton's divided difference formula is

$$y = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \cdots$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	-21			
		18		
1	15		-7	
		-3		1
2	12		-3	
		-9		
3	3			

$$y = -21 + (x+1)\,18 + (x+1)(x-1)(-7) + (x+1)(x-1)(x-2)(1)$$

$$= -21 + 18x + 18 - 7(x^2-1) + (x^2-1)(x-2)$$

$$= -21 + 18x + 18 - 7x^2 + 7 + x^3 - 2x^2 - x + 2$$

$$y = x^3 - 9x^2 + 17x + 6$$

$$y' = 3x^2 - 18x + 17$$

$$y'(2) = -7$$

(2) find $f'(10)$ from the following data

x	3	5	11	27	34
$f(x)$	-13	23	899	17315	35606

The newtons divided difference formula is

$$y = f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \cdots$$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
3	-13				
		18			
5	23		16		
		146		1	
11	899		40		0
		1026		1	
27	17315		69		
		2613			
34	35606				

$$y = f(x) = -13 + 18(x-3) + 16(x-3)(x-5) + (x-3)(x-5)(x-11)$$

$$= -13 + 18x - 54 + 16[x^2 - 8x + 15] + (x^2 - 8x + 15)(x-11)$$

$$= -13 + 18x - 54 + 16x^2 - 128x + 240 + x^3 - 11x^2 - 8x^2 + 88x + 15x - 165$$

$$f(x) = x^3 - 3x^2 - 7x + 8$$

$$f'(x) = 3x^2 - 6x - 7$$

$$f'(10) = 233.$$

Newton's forward formula for derivatives

$$y = f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \cdots \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y''' = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{(24u-36)}{4!}\Delta^4 y_0 + \cdots\right]$$

① Find the first three derivatives of $f(x)$ at $x = 1.5$ & at $x = 4.0$ using Newton's forward interpolation formula to the data given below.

x	1.5	2	2.5	3	3.5	4
y	3.375	7	13.625	24	38.875	59

Soln

$$f'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$f'''(x) = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{(24u-36)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{x-1.5}{0.5}$$

When $x = 1.5$ $\boxed{u=0}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375					
		3.625				
2	7		3			
		6.625		0.75		
2.5	13.625		3.75		0	
		10.375		0.75		0
3	24		4.5		0	
		14.875		0.75		
3.5	38.875		5.25			
		20.125				
4	59					

$$f'(1.5) = \frac{1}{0.5}\left[3.625 + (0-1)\cdot\frac{3}{2} + \frac{2}{6}(0.75)\right]$$

$$= \frac{1}{0.5}\left[3.625 - 1.5 + 0.25\right]$$

$$= 4.75$$

$$f''(1.5) = \frac{1}{0.5^2}\left[3 + (-6)\times\frac{0.75}{6}\right]$$

$$= \frac{1}{0.5^2}\left[3 - 0.75\right] = 9$$

$$f'''(1.5) = \frac{1}{0.5^3}\left[0.75\right] = 6$$

Newton's Backward Interpolation formula

$$y' = \frac{1}{h}\left[\nabla y_n + \frac{(2v+1)}{2!}\nabla^2 y_n + \frac{(3v^2+6v+2)}{3!}\nabla^3 y_n + \frac{(4v^3+18v^2+22v+6)}{4!}\nabla^4 y_n + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\nabla^2 y_n + (6v+6)\frac{\nabla^3 y_n}{3!} + \frac{(12v^2+36v+22)}{4!}\nabla^4 y_n + \cdots\right]$$

$$y''' = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{(24v+36)}{4!}\frac{\nabla^4 y_n}{\cancel{\ }} + \cdots\right]$$

$$v = \frac{x - x_n}{h} = \frac{x-4}{0.5}$$

When $x = 4 \Rightarrow \boxed{v = 0}$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$f'''(x) = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{(24u-36)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{x-1.5}{0.5}$$

when $x = 1.5$ $\boxed{u=0}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375					
		3.625				
			3			
2	7			0.75		
		6.625			0	
2.5	13.625		3.75			0
		10.375		0.75		
3	24		4.5		0	
		14.875		0.75		
3.5	38.875		5.25			
		20.125				
4	59					

$$y' = \frac{1}{0.5}\left[20.125 + \frac{1}{2}\times 5.25 + \frac{2}{6}\times 0.75\right]$$

$$= 46$$

$$y'' = \frac{1}{0.5^2}\left[5.25 + 6\times\frac{0.75}{6}\right] = 24$$

$$y''' = \frac{1}{0.5^3}\left[0.75\right] = 6.$$

(2) For the given data, find the first two derivatives at $x = 1.1$

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Soln

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{x-1.0}{0.1}$$

At $x = 1.1$ $\quad u = \frac{1.1-1.0}{0.1} = 1.$

$y' = \frac{1}{0.1}[0.4$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.0	7.989					
		0.4140				
1.1	8.403		-0.0360			
		0.3780		0.0060		
1.2	8.781		-0.03		-0.0020	
		0.3480		0.0040		0.001
1.3	9.129		-0.0260		-0.0010	
		0.3220		0.003		0.003
1.4	9.451		-0.0230		0.002	
		0.2990		0.0050		
1.5	9.750		-0.0180			Δ^6
		0.2810				
1.6	10.031					0.00

$$y'(1.1) = \frac{1}{0.1}\left[0.414 + \frac{(2-1)}{2}(-0.0360) + \frac{(3-6+2)}{6}(0.006) + \frac{(4-18+22-6)}{24}(-0.002\right]$$

$$= \frac{1}{0.1}\left[0.414 - 0.0180 - 0.0010 - 0.0002\right]$$

$$= 3.9480$$

$$y''(1.1) = \frac{1}{(0.1)^2}\left[(-0.0360) + \left(\frac{6-6}{6}\right)(0.0060)\right] + \left(\frac{12-36+22}{24}\right)(-0.0020)\Big]$$

$$= 100\left[-0.0360 + 0\right] + \frac{(-2)}{24}(-0.0020)\Big]$$

$= -36 + 0.00016$

$= \cancel{-35.9998}\ -3.584$

(3) find the first two derivatives of $x^{1/3}$ at $x = 50$ and $x = 56$ for the given data

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Soln

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
50	3.6840						
		0.0244					
51	3.7084		-0.0003				
		0.0241		0			
52	3.7325		-0.0003		0		
		0.0238		0		0	
53	3.7563		-0.0003		0		0
		0.0235		0		0	
54	3.7798		-0.0003		0		
		0.0232		0			
55	3.8030		-0.0003				
		0.0229					
56	3.8259						

Newton's forward formula:

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$= -36 + 0.00016$

$= \cancel{-35.9998}$ -3.584

(3) find the first two derivatives of $x^{1/3}$ at $x = 50$ and $x = 56$ for the given data

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Soln

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
50	3.6840						
		0.0244					
51	3.7084		-0.0003				
		0.0241		0			
52	3.7325		-0.0003		0		
		0.0238		0		0	
53	3.7563		-0.0003		0		0
		0.0235		0		0	
54	3.7798		-0.0003		0		
		0.0232		0			
55	3.8030		-0.0003				
		0.0229					
56	3.8259						

Newton's forward formula:

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{50-50}{1} = 0$$

$$y' = \frac{1}{1}\left[0.02414 + \frac{(-1)}{2}(-0.0003)\right]$$

$$= 0.0244 + 0.0002$$

$$= 0.0246$$

$$y'' = \frac{1}{1}\left[-0.0003\right] = -0.0003$$

Newton's Backward Interpolation formula.

$$y' = \frac{1}{h}\left[\nabla y_n + \frac{(2v+1)}{2!}\nabla^2 y_n + \frac{(3v^2+6v+2)}{3!}\nabla^3 y_n + \frac{(4v^3+18v^2+22v+6)}{4!}\nabla^4 y_n + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{(6v+6)}{3!}\nabla^3 y_n + \frac{(12v^2+36v+22)}{4!}\nabla^4 y_n + \cdots\right]$$

$$v = \frac{x-x_n}{h} = \frac{x-56}{0.5}$$

$$v = \frac{56-56}{0.5} = 0$$

$$y' = \frac{1}{0.5}\left[0.0299 + \frac{(0+1)}{2!}(-0.0003) + \frac{2}{3!}(0) + 0\right]$$

$$= \frac{1}{0.5}\left[0.0299 - \frac{0.0003}{2}\right]$$

$$= 0.0595$$

$$y'' = \frac{1}{0.5^2}\left[-0.0003\right] = -0.0012$$

Numerical Integration

Trapezoidal rule

$$I = \int_a^b f(x)\,dx = \frac{h}{2}\left[(\text{Sum of first and last ordinate}) + 2(\text{Sum of remaining ordinates})\right]$$

$$h = \frac{b-a}{n}$$

Simpson's 1/3 rule

$$I = \int_a^b f(x)\,dx = \frac{h}{3}\left[(\text{first} + \text{Last}) + 4(\text{Sum of odd ordinates}) + 2(\text{Sum of even ordinates})\right]$$

$$h = \frac{b-a}{n} - [\text{multiples of } 2]$$

Simpson's 3/8 rule

$$I = \frac{3h}{8}\left[(\text{first Last}) + 2(\text{Sum of multiples of } 3) + 3(\text{Sum of non-multiples of } 3)\right]$$

$$h = \frac{b-a}{n} \quad [\text{multiples of } 3]$$

1. Using Trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

Soln

$$h = \frac{b-a}{n} = \frac{1+1}{8} = \frac{2}{8} = 0.25$$

x	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
y	0.5	0.65	0.8	0.9412	1	0.9412	0.8	0.64	0.5

$$I = \frac{h}{2}\left[(y_0+y_8) + 2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]$$

$$= \frac{0.25}{2}\left[(0.5+0.5) + 2(0.65+0.8+0.9412+1+0.9412+0.8+0.64)\right]$$

$$= \frac{0.25}{2}\left[1 + 2(5.7624)\right]$$

$$= \frac{0.25}{2}\left[12.5248\right]$$

$$= 1.5656$$

2) Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ with $h = 1/6$ by Trapezoidal rule.

Soln

$$f(x) = \frac{1}{1+x^2} \qquad h = 1/6$$

x	0	$1/6$	$2/6$	$3/6$	$4/6$	$5/6$	1
y	1	$36/37$	$9/10$	$4/5$	$9/13$	$36/61$	$1/2$

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{(1/6)}{2}\left[(1+1/2) + 2\left(\frac{36}{37} + \frac{9}{10} + \frac{4}{5} + \frac{9}{13} + \frac{36}{61}\right)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 2(3.9554)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 7.9108\right]$$

$$= 0.7842$$

(3) Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ by Trapezoidal rule. Also check up the results by actual Integration

Soln

$f(x) = \frac{1}{1+x^2}$, $h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
y	1.00	0.500	0.200	0.100	0.058824	0.038426	0.27026

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.027027) + 2(0.5+0.2+0.1+0.058824+0.038462)\right]$$

$$= 1.41079950$$

By actual Integration

$$I = \int_0^6 \frac{1}{1+x^2}\,dx = \left[\tan^{-1}x\right]_0^6 = \tan^{-1}6 - \tan^{-1}0$$

$$= 1.40564765$$

(4) Evaluate $\int_{1.0}^{1.3} \sqrt{x}\,dx$ taking $h = 0.05$ by trapezoidal rule

Soln

$f(x) = \sqrt{x}$

$h = \frac{b-a}{n} = 0.05$

x	1.0	1.05	1.1	1.15	1.2	1.25	1.3
y	1	1.0247	1.0488	1.0724	1.0954	1.1180	1.1402

$$I = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{0.05}{2}\left[(1 + 1.402) + 2(1.0247 + 1.0488 + 1.0724 + 1.0954 + 1.1180)\right]$$

$$= 0.1\left[2.1402 + 2(5.3593)\right]$$

$$= 0.1\left[2.1402 + 10.7186\right]$$

$$= \cancel{0.1}\ 0.025\ [12.8588]$$

$$= \cancel{1.28588}\ 0.3214$$

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{(1/6)}{2}\left[(1+1/2) + 2\left(\frac{36}{37}+\frac{9}{10}+\frac{4}{5}+\frac{9}{13}+\frac{36}{61}\right)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 2(3.9554)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 7.9108\right]$$

$$= 0.7842$$

(3) Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ by Trapezoidal rule. Also check up the results by actual Integration

Soln

$f(x) = \frac{1}{1+x^2}$, $h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
y	1.00	0.500	0.200	0.100	0.058824	0.038426	0.27026

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.027027) + 2(0.5+0.2+0.1 + 0.058824 + 0.038462)\right]$$

$$= 1.41079950$$

By actual Integration

$$I = \int_0^6 \frac{1}{1+x^2}\,dx = \left[\tan^{-1}x\right]_0^6 = \tan^{-1}6 - \tan^{-1}0$$

$$= 1.40564765$$

(4) Evaluate $\int_{1.0}^{1.3} \sqrt{x}\,dx$ taking $h = 0.05$ by trapezoidal rule

Soln

$f(x) = \sqrt{x}$

$h = \frac{b-a}{n} = 0.05$

x	1.0	1.05	1.1	1.15	1.2	1.25	1.3
y	1	1.0247	1.0488	1.0724	1.0954	1.1180	1.1402

$$I = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{0.05}{2}\left[(1 + 1.402) + 2(1.0247 + 1.0488 + 1.0724 + 1.0954 + 1.1180)\right]$$

$$= 0.1\left[2.1402 + 2(5.3593)\right]$$

$$= 0.1\left[2.1402 + 10.7186\right]$$

$$= \cancel{0.1}\ 0.025\ (12.8588)$$

$$= \cancel{1.28588}\ 0.3214$$

(5) Dividing the range into 10 equal parts find the value of $\int_0^{\pi/2} \sin x \, dx$ by Simpsons 1/3 rule.

Soln

$f(x) = \sin x \qquad h = \frac{b-a}{n} = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$

x	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$	$6\pi/20$	$7\pi/20$	$8\pi/20$
$f(x)$	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511

$$I = \frac{h}{3}\left[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

$$= \frac{\pi/20}{3}\left[(0+1) + 4(0.1564 + 0.4540 + 0.7071 + 0.8910) + 2(0.3090 + 0.5878 + 0.8090)\right]$$

$$= \pi/60 * 19.0986 = 1$$

(6) The velocity v of a particle at a distance s from a point on its path is gn by the table below.

s	0	10	20	30	40	50	60
v	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by Simpsons 1/3 rule.

Soln

$$\text{Velocity} = \frac{\text{distance}}{\text{time}}$$

$$v = \frac{ds}{dt}$$

$$dt = \frac{1}{v}\, ds$$

$$t = \int_0^{60} \frac{1}{v}\, ds, \quad h = 10$$

$$I = \int_0^{60} \frac{1}{v}\, ds = \frac{h}{3}\Big[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\Big]$$

v	47	58	64	65	61	52	38
$1/v$	0.02127	0.01724	0.015625	0.01538	0.01639	0.01923	0.026316

$$I = \frac{10}{3}\Big[(0.02127 + 0.026316) + 4(0.07124 + 0.01538 + 0.01923) + 2(0.015625 + 0.01639)\Big]$$

$$I = 1.06338$$

⑦ Compute $\int_0^{\pi/2} \sin x \, dx$ using Simpson's $3/8^{th}$ rule of numerical integration.

Soln

$$I = \int_0^{\pi/2} \sin x \, dx$$

$f(x) = \sin x \qquad h = \frac{\pi/2 - 0}{9} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
$f(x)$	0	0.1736	0.3420	0.50	0.6428	0.7660

$6\pi/18$	$7\pi/18$	$8\pi/18$	$9\pi/18$
0.8660	0.9397	0.9848	1

$$I = \frac{3h}{8}\left[(y_0 + y_9) + 3(y_1 + y_2 + y_4 + y_5 + y_7 + y_8) + 2(y_3 + y_6)\right]$$

$$= \frac{3\pi}{8 \times 18}\left[(0 + 1) + 3(0.1736 + 0.3428 + 0.6428 + 0.7660 + 0.9397 + 0.9848) + 2(0.5 + 0.8660)\right]$$

$I = 0.99998857 6$

$I \sim 1$

(7) The velocities of a car running on a straigh road at intervals of 2 minutes are gn below

Time (min)	0	2	4	6	8	10	12
Velocity (km/hr)	0	22	30	27	18	7	0

using Simpsons 1/3 rule find the distance covered by the car.

Soln

$$\text{Velocity} = \frac{dx}{dt} \quad \text{(ie)} \quad v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$x = \int v\,dt$$

t	0	2	4	6	8	10	12
v	0	$\frac{22}{60}$	$\frac{30}{60}$	$\frac{27}{60}$	$\frac{18}{60}$	$\frac{7}{60}$	0

$$I = \frac{h}{3}\Big[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)\Big]$$

$$= \frac{2}{3}\Big[0 + 0 + 2\left(\frac{30}{60} + \frac{18}{60}\right) + 4\left(\frac{22}{60} + \frac{27}{60} + \frac{7}{60}\right)\Big]$$

$$= 3.5556 \text{ km}$$

Romberg Method

$$I = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

I_1 – Value of integral with $h = \frac{b-a}{2}$

I_2 – Value of integral with $h = \frac{b-a}{4}$

I_3 – " " " $h = \frac{b-a}{8}$.

① Compute $I = \int_0^{1/2} \frac{x}{\sin x}\,dx$, using Simpsons rule with $h = 1/4,\ 1/8,\ 1/16$ and then Romberg's Method.

Soln

$$I = \int_0^{1/2} \frac{x}{\sin x}\,dx$$

$$f(x) = \frac{x}{\sin x}$$

i) Take $h = 1/4$

x	0	1/4	1/2
$f(x)$	y_0 1	y_1 1.0105	y_2 1.0429

By Simpsons 1/3 rule,

$$I_1 = \frac{h}{3}\left[(y_0 + y_2) + 4(y_1) + 0\right]$$

$$= \frac{1}{12}\left[(1 + 1.0429) + 4(1.0105)\right]$$

$$I_1 = 0.507075$$

(ii) Take $h = 1/8$

x	0	$1/8$	$2/8$	$3/8$	$4/8$
$f(x)$	1 y_0	1.0026 y_1	1.0105 y_2	1.0238 y_3	1.0429 y_4

$$I_2 = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$$

$$= \frac{1}{24}\left[(1 + 1.0429) + 4(1.0026 + 1.0236) + 2(1.01051)\right]$$

$$I_2 = 0.5070625$$

(iii) Take $h = \frac{1}{16}$

x	0	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{6}{16}$	$\frac{7}{16}$	$\frac{8}{16}$
$f(x)$	1 y_0	1.0007 y_1	1.0026 y_2	1.0059 y_3	1.0105 y_4	1.0165 y_5	1.0238 y_6	1.0326 y_7	1.0429 y_8

$$I_3 = \frac{h}{3}\left[(y_0+y_8) + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$$

$$= \frac{1}{48}\left[(1+1.0429) + 4(1.0007+1.0059+1.0165+1.0326) + 2(1.0026+1.0105+1.0238)\right]$$

$$I_3 = 0.5070729.$$

for I_1, I_2

Romberg formula is

$$I_4 = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

$$= 0.5070625 + \left(\frac{0.5070625 - 0.507075}{3}\right)$$

$$I = 0.507058$$

for I_2, I_3

$$I_5 = I_3 + \left(\frac{I_3 - I_2}{3}\right)$$

$$= 0.5070729 + \left(\frac{0.5070729 - 0.5070625}{3}\right)$$

$$= 0.50707633366.$$

Romberg for I_4 & I_5

$$I = I_5 + \left(\frac{I_5 - I_4}{3}\right)$$

(3) Evaluate $I = \int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method. Hence deduce an approximate value of π

Soln

$a = 0$; $b = 1$

$f(x) = \frac{1}{1+x^2}$

I $h = \frac{b-a}{2} = \frac{1-0}{2} = 0.5$

x	0	0.5	1
$f(x)$	1	0.8	0.5

$$I_1 = \frac{h}{2}\left[(y_0+y_2) + 2(y_1)\right]$$

$$= \frac{0.5}{2}\left[(1+0.5) + 2\times 0.8\right]$$

$$I_1 = 0.7750$$

II $h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.9412	0.8	0.64	0.5

(3) Evaluate $I = \int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method. Hence deduce an approximate value of π

Soln

$a = 0 \; ; \; b = 1$

$f(x) = \frac{1}{1+x^2}$

I $h = \frac{b-a}{2} = \frac{1-0}{2} = 0.5$

x	0	0.5	1
$f(x)$	1	0.8	0.5

$I_1 = \frac{h}{2}\left[(y_0 + y_2) + 2(y_1)\right]$

$= \frac{0.5}{2}\left[(1 + 0.5) + 2 \times 0.8\right]$

$I_1 = 0.7750$

II $h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.9412	0.8	0.64	0.5

$$I_2 = \frac{0.25}{2}\left[(1+0.5)+2(0.9412+0.8+0.64)\right]$$

$$\boxed{I_2 = 0.7828}$$

(iii) $h = \frac{b-a}{8} = \frac{1-0}{8} = 0.125$

x	0	0.12	0.25	0.375	0.5
$f(x)$	1	0.9846	0.9412	0.8767	0.8

0.625	0.75	0.875	1
0.7191	0.64	0.5664	0.5

$$I_3 = \frac{0.5}{2}\left[(1+0.5)+2(0.9846+0.9412+0.8767+0.8+0.7191+0.64+0.5664)\right]$$

$$\boxed{I_3 = 0.7848}$$

Romberg for I_1, I_2

$$I_4 = I_2 + \left(\frac{I_2 - I_1}{3}\right) = 0.7854$$

Romberg for I_2, I_3

$$I_5 = I_3 + \left(\frac{I_3 - I_2}{3} \right) = 0.7855$$

Romberg for I_4, I_5

$$I = I_5 + \left(\frac{I_5 - I_4}{3} \right) = 0.7855$$

$$I = \int_0^1 \frac{dx}{1+x^2}$$

$$0.7855 = [\tan^{-1} x]_0^1$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$\frac{\pi}{4} = 0.7855$$

$$\pi = 3.1420.$$

③ Using Romberg Integration, evaluate

$$\int_0^1 \frac{dx}{1+x}$$

Soln

I) Here $a=0,\ b=1$

x	0	0.5	1
$\frac{1}{1+x}$	1	0.6667	0.5

$$I_1 = \frac{h}{2}\left[(y_0+y_2)+2(y_1)\right]$$

$$= \frac{0.5}{2}\left[(1+0.5)+2(0.6667)\right]$$

$$\boxed{I_1 = 0.7084}$$

II) $h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.8	0.6667	0.5714	0.5

$$I_2 = \frac{0.25}{2}\left[(1+0.5)+2(0.8+0.6667+0.5714)\right]$$

$$\boxed{I_2 = 0.6970}$$

III $h = \frac{b-a}{8} = \frac{1-0}{8} = 0.125$

x	0	0.125	0.25	0.375	0.5
$f(x)$	1	0.8889	0.8	0.7273	0.6667
		0.625	0.75	0.875	1
		0.6154	0.5714	0.5333	0.5

$$I_3 = \frac{0.125}{2}\left[(1+0.5) + 2(0.8889 + 0.8 + 0.7273 + 0.6667 + 0.6154 + 0.5714 + 0.5333)\right]$$

$$\boxed{I_3 = 0.6941.}$$

Romberg for I_1, I_2

$$I_4 = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

$$= 0.6970 + \left(\frac{0.6970 - 0.7084}{3}\right)$$

$$\boxed{I_4 = 0.6932}$$

Romberg for I_2, I_3

$$I_5 = I_3 + \left(\frac{I_3 - I_2}{3}\right)$$

$$= 0.6941 + \left(\frac{0.6941 - 0.6970}{3} \right)$$

$$\boxed{I_5 = 0.6931}$$

Romberg for I_4, I_5

$$I_6 = I_5 + \left(\frac{I_5 - I_4}{3} \right)$$

$$\boxed{I_6 = 0.6931}$$

Gauss Quadrature formula

Quadrature:

The process of finding a definite integral from a tabulated values of a function is known as Quadrature.

Gaussian two point Quadrature formula

$$\text{Let} \quad I = \int_a^b f(x)\, dx$$

$$\text{Take} \quad x = \left(\frac{a+b}{2} \right) + \left(\frac{b-a}{2} \right) t$$

$$dx = \left(\frac{b-a}{2} \right) dt$$

By using this transformation

$$I = \int_{-1}^{1} g(t)\, dt = g\left(\frac{-1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

① Evaluate $\int_{-1}^{1} e^{-x^2} \cos x \, dx$ by Gauss two Point Quadrature formula.

Soln

$$I = \int_{-1}^{1} e^{-x^2} \cos x \, dx$$

$$f(x) = e^{-x^2} \cos x$$

$$I = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$= e^{-\left(\frac{-1}{\sqrt{3}}\right)^2} \cos\left(\frac{-1}{\sqrt{3}}\right) + e^{-\left(\frac{1}{\sqrt{3}}\right)^2} \cos\left(\frac{1}{\sqrt{3}}\right)$$

$$= e^{-1/3} \cos\left(\frac{-1}{\sqrt{3}}\right) + e^{-1/3} \cos\left(\frac{1}{\sqrt{3}}\right)$$

$$= e^{-1/3} \left[\cos\left(-1/\sqrt{3}\right) + \cos\left(1/\sqrt{3}\right) \right]$$

$$I = 1.2008$$

② Apply Gauss two point formula to evaluate $\int_{-1}^{1} \frac{1}{1+x^2} dx$

Soln

$$I = \int_{-1}^{1} \frac{1}{1+x^2} dx$$

$$f(x) = \frac{1}{1+x^2}$$

$$I = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{1+\left(\frac{-1}{\sqrt{3}}\right)^2} + \frac{1}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{3}{4} + \frac{3}{4}$$

$$= \frac{6}{4}$$

$$= 1.5$$

(3) Evaluate the integral $I = \int_1^2 \frac{2x}{1+x^4}\,dx$ using Gaussian two point formula

Soln

$$I = \int_1^2 \frac{2x}{1+x^4}\,dx$$

$$f(x) = \frac{2x}{1+x^4}, \quad a = 1, \quad b = 2$$

$$x = \frac{a+b}{2} + \left(\frac{b-a}{2}\right)t$$

$$x = \frac{3}{2} + \frac{1}{2}t$$

$$dx = \tfrac{1}{2}\,dt$$

$$I = \int_{-1}^{1} \frac{\cancel{2}\left(\frac{3}{2}+\frac{1}{2}t\right)}{1+\left(\frac{3}{2}+\frac{1}{2}t\right)^4} \cdot \frac{dt}{\cancel{2}}$$

$$= \int_{-1}^{1} \frac{\left(\frac{3+t}{2}\right)}{1+\left(\frac{3+t}{2}\right)^4}\, dt$$

$$g(t) = \frac{\frac{3+t}{2}}{1+\left(\frac{3+t}{2}\right)^4}$$

$$I = g\left(\frac{-1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\frac{3-\frac{1}{\sqrt{3}}}{2}}{1-\left(\frac{3-\frac{1}{\sqrt{3}}}{2}\right)^4} + \frac{\frac{3+\frac{1}{\sqrt{3}}}{2}}{1+\left(\frac{3+\frac{1}{\sqrt{3}}}{2}\right)^4}$$

$$= \frac{1.2113}{3.1530} + \frac{1.7887}{11.2359}$$

$$= 0.3842 + 0.1592$$

$$= 0.5434.$$

$$= \frac{\pi}{4}\left[0.3259 + 0.9454\right]$$

$$= 0.9985$$

Gaussian Three Point Quadrature formula:

$$I = \int_a^b f(x)\,dx$$

Take $x = \left(\frac{a+b}{2}\right) + \left(\frac{b-a}{2}\right)t$

$$dx = \left(\frac{b-a}{2}\right)dt$$

$$I = \int_{-1}^{1} g(t)\,dt = \frac{5}{9}\left[g\left(-\sqrt{\frac{3}{5}}\right) + g\left(\sqrt{\frac{3}{5}}\right)\right] + \frac{8}{9}g(0)$$

(1) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using 3 point Quadrature formula.

Soln

$$I = \int_0^1 \frac{dx}{1+x^2}$$

$$f(x) = \frac{1}{1+x^2}, \quad a = 0, \quad b = 1$$

Take $x = \left(\frac{a+b}{2}\right) + \left(\frac{b-a}{2}\right)t$

$$dx = \left(\frac{b-a}{2}\right) dt$$

$$\Rightarrow x = \frac{1}{2} + \frac{1}{2}t$$

$$dx = \frac{1}{2} dt$$

$$I = \int_{-1}^{1} \frac{\frac{1}{2} dt}{1 + \left(\frac{1+t}{2}\right)^2} = \frac{1}{2}\int_{-1}^{1} \frac{dt}{1 + \left(\frac{1+t}{2}\right)^2}$$

$$\therefore \quad g(t) = \frac{1}{1 + \left(\frac{1+t}{2}\right)^2}$$

$$I = \frac{1}{2}\left[\frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9} g(0)\right.$$

$$= \frac{1}{2}\left[\frac{5}{9}\left(\frac{1}{1 + \left(\frac{1+(-\sqrt{3/5})}{2}\right)^2} + \frac{1}{1 + \left(\frac{1+(\sqrt{\frac{3}{5}})}{2}\right)^2}\right.\right. + \frac{8}{9}\left[\frac{1}{1 + (1/2)^2}\right]\Bigg]\Bigg]$$

$$= \frac{1}{2}\left[\frac{5}{9}\left(0.9875 + 0.5595 + 0.7111\right]\right.$$

$$= 0.7853.$$

(2) Apply three point Gaussian Quadrature formula to evaluate $\int_0^1 \frac{\sin x}{x}\, dx$

Soln

$$I = \int_0^1 \frac{\sin x}{x}\, dx$$

$$f(x) = \frac{\sin x}{x}, \quad a = 0, \; b = 1$$

$$x = \left(\frac{b+a}{2}\right) + \left(\frac{b-a}{2}\right) t$$

$$dx = \left(\frac{b-a}{2}\right) dt$$

$$\Rightarrow x = \tfrac{1}{2} + \tfrac{1}{2} t = \tfrac{1}{2}(1+t)$$

$$dx = \tfrac{1}{2}\, dt$$

$$I = \int_{-1}^{1} \frac{\sin \frac{1}{2}(1+t)}{\cancel{\frac{1}{2}}(1+t)} \cdot \cancel{\tfrac{1}{2}}\, dt$$

$$= \int_{-1}^{1} \frac{\sin \frac{1}{2}(1+t)}{(1+t)}\, dt$$

$$\therefore g(t) = \frac{\sin \frac{1+t}{2}}{1+t}$$

$$g(0) = \sin \tfrac{1}{2} = 0.47943$$

$$g\left(\sqrt{\tfrac{3}{5}}\right) = \sin\left[\frac{\sqrt{\frac{3}{5}}+1}{2}\right] \Big/ \sqrt{\tfrac{3}{5}}+1 = \frac{0.7754}{1.7746} = 0.437$$

$$g\left(-\sqrt{\tfrac{3}{5}}\right) = \frac{\sin\left[\frac{-\sqrt{\frac{3}{5}}+1}{2}\right]}{-\sqrt{\frac{3}{5}}+1} = \frac{0.1125}{0.2254} = 0.499$$

$$I = \frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9}g(0)$$

$$= \frac{5}{9}\left[0.499 + 0.437\right) + \frac{8}{9}(0.47943)$$

$$= 0.52 + 0.42616$$

$$= 0.94616.$$

③ Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(x+1)^4}\,dx$ by Gaussian three point formula.

Soln

$$I = \int_0^2 \frac{x^2+2x+1}{1+(x+1)^4}$$

$$f(x) = \frac{x^2+2x+1}{1+(x+1)^4}, \quad a=0, \; b=2$$

$$x = \frac{b+a}{2} + \left(\frac{b-a}{2}\right)t$$

$$dx = \left(\frac{b-a}{2}\right)dt$$

$$\Rightarrow \quad x = 1 + t$$

$$dx = dt$$

$$I = \int_{-1}^{1} \frac{(z+1)^2 + 2(z+1) + 1}{1 + [(z+1)+1]^4}\, dz$$

$$g(t) = \frac{(z+1)^2 + 2(z+1) + 1}{1 + [(z+1)+1]^4}$$

$$= \frac{z^2 + 2z + 1 + 2z + 2 + 1}{1 + (z+2)^4}$$

$$g(t) = \frac{(z+2)^2}{1+(z+2)^4}$$

$$g(0) = \frac{4}{17}$$

$$g\left(-\sqrt{\tfrac{3}{5}}\right) = \frac{\left(-\sqrt{\frac{3}{5}}+2\right)^2}{1+\left(-\sqrt{\frac{3}{5}}+2\right)^4} = \frac{1.50161}{3.2548} = 0.4614$$

$$g\left(\sqrt{\tfrac{3}{5}}\right) = \frac{\left(\sqrt{\frac{3}{5}}+2\right)^2}{1+\left(\sqrt{\frac{3}{5}}+2\right)^4} = \frac{7.69839}{60.2652} = 0.12774$$

$$I = \frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9}\, g(0).$$

$$= \frac{5}{9}\left[0.4614 + 0.12774\right] + \frac{8}{9}\left(\frac{4}{17}\right)$$

$$= \frac{5}{9}\left[0.4614\right.$$

$$= 0.5364 //$$

Double Integration

Trapezoidal rule:

$$I = \int_c^d \int_a^b f(x,y)\, dx\, dy$$

$I = \frac{hk}{4}$ [Sum of four Corners + 2 (Sum of remaining boundary values) + 4 (Sum of interior Values)]

Simpson's rule

$I = \frac{hk}{9}$ [Sum of four corners +

$\underbrace{2\text{(Sum of odd position values)} + 4\text{(Sum of even position values)}}_{\text{Boundary}}$

$+ \underbrace{4\text{(Sum of odd position values)} + 8\text{(Sum of even position values)}}_{\text{odd rows}}$

$+ \underbrace{8\text{(Sum of odd position values)} + 16\text{(Sum of even position values)}}_{\text{even rows}}$]

$\cancel{I} = \frac{hk}{4}$ [~~Sum of four corn~~

$$I = \frac{0.1 \times 0.1}{4}\Big[0.5 + 0.4167 + 0.4545 + 0.3846$$
$$+ 2(0.4762 + 0.4545 + 0.4348 + 0.4762$$
$$+ 0.4 + 0.4348 + 0.4167 + 0.4)$$
$$+ 4(0.4545 + 0.4348 + 0.4167)\Big]$$

$$= \frac{0.1 \times 0.1}{4}\left[1.7558 + 6.9864 + 5.2240\right]$$

$$= \frac{0.1 \times 0.1}{4} \times 13.9662 = 0.0349.$$

(2) Evaluate $\int_1^2\int_1^2 \frac{1}{x^2+y^2}\,dx\,dy$, numerically with $h = 0.2$, along x-direction and $k = 0.25$ along y-direction.

Soln

$$I = \int_1^2\int_1^2 \frac{1}{x^2+y^2}\,dx\,dy$$

$$f(x, y) = \frac{1}{x^2+y^2}$$

By Trapezoidal

$$I = \frac{hk}{4}\Big[\text{Sum of four corners} + 2(\text{Sum of remaining boundary}) + 4(\text{Sum of interiors})\Big]$$

y \ x	1	1.2	1.4	1.6	1.8	2
1	0.5	0.4098	0.3378	0.2809	0.2359	0.2
1.25	0.3902	0.3331	0.2839	0.2426	0.2082	0.1798
1.5	0.3077	0.2710	0.2375	0.2079	0.1821	0.16
1.75	0.2462	0.2221	0.1991	0.1779	0.1587	0.1416
2	0.2	0.1838	0.1679	0.1524	0.1381	0.125

$$I = \frac{(0.2)(0.25)}{4}\Big[0.5 + 0.2 + 0.2 + 0.125$$
$$+ 2(0.4098 + 0.3378 + 0.2809 + 0.2359$$
$$+ 0.1798 + 0.16 + 0.1416$$
$$+ 0.1381 + 0.1524 + 0.1679 + 0.1838$$
$$+ 0.2462 + 0.2710 + 0.3331)$$
$$+ 4(0.3331 + 0.2839 + 0.2426$$
$$+ 0.2082 + 0.2710 + 0.2375$$
$$+ 0.2079 + 0.1821 + 0.2221$$
$$+ 0.1991 + 0.1779 + 0.1587)\Big]$$

$$= \frac{(0.2)(0.25)}{4}\left[1.025 + 6.6642 + 10.8964)\right]$$

$$= 0.2323.$$

3. Evaluate $I = \int_0^{1/2}\int_0^{1/2} \frac{\sin(xy)}{1+xy}\,dx\,dy$ using Simpson's rule with $h = k = 1/4$

Soln

$$I = \int_0^{1/2}\int_0^{1/2} \frac{\sin(xy)}{1+xy}\,dx\,dy$$

$$f(x,y) = \frac{\sin xy}{1+xy}$$

By Simpsons 1/3 rule,

$$I = \frac{hk}{9}\Big[\text{Sum of four corners} + 2(\underbrace{\text{Sum of odd position}) + 4(SEP)}_{\text{Boundary}} + \underbrace{4(SOP) + 8(SEP)}_{\text{odd rows}} + \underbrace{8(SOP) + 16(SEP)}_{\text{even rows}}\Big]$$

	y \ x	0	1/4	1/2
I	0	0	0	0
II	1/4	0	0.0588	0.1108
III	1/2	0	0.1108	0.1979

$$I = \frac{0.1 \times 0.1}{9}\Big[0.5 + 0.4167 + 0.3571 + 0.2976$$
$$+ 2[0.4545 + 0.4167 + 0.3247 + 0.3478]$$
$$+ 4[0.4762 + 0.4348 + 0.3788 + 0.3205$$
$$+ 0.3401 + 0.3106 + 0.4545 + 0.3846]$$
$$+ 4(0.3788) + 8(0.3968 + 0.3623)$$
$$+ 8(0.4132 + 0.3497)$$
$$+ 16(0.4329 + 0.3953$$
$$+ 0.3663 + 0.3344)\Big]$$

$$= \frac{0.1 \times 0.1}{9}\Big[1.5714 + 3.0862 + 12.4004$$
$$+ 1.5152 + 6.0728 + 6.1032$$
$$+ 24.4624\Big]$$

$I = 0.0613$

5 Evaluate $\int_0^2 \int_0^1 4xy \, dx\, dy$ using Simpsons rule by taking $h = 1/4$ & $k = \frac{1}{2}$

Soln

$$I = \int_0^2 \int_0^1 4xy \, dx\, dy$$

Here $f(x, y) = 4xy$

$h = 0.25 \quad k = 0.5$

y \ x	0	0.25	0.5	0.75	1
0	0	0	0	0	0
0.5	0	0.5	1	1.5	2
1	0	1	2	3	4
1.5	0	1.5	3	4.5	6
2	0	2	4	6	8.

$$I = \frac{0.25 \times 0.5}{9} [8 + 16 + 64 + 8 + 32 + 32 + 128]$$

$I = 4.$