Numerical Methods

Unit - 2

Interpolation and Approximation

Lagrange's interpolation formula (unequal intervals)

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0$$
$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

① Using Lagrange's formula, find the polynomial to the given data

x	0	1	3
y	5	6	50

Soln

$$y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \cdot y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \cdot y_1$$
$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \cdot y_2$$

Here $x_0 = 0 \quad x_1 = 1 \quad x_2 = 3$
$y_0 = 5 \quad y_1 = 6 \quad y_2 = 50$

$$y = f(x) = \frac{(x-1)(x-3)}{(0-1)(0-3)}(5) + \frac{(x-0)(x-3)}{(1-0)(1-3)} \cdot (6)$$
$$+ \frac{(x-0)(x-1)}{\ldots} \cdot (50)$$

$$= \frac{(x-1)(x-3)}{3}(5) + \frac{x(x-3)}{-2}(6) + \frac{x(x-1)}{6}(50)$$

$$= \frac{5}{3}[x^2-4x+3] - 3[x^2-3x] + \frac{50}{6}[x^2-x]$$

$$= x^2\left[\frac{5}{3} - 3 + \frac{50}{6}\right] + x\left[-\frac{20}{3} + 9 - \frac{50}{6}\right] + \left[\frac{15}{3}\right]$$

$$= 7x^2 + (-6)x + 5$$

$$\boxed{y = f(x) = 7x^2 - 6x + 5}$$

② Using Lagrange's interpolation find $y(2)$ from the following data

x	0	1	3	4	5
y	0	1	81	256	625

Soln

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 4 \quad x_4 = 5$

$y_0 = 0 \quad y_1 = 1 \quad y_2 = 81 \quad y_3 = 256 \quad y_4 = 625$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \cdot y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \cdot y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \cdot y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \cdot y_3$$

Put $x=2$

$$y(2) = \frac{(2-1)(2-3)(2-4)(2-5)}{(0-1)(0-3)(0-4)(0-5)}(0) + \frac{(2-0)(2-3)(2-4)(2-5)}{(1-0)(1-3)(1-4)(1-5)}(1) + \frac{(2-0)(2-1)(2-4)(2-5)}{(3-0)(3-1)(3-4)(3-5)}(81) + \frac{(2-0)(2-1)(2-3)(2-5)}{(4-0)(4-1)(4-3)(4-5)}(256) + \frac{(2-0)(2-1)(2-3)(2-4)}{(5-0)(5-1)(5-3)(5-4)}(625)$$

$$= \frac{(2)(-1)(-2)(-3)}{(1)(-2)(-3)(-4)} + \frac{(2)(1)(-2)(-3)}{(3)(2)(-1)(-2)}(81) + \frac{(2)(1)(-1)(-3)}{(4)(3)(1)(-1)}(256) + \frac{(2)(1)(-1)(-2)}{(5)(4)(2)(1)}(625)$$

$$= \frac{12}{24} + \frac{12}{12}(81) - \frac{6}{12}(256) + \frac{4}{40}(625)$$

$$= \frac{1}{2} + 81 - 128 + 62.5$$

$$= 0.5 + 81 - 128 + 62.5 = 16.$$

3) Use Lagrange's Method to find $\log_{10} 656$, given that $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$ and $\log_{10} 661 = 2.8202$.

Soln

x	654	658	659	661
$y = \log_{10} x$	2.8156	2.8182	2.8189	2.8202

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 656$

$$y = f(656) = \frac{(656-658)(656-659)(656-661)}{(654-658)(654-659)(654-661)} \cdot (2.8156)$$

$$+ \frac{(656-654)(656-659)(656-661)}{(658-654)(658-659)(658-661)} (2.8182)$$

$$+ \frac{(656-654)(656-658)(656-661)}{(659-654)(659-658)(659-661)} (2.8189)$$

$$+ \frac{(656-654)(656-659)(656-659)}{(661-654)(661-659)(661-659)} \cdot (2.8202)$$

$$= \frac{(-2)(-3)(-5)}{(-4)(-5)(-7)} (2.8156) + \frac{2(-3)(-5)}{4(-1)(-3)} (2.8182)$$

$$+ \frac{(2)(-2)(-5)}{(5)(1)(-2)} (2.8189) + \frac{(2)(-2)(-3)}{(7)(3)(2)} (2.8202)$$

$$= 0.6033 + 7.0455 - 5.6378 + 0.8058$$

$$= 2.8168.$$

4) Use Lagrange's formula to find the value of y at $x = 6$ from the following data

x :	3	7	9	10

Soln

$x_0 = 3 \quad x_1 = 7 \quad x_2 = 9 \quad x_3 = 10$

$y_0 = 168 \quad y_1 = 120 \quad y_2 = 72 \quad y_3 = 63.$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 6$

$$y = f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)}(168) + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)}(120) + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)}(72) + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)}(63)$$

$$= \frac{(-1)(-3)(-4)}{(-4)(-6)(-7)}(168) + \frac{(3)(-3)(-4)}{4(-2)(-3)}(120) + \frac{(3)(-1)(-4)}{(6)(2)(-1)}(72) + \frac{(3)(-1)(-3)}{(7)(3)(1)}(63)$$

$= 12 + 180 - 72 + 27$

$= 147$

5) Given the values

x	14	17	31	35
$f(x)$	68.7	64.0	44.0	39.1

find $f(27)$ by using Lagrange's interpolation formula.

Soln

$x_0 = 14 \quad x_1 = 17 \quad x_2 = 31 \quad x_3 = 35$

$y_0 = 68.7 \quad y_1 = 64 \quad y_2 = 44 \quad y_3 = 39.1$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 27$

$$y = f(27) = \frac{(27-17)(27-31)(27-35)}{(14-17)(14-31)(14-35)} \cdot (68.7)$$
$$+ \frac{(27-14)(27-31)(27-35)}{(17-14)(17-31)(17-35)} (64.0)$$
$$+ \frac{(27-14)(27-17)(27-35)}{(31-14)(31-17)(31-35)} (44.0)$$
$$+ \frac{(27-14)(27-17)(27-31)}{(35-14)(35-17)(35-31)} (39.1)$$

$$= \frac{(10)(-4)(-8)}{(-3)(-17)(-21)} (68.7) + \frac{13(-4)(-8)}{(3)(-14)(-8)} (6$$
$$+ \frac{(13)(10)(-8)}{(17)(14)(-4)} (44) + \frac{(13)(10)(-4)}{(21)(18)(4)} (3$$

$$= -20.52 + 35.22 + 48.07 - 13.45$$

6) Find the Missing term in the following table using Lagranges interpolation

x	0	1	2	3	4
y	1	3	9	—	81

Soln

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 4$

$y_0 = 1 \quad y_1 = 3 \quad y_2 = 9 \quad y_3 = 81$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 3$

$$y = f(3) = \frac{(3-1)(3-2)(3-4)}{(0-1)(0-2)(0-4)}(1) + \frac{(3-0)(3-2)(3-4)}{(1-0)(1-2)(1-4)}(3) + \frac{(3-0)(3-1)(3-4)}{(2-0)(2-1)(2-4)}(9) + \frac{(3-0)(3-1)(3-2)}{(4-0)(4-1)(4-2)}(81)$$

$$= \frac{-2}{-8} - 3 + \frac{27}{2} + \frac{81}{4}$$

$$= 31$$

7) Using Lagranges formula prove

$y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} + y_{-5})$

Soln y_{-5}, y_{-3}, y_3, y_5 occur in the answers.

So we can have the table

x	-5	-3	3	5
y	y_{-5}	y_{-3}	y_3	y_5

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_{-5} + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_{-3} + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_3 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_5$$

put $x = 1$

$$y_1 = f(1) = \frac{(1+3)(1-3)(1-5)}{(-5+3)(-5-3)(-5-5)} \cdot y_{-5} + \frac{(1+5)(1-3)(1-5)}{(-3+5)(-3-3)(-3-5)} \cdot y_{-3} + \frac{(1+5)(1+3)(1-5)}{(3+5)(3+3)(3-5)} y_3 + \frac{(3+5)(3+3)(3-3)}{(5+5)(5+3)(5-3)} \cdot y_5$$

$$= \frac{(4)(-2)(-4)}{(-2)(-8)(-10)} \cdot y_{-5} + \frac{(6)(-2)(-4)}{(2)(-6)(-8)} \cdot y_{-3} + \frac{(6)(4)(-4)}{(8)(6)(-2)} \cdot y_3 + \frac{(6)(4)(-2)}{(10)(8)(2)} y_5$$

$$= -0.2y + 0.5y + y - 0.3y$$

$$+\frac{(0+30)(0+13)(0-18)}{(3+30)(3+13)(3-18)}.(38)$$

$$+\frac{(0+30)(0+13)(0-3)}{(18+30)(18+13)(18-3)}(42)$$

$$=37.23.$$

(2) Find the value of θ given $f(\theta)=0.3887$ where $f(\theta)=\int_0^{\theta}\frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}}$ using the table

θ	21°	23°	25°
$f(\theta)$	0.3706	0.4068	0.4433.

Soln

Let $\theta = x$

$f(\theta)=f(x)=y$

x	21°	23°	25°
y	0.3706	0.4068	0.4433

$$x=f(y)=\frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)}.x_0+\frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)}.x_1$$

$$+\frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)}.x_2$$

Put $y=0.3887$

$$x=f(0.3887)=\frac{(0.3887-0.4068)(0.3887-0.4433)}{(0.3706-0.4068)(0.3706-0.4433)}(21^\circ)$$

$$+\frac{(0.3887-0.3706)(0.3887-0.4433)}{(0.4068-0.3706)(0.4068-0.4433)}(23^\circ)$$

$$+\frac{(0.3887-0.3706)(0.3887-0.4068)}{}$$

Newton's divided difference formula : (unequal)

$$y = f(x) = y_0 + (x-x_0)\,\Delta f(x_0) + (x-x_0)(x-x_1)\,\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\,\Delta^3 f(x_0) + \cdots$$

① Using Newton's divided difference formula find $f(x)$ and $f(6)$ from the following data.

x :	1 x_0	2 x_1	7 x_2	8 x_3
$f(x)$:	1	5	5	4

Soln

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	1			
		$\frac{5-1}{2-1} = 4$		
2	5		$\frac{0-4}{7-1} = -\frac{4}{6}$	
		$\frac{5-5}{7-2} = 0$		$\frac{-\frac{1}{6}+\frac{4}{6}}{8-1} = \frac{1/2}{7} = \frac{1}{14}$
7	5		$\frac{-1-0}{8-2} = \frac{-1}{6}$	
		$\frac{4-5}{8-7} = -1$		
8	4			

$$y = f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\,\Delta^3 f(x_0) + \cdots$$

$$= 1 + (x-1)(4) + (x-1)(x-2)\left(-\frac{4}{6}\right)$$

$$= x^3\left[\frac{1}{14}\right] + x^2\left[-\frac{4}{6}\right] - \frac{3}{14} - \frac{7}{14}\Big]$$

$$+ x\left[4 + \frac{12}{6} + \frac{2}{14} + \frac{21}{14}\right] + \left[1 - 4 - \frac{8}{6} - \frac{14}{14}\right]$$

$$f(x) = \frac{1}{14}x^3 - \frac{29}{21}x^2 + \frac{107}{14}x - \frac{16}{63}$$

Put $x = 6$

$$y = f(6) = \frac{1}{14}(6)^3 - \frac{29}{21}(6)^2 + \frac{107}{14}(6) - \frac{16}{36}$$

$$\cancel{= 54 - 114 + 40.4 - 7.833}$$

$$= 15.428 - 49.714 + 45.857 - 0.444$$

$$= 11.127$$

2) Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

Soln

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		$\frac{33-1245}{-1+4} = -404$			
-1	33		$\frac{-28+404}{0+4} = 94$		
		$\frac{5-33}{0+1} = -28$		$\frac{10-94}{2+4} = -14$	
0	5		$\frac{2+28}{2+1} = 10$		$\frac{13+14}{5+4} = 3$
		$\frac{9-5}{2-0} = 2$		$\frac{88-10}{5+1} = 13$	
2	9		$\frac{442-2}{5-0} = 88$		
		$\frac{1335-9}{5-2} = 442$			
5	1335				

$$y = f(x) = f(x_0) + (x-x_0)\Delta f(x) + (x-x_0)(x-x_1)\Delta^2 f(x)$$
$$+ (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x)$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 f(x)$$
$$= 1245 + (x+4)(-404) + (x+4)(x+1)(94) + (x+4)(x+1)(x-0)(x-2)(3)$$
$$+ (x+4)(x+1)(x-0)(-14)$$
$$= 1245 - 404x - 1616 + (x^2+5x+4)\,94 + (x^2+5x+4)(3x^2-6x)$$
$$+ (x^2+5x+4)(-14x)$$
$$= 1245 - 404x - 1616 + 94x^2 + 470x + 376 + 3x^4 + 15x^3 + 12x^2$$
$$-14x^3 - 70x^2 - 56x + 3x^4 + 15x^3 + 12x^2$$
$$-6x^3 - 30x^2 - 24x$$
$$= \cancel{1245}\ x(-404 + 470 - \cancel{16}) + x^2[94 - 70 + 12 - 30]$$
$$= x^4[3] + x^3[-14 + 15 - 6] + x^2[94 - 70 + 12 - 30] + [1245 - 1616 + 376]$$
$$+ x[-404 + 470 - 56 - 24]$$
$$= 3x^4 + 5x^3 + 6x^2 - 14x + 5$$

(3) Find the cubic polynomial from the following table using Newton's divided difference formula and hence find $f(4)$

x	$0\ x_0$	$1\ x_1$	$2\ x_2$	$5\ x_3$
y	2	3	12	147

Soln

x	$y=f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	2			
		$\frac{3-2}{1-0}=1$		
1	3		$\frac{9-1}{2-0}=4$	
		$\frac{12-3}{2-1}=9$		$\frac{9-4}{5-0}=1$
2	12		$\frac{45-9}{5-1}=9$	
		$\frac{147-12}{5-2}=45$		
5	147			

$$y=f(x)=y_0+(x-x_0)\Delta f(x)+(x-x_0)(x-x_1)\Delta^2 f(x)+(x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x)$$

$$=2+(x-0)(1)+(x-0)(x-1)(4)+(x-0)(x-1)(x-2)(1)$$

$$=2+x+4x^2-4x+(x^2-x)(x-2)$$

$$=2+x+4x^2-4x+x^3-x^2-2x^2+2x$$

$$=x^3+x^2-x+2$$

Put $x=4$

$$y=f(4)=4^3+4^2-4+2$$

$$=78$$

Cubic Spline Interpolation formula.

$$g(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right]$$
$$+\frac{1}{h}(x_i - x)\left[y_{i-1} - \frac{h^2}{6}M_{i-1}\right]$$
$$-\frac{1}{h}(x_{i-1} - x)\left[y_i - \frac{h^2}{6}M_i\right]$$

where, $M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]$

with $M_0 = M_n = 0$

(1) Obtain cubic spline polynomial which best fits with the following data, given that $y_0'' = y_3'' = 0$

x	-1 x_0	0 x_1	1 x_2	x_3 2
y	-1 y_0	1 y_1	3 y_2	35 y_3

Soln

Given $M_0 = M_3 = 0$, $h = 1$

WKT $M_{i-1} + 4M_i + M_{i+1} = 6\left[y_{i-1} - 2y_i + y_{i+1}\right]$

Put $i = 1$

$$M_0 + 4M_1 + M_2 = 6\left[y_0 - 2y_1 + y_2\right]$$
$$4M_1 + M_2 = 6\left[-1 - 2 + 3\right]$$
$$4M_1 + M_2 = 0 \quad \text{——— (1)}$$

Put $i = 2$

$$M_1 + 4M_2 + M_3 = 6\left[y_1 - 2y_2 + y_3\right]$$
$$M_1 + 4M_2 = 6\left[1 - 6 + 35\right]$$

Solve ① & ②

$M_1 = -12 \quad M_2 = 48$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right]$$
$$+ (x_i - x)\left[y_{i-1} - \frac{1}{6}M_{i-1}\right]$$
$$- (x_{i-1} - x)\left[y_i - \frac{1}{6}M_i\right]$$

Case(i) $-1 < x < 0$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1\right]$$
$$+ (x_1 - x)\left[y_0 - \frac{1}{6}M_0\right] - (x_0 - x)\left[y_1 - \frac{1}{6}M_1\right]$$

$$= \frac{1}{6}\left[-(-1-x)^3(-12)\right] + (0-x)(-1)$$
$$-(-1-x)\left[1 + \frac{12}{6}\right]$$

$$= \frac{1}{6}\left[-12(1+x)^3\right] + x + (1+x)(3)$$

$$= -2\left[1 + x^3 + 3x + 3x^2\right] + x + 3 + 3x$$

$$= -2 - 2x^3 - 6x - 6x^2 + x + 3 + 3x$$

$$\boxed{S(x) = -2x^3 - 6x^2 - 2x + 1, \quad -1 < x < 0}$$

Case(ii) $0 < x < 1$

Put $i = 2$

$$S(x) = \frac{1}{6}\left[(x_2 - x_1)^3 M_1 - (x_1 - x)^3 M_2 \right.$$
$$+ (x_2 - x)\left[y_1 - \frac{1}{6} M_1 \right]$$
$$- (x_1 - x)\left[y_2 - \frac{1}{6} M_2 \right]$$
$$= \frac{1}{6}\left[(1-x)^3 (-12) - (0-x)^3 (48) \right]$$
$$+ (1-x)\left[1 - \frac{1}{6}(-12) \right] - (0-x)\left[3 - \frac{1}{6} \times 48 \right]$$
$$= \frac{1}{6}\left[-12(1-x)^3 + 48x^3 \right] + 3(1-x) - 5x$$
$$= \frac{1}{6}\left[-12(1 - x^3 - 3x + 3x^2) + 48x^3 \right] + 3 - 3x - 5x$$
$$= \frac{1}{6}\left[-12 + 12x^3 + 36x - 36x^2 + 48x^3 \right] + 3 - 3x - 5x$$
$$= x^3[2+8] + x^2[-6] + x[6-8] - 2 + 3$$

$$\boxed{S(x) = 10x^3 - 6x^2 - 2x + 1, \quad 0 < x < 1}$$

Case (iii) $\quad 1 < x < 2$

Put $i = 3$

$$S(x) = \frac{1}{6}\left[(x_3 - x)^3 M_2 - (x_2 - x)^3 M_3 \right]$$
$$+ (x_3 - x)\left[y_2 - \frac{1}{6} M_2 \right] - (x_2 - x)\left[y_3 - \frac{1}{6} M_3 \right]$$
$$= \frac{1}{6}\left[(2-x)^3 48 \right] + (2-x)\left[3 - \frac{1}{6} \times 48 \right]$$

$= 8(2-x)^3 + (2-x)(-5) - 35(1-x)$

$= 8[8 - x^3 - 12x + 6x^2] - 10 + 5x - 35 + 35x$

$= 64 - 8x^3 - 96x + 48x^2 - 10 + 5x - 35 + 35x$

$$\boxed{S(x) = -8x^3 + 48x^2 - 56x + 19, \quad 1 < x < 2}$$

The cubic Spline polynomial is

$$S(x) = \begin{cases} -2x^3 - 6x^2 - 2x + 1, & -1 < x < 0 \\ 10x^3 - 6x^2 - 2x + 1, & 0 < x < 1 \\ -8x^3 + 48x^2 - 56x + 19, & 1 < x < 2 \end{cases}$$

(2) From the following table

x	1 x_0	2 x_1	3 x_2
y	-8 y_0	-1 y_1	18 y_2

Compute $y(1.5)$ and $y'(1)$ using cubic Spline.

Soln

Take $M_0 = M_2 = 0$, $h = 1$

W.k.T $M_{i-1} + 4M_i + M_{i+1} = 6[y_{i-1} - 2y_i + y_{i+1}]$

Put $i = 1$

$M_0 + 4M_1 + M_2 = 6[y_0 - 2y_1 + y_2]$

$4M_1 = 6[-8 + 2 + 18]$

$4M_1 = 72$

$$\boxed{M_1 = 18}$$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i \right] + (x_i - x)\left[y_{i-1} - \frac{1}{6} M_{i-1} \right] - (x_{i-1} - x)\left[y_i - \frac{1}{6} M_i \right]$$

Case (i) $1 < x < 2$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1 \right] + (x_1 - x)\left[y_0 - \frac{1}{6} M_0 \right] - (x_0 - x)\left[y_1 - \frac{1}{6} M_1 \right]$$

$$= \frac{1}{6}\left[(2-x)^3 (0) - (1-x)^3 (18) \right] + (2-x)\left[-8 - \frac{1}{6}(0) \right] - (1-x)\left[-1 - \frac{1}{6}(18) \right]$$

$$= \frac{1}{6}\left[-(1-x)^3 (18) \right] + (2-x)(-8) - (1-x)\left[-1 - 3 \right]$$

$$= -18(1-x)^3 - 8(2-x) + 4(1-x)$$

$$= -18(1-x)^3 - 16 + 8x + 4 - 4x$$

$$\boxed{S(x) = -18(1-x)^3 + 4x - 12, \quad 1 < x < 2}$$

Put $x = 1.5$

$$y(1.5) = S(1.5) = -18(1-1.5)^3 + 4(1.5) - 12$$

$$= -5.625$$

$y'(1) = 9(0) + 4 = 4$

$\boxed{y'(1) = 4}$ $\boxed{y(1.5) = -5.625}$

(3) Find the cubic spline interpolation

x :	1	2	3	4	5
f :	1 y_0	0 y_1	1 y_2	0 y_3	1 y_4

soln

Take $M_0 = M_{\cancel{4}} = 0$, $h = 1$

WKT $M_{i-1} + 4M_i + M_{i+1} = 6[y_{i-1} - 2y_i + y_{i+1}]$

Put $i=1$ $M_0 + 4M_1 + M_2 = 6[y_0 - 2y_1 + y_2]$

$4M_1 + M_2 = 6[1-0+1] = 12$

$4M_1 + M_2 = 12$ ——— (1)

Put $i=2$ $M_1 + 4M_2 + M_3 = 6[y_1 - 2y_2 + y_3]$

$= 6[0-2+0]$

$M_1 + 4M_2 + M_3 = -12$ ——— (2)

Put $i=3$ $M_2 + 4M_3 + M_4 = 6[y_2 - 2y_3 + y_4]$

$M_2 + 4M_3 = 6[1-0+1]$

$M_2 + 4M_3 = 12$ ——— (3)

from (1) & (2)

$4 \times (1) \Rightarrow 16M_1 + \cancel{4M_2} = 48$

from ② & ③

$$② \Rightarrow M_1 + 4M_2 + M_3 = -12$$

$$4 \times ③ \Rightarrow 4M_2 + 16M_3 = 48$$

$$M_1 - 15M_3 = -60 \quad \text{—— ⑤}$$

Solve ④ & ⑤

$$\boxed{M_3 = \frac{30}{7}}$$

$$⑤ \Rightarrow M_1 = -60 + 15M_3$$

$$M_1 = -60 + \frac{450}{7}$$

$$\boxed{M_1 = \frac{30}{7}}$$

$$① \Rightarrow 4M_1 + M_2 = 12$$

$$M_2 = 12 - 4M_1$$

$$= 12 - 4\left(\frac{30}{7}\right)$$

$$\boxed{M_2 = \frac{-36}{7}}$$

The cubic Spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right] + (x_i - x)\left[y_{i-1} - \frac{1}{6}M_{i-1}\right] - (x_{i-1} - x)\left[y_i - \frac{1}{6}M_i\right]$$

Case (i) $-1 < x < 2$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1\right] + (x_1 - x)\left[y_0 - \frac{1}{6}M_0\right] - (x_0 - x)\left[y_1 - \frac{1}{6}M_1\right]$$

$$= \frac{1}{6}\left[(2-x)^3(0) - (1-x)^3\left(\frac{30}{7}\right)\right] + (2-x)\left[1 - \frac{1}{6}(0)\right]$$

$(a-b)$
$= a^3 - 3$

$$= \frac{1}{6}\left[-(1-x)^3\left(\frac{30}{7}\right)\right] + (2-x)[1] - (1-x)\left[-\frac{1}{6}\,\frac{30}{7}\right]$$

$$= \frac{1}{6}\left[-\frac{30}{7}(1-x)^3 + (2-x) + \frac{5}{7}(1-x)\right]$$

$$= \frac{1}{6}\left[-\frac{30}{7}\left[1^3 - x^3 - 3x + 3x^2\right] + 2 - x + \frac{5}{7} - \frac{5}{7}x\right]$$

$$= -\cancel{\frac{5}{7}} + \frac{5}{7}x^3 + \frac{15}{7}x + 15x^2 + 2 - x + \cancel{\frac{5}{7}} - \frac{5}{7}x$$

$$= \frac{5}{7}x^3 + 15x^2 + x\left(\frac{15}{7} - 1 - \frac{5}{7}\right)$$

$$S(x) = \frac{5}{7}x^3 + 15x^2 + \frac{3}{7}x + 2, \quad 1 \le x \le 2$$

Case (ii) $2 < x < 3$.

Put $i = 2$.

$$S(x) = \frac{1}{6}\left[(x_2 - x)^3 M_1 - (x_1 - x)^3 M_2\right] + (x_2 - x)\left[y_1 - \frac{1}{6}M_1\right] - (x_1 - x)\left[y_2 - \frac{1}{6}M_2\right]$$

$$= \frac{1}{6}\left[(3-x)^3\frac{30}{7} - (2-x)\left(-\frac{36}{7}\right)\right] + (3-x)\left[0 - \frac{1}{6}\left(\frac{30}{7}\right)\right] - (2-x)\left[1 - \frac{1}{6}\left(-\frac{36}{7}\right)\right]$$

$$= \frac{1}{6}\left[\frac{30}{7}(3-x)^3 + \frac{36}{7}(2-x)\right] + (3-x)\left(-\frac{5}{7}\right)$$

$$-(2-x)\left[1+\frac{5}{7}\right]$$

$$= \frac{5}{7}\left[27-27x+9x^2-x^3\right] + \frac{6}{7}\left[4+x^2-4x\right]$$

$$-\frac{15}{7} + \frac{5x}{7} - \frac{26}{7} + \frac{13x}{7}$$

$$= x^3\left[-\frac{5}{7}\right] + x^2\left[\frac{45}{7} + \frac{6}{7} + \cancel{\frac{5}{7} + \frac{13}{7}}\right]$$

$$+ x\left[-135 - \frac{24}{7} + \frac{5}{7} + \frac{13}{7}\right] + \frac{135}{7} + \frac{24}{7} - \frac{15}{7}$$

$$-\frac{26}{7}$$

$$S(x) = -\frac{5}{7}x^3 + \frac{51}{7}x^2 - \frac{951}{7} + \frac{118}{7}, \quad 2<x<3$$

case (iii) $3<x<4$

Put $i=3$

$$S(x) = \frac{1}{6}\left[(x_3-x)^3 M_2 - (x_2-x) M_3\right]$$

$$+ (x_3-x)\left[y_2 - \frac{1}{6}M_2\right] - (x_2-x)\left[y_3 - \frac{1}{6}M_3\right]$$

$$= \frac{1}{6}\left[(4-x)^3\left(-\frac{36}{7}\right) + (3-x)^3\left(\frac{30}{7}\right)\right]$$

$$+ (4-x)\left[1 - \frac{1}{6}\left(-\frac{36}{7}\right)\right]$$

$$-(3-x)\left[0 - \frac{1}{6}\left(\frac{30}{7}\right)\right]$$

$$= \frac{1}{6}\left[-\frac{36}{7}\left[64 - 48x + 12x^2 - x^3\right] \right.$$
$$-\frac{30}{7}\left[27 - 27x + 9x^2 - x^3\right]$$
$$+(4-x)\left(1 + \frac{6}{7}\right) - (3-x)\left(-\frac{5}{7}\right)$$

$$= \cancel{\left[-\frac{384}{7} \right.} \quad \frac{13}{7}$$

$$= \frac{1}{7}\left[-384 + 288x - 72x^2 + 6x^3 - 810 \right.$$
$$+ 810x - 270x^2 + 30x^3.$$
$$\left. + 52 - 13x + 15 - 5x \right]$$

$$= \frac{1}{7}\left[x^3[30+6] + x^2[-72-270] \right] +$$
$$+ x[288 + 810 - 13 - 5] +$$
$$[-384 - 810 + 52 + 15]$$

$$S(x) = \frac{1}{7}\left[36x^3 - 342x^2 + 1080x - 1127, \quad 3 \le x \le 4 \right.$$

case (iv) $4 < x < 5$

Put $i = 4$.

$$S(x) = \frac{1}{6}\left[(x_3 - x)^3 M_3 - (x_2 - x) M_4 \right]$$
$$+ (x_4 - x)\left[y_3 - \frac{1}{6} M_3 \right]$$
$$- (x_3 - x)\left[y_4 - \frac{1}{6} M_4 \right]$$

$$= \frac{1}{6}\left[(5-x)^3 \left(\frac{30}{7}\right) - 0 \right] + (5-x)\left[0 - \frac{1}{6}\left(\frac{30}{7}\right)\right]$$
$$+(x-4)[1-0]$$

4) Find the cubic spline for the data

x	1	2	3
y	-6	-1	16

Hence evaluate $y(1.5)$ given that $y_0'' = y_2'' = 0$.

Soln

Given $h = 1 \quad M_0 = M_2 = 0$

W.K.T

$$M_{i-1} + 4M_i + M_{i+1} = 6\left[y_{i-1} - 2y_i + y_{i+1}\right]$$

Put $i = 1$

$$M_0 + 4M_1 + M_2 = 6\left[y_0 - 2y_1 + y_2\right]$$

$$4M_1 = 6\left[-6 - 2(-1) + 16\right]$$

$$4M_1 = 72$$

$$\boxed{M_1 = 18}$$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right] + (x_i - x)\left[y_{i-1} - \frac{1}{6}M_{i-1}\right] - (x_{i-1} - x)\left[y_i - \frac{1}{6}M_i\right]$$

Case(i) $1 \le x \le 2$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1\right]$$

$$= \frac{1}{6}\left[(2-x)^3(0) + (x-1)^3(18)\right]$$
$$+ (2-x)\left[-6 - \frac{1}{6}(0)\right]$$
$$+ (x-1)\left[-1 - \frac{1}{6}(18)\right]$$
$$= \frac{1}{6}\left[(x-1)^3(18)\right] + (2-x)(-6-0)$$
$$+ (x-1)(-1-3)$$
$$= 3(x^3 - 3x^2 + 3x - 1) - 12 + 6x - 4x + 4$$
$$S(x) = 3x^3 - 9x^2 + 11x - 11$$

Case (ii) $2 \le x \le 3$

Put $i = 2$.

$$S(x) = \frac{1}{6}\left[(x_2 - x)^3 M_1 - (x_1 - x)^3 M_2\right]$$
$$+ (x_2 - x)\left[y_1 - \frac{1}{6}M_1\right]$$
$$- (x_1 - x)\left[y_2 - \frac{1}{6}M_2\right]$$
$$= \frac{1}{6}\left[(3-x)^3\, 18 - (2-x)^3(0)\right]$$
$$+ (3-x)\left[-1 - \frac{1}{6}(18)\right]$$
$$- (x-2)\left[16 - \frac{1}{6}(0)\right]$$
$$= \frac{18}{6}\left[27 - 27x + 9x^2 - x^3\right]$$
$$- 12 + 4x + 16x - 32$$

$$S(x) = -3x^3 + 27x^2 - 61x + 37$$

$$y = S(x) = \begin{cases} 3x^3 - 9x^2 + 11x - 11, & 1 \le x \le 2 \\ -3x^3 + 27x^2 - 61x + 37, & 2 \le x \le 3 \end{cases}$$

To find $y(1\cdot5)$

$$S(1\cdot5) = 3(1\cdot5)^3 - 9(1\cdot5)^2 + 11(1\cdot5) - 11$$
$$= -4\cdot625$$

Newton's forward interpolation formula (equal intervals).

$$y = f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

where $u = \frac{x - x_0}{h}$

① Using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence evaluate y at $x=5$.

x	4	6	8	10
y	1	3	8	10

Soln

$u = \frac{x - x_0}{h}$, $h = 2$

$u = \frac{x-4}{2}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
4	1			
		2		
6	3		3	
		5		-6
8	8		-3	
		2		

The Newton's forward interpolation form is

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$= 1 + \left(\frac{x-4}{2}\right)(2) + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}-1\right)}{2} \times 3$$

$$+ \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}-1\right)\left(\frac{x-4}{2}-2\right)}{3!} \times -6$$

$$= 1 + (x-4) + \frac{3(x-4)(x-6)}{8} - \frac{(x-4)(x-6)(x-8)}{8}$$

$$= \frac{1}{8}\left[8 + 8x - 32 + 3[x^2 - 10x + 24] - [x^3 - 18x^2 + 104x - 192]\right]$$

$$y = \frac{1}{8}\left[-x^3 + 21x^2 - 126x + 240\right].$$

Put $x = 5$

$$y(5) = \frac{1}{8}\left[-5^3 + 21 \times 5^2 - 126 \times 5 + 240\right]$$

$$\boxed{y(5) = 1 \cdot 25}$$

(2) Fit a polynomial, by using Newtons forward interpolation formula to the data given below.

x	0 (x_0)	1 (x_1)	2 (x_2)	3 (x_3)
y	1 (y_0)	2 (y_1)	1 (y_2)	10 (y_3)

Soln

$$u = \frac{x - x_0}{h}, \quad h = 1$$

$$u = \frac{x - 0}{1} = x$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$= 1 + \frac{x}{1!}(2) + \frac{x(x-1)}{2!}(-1) + \frac{x(x-1)(x-2)}{3!}(10)$$

$$= 1 + 2x + \frac{(x^2 - x)}{2} + \frac{10}{6}\left[(x^2 - x)(x-2)\right]$$

$$= 1 + 2x + \frac{x^2}{2} - \frac{x}{2} + \frac{5}{3}\left[x^3 - 2x^2 - x^2 + 2x\right]$$

$$= \frac{5}{3}x^3 + x^2\left[\frac{1}{2} - \frac{10}{3}\right] + x\left[2 - \frac{1}{2} + \frac{10}{3}\right] + 1$$

(3) From the data given below find the number of students whose weight is between 60 to 70.

Weight in kgs	0-40	40-60	60-80	80-100	100-120
No. of Students	250	120	100	70	50

Soln

$$u = \frac{x - x_0}{h}, \quad h = 20$$

$$u = \frac{x - 40}{20}$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250				
		120			
Below 60	370		-20		
		100		-10	
Below 80	470		-30		20
		70		10	
Below 100	540		-20		
		50			
Below 120	590				

The Newton's forward interpolation formula is

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{\quad}\Delta^4 y_0 + \cdots$$

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 +$$

$$= 250 + \left(\frac{x-40}{20}\right)120 + \frac{\left(\frac{x-40}{20}\right)\left(\frac{x-40}{20}-1\right)}{2} \times -20$$

$$+ \frac{\left(\frac{x-40}{20}\right)\left(\frac{x-40}{20}-1\right)\left(\frac{x-40}{20}-2\right)}{6} \times -10$$

$$+ \frac{\left(\frac{x-40}{20}\right)\left(\frac{x-40}{20}-1\right)\left(\frac{x-40}{20}-2\right)\left(\frac{x-40}{20}-3\right)}{24} \times 20$$

$$y = 250 + 6(x-40) - 10\left(\frac{x-40}{20}\right)\left(\frac{x-60}{20}\right)$$

$$-\frac{5}{3}\left(\frac{x-40}{20}\right)\left(\frac{x-60}{20}\right)\left(\frac{x-80}{20}\right)$$

$$+\frac{5}{6}\left(\frac{x-40}{20}\right)\left(\frac{x-60}{20}\right)\left(\frac{x-80}{20}\right)\left(\frac{x-100}{20}\right)$$

$$y(70) = 250 + 6(70-40) - 10\left(\frac{70-40}{20}\right)\left(\frac{70-60}{20}\right) - \frac{5}{3}\left(\frac{70-40}{20}\right)\left(\frac{70-60}{20}\right)\left(\frac{70-80}{20}\right)$$

$$+\frac{5}{6}\left(\frac{70-40}{20}\right)\left(\frac{70-60}{20}\right)\left(\frac{70-80}{20}\right)\left(\frac{70-100}{20}\right)$$

$$= 250 + 180 - \frac{15}{2} + \frac{5}{8} + \frac{15}{32}$$

$$y(70) = 423.59 \simeq 424.$$

$$y(60) = 370.$$

No. of Students whose weight between 60–70 $\Big\} = y(70) - y(60)$

$= 424 - 370$

Newton's Backward Interpolation formula

$$y = y_0 + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n$$

Where $v = \frac{x - x_n}{h}$

(1) Use Newton's backward difference formula to construct an interpolating polynomial of degree 3 for the data.

$f(-0.75) = -0.07181250$, $f(-0.5) = -0.024750$

$f(-0.25) = 0.33493750$, $f(0) = 1.10100$.

Hence find $f(-\frac{1}{3})$.

Soln.

$$v = \frac{x - x_n}{h} \quad \& \quad h = 0.25$$

$$v = \frac{x - 0}{0.25} = \frac{x}{0.25}$$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
-0.75	-0.07181250			
		0.0470625		
-0.50	-0.024750		0.312625	
		0.3596875		0.09375
-0.25	0.33493750		0.40625	
		0.7660625		
0	1.10100			

The Newton's backward interpolation formula is

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

$$= 1.10100 + \left(\frac{x}{0.25}\right)(0.7660625) + \left(\frac{x}{0.25}\right)\left(\frac{x}{0.25}+1\right)(0.406375) + \frac{\left(\frac{x}{0.25}\right)\left(\frac{x}{0.25}+1\right)\left(\frac{x}{0.25}+2\right)}{3!}(0.09375)$$

$$= 1.10100 + (-1.33333)(0.7660625) + \frac{(-1.33333)(-0.33333)}{2}(0.40637) + \frac{(-1.33333)(-0.33333)(-0.66666)}{6}(0.09375)$$

$$= 1.10100 - 1.021414 + 0.09030426 + 0.0046295$$

$$y(-1/3) = 0.165260.$$

② The amount A of a substance remaining in a reacting system after an interval of time t in a certain chemical experiment

T (min)	2	5	8	11
A (gm)	94.8	87.9	81.3	75.1

Obtain the value of A where $t = 9$ mins using Newton's interpolation formula.

Soln

T x	A y	∇y	$\nabla^2 y$	$\nabla^3 y$
2	94.8			
		-6.9		
5	87.9		0.3	
		-6.6		0.1
8	81.3		0.4	
		-6.2		
11	75.1			

$$v = \frac{x - x_n}{h}, \quad h = 3$$

The Newton's Backward interpolation formula is

$$y = y_n + \frac{v}{1!} \nabla y_n + \frac{v(v+1)}{2!} \nabla^2 y_n + \frac{v(v+1)(v+2)}{3!} \nabla^3 y_n + \cdots$$

$$y = 75.1 + \left(\frac{x-11}{3}\right)(-6.2) + \frac{\left(\frac{x-11}{3}\right)\left(\frac{x-11}{3}+1\right)}{} (0.4)$$

$$y = 75.1 - 6.2\left(\frac{x-11}{3}\right) + \left(\frac{(x-11)(x-8)}{8}\right) \times 0.4$$

$$+ \frac{(x-11)(x-8)(x-5)}{162} \times 0.1$$

Put $x = 9$

$$y(9) = 75.1 - \frac{6.2(9-11)}{3} + \frac{(9-11)(9-8)}{18} \times 0.5$$

$$+ \frac{(9-11)(9-8)(9-5)}{162} \times 0.1$$

$$= 75.1 + \frac{62}{15} - \frac{2}{45} - \frac{2}{405}$$

$$y(9) = 79.1839.$$