Unit - V

Analysis of Trusses

Determinate truss:-

The structure is said to be statically determinate. If $m+r < 2j$, the truss is unstable since there are an insufficient member forces or reactions or possibly both to equilibrate the applied loads. It follows that plane truss is statically determinate truss.

Indeterminate truss:-

This means that the internal forces in the members can either be calculated using statics only (hence, internally determinate) or cannot be calculated using statics only (hence internally indeterminate) and it can be externally determinate or indeterminate truss.

(1)

Plane trusses:-

A structure made up of several bars (or members) riveted or welded together is known as frame. The frame composed of such members which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external loads, then the frame is known as perfect frame. Though in actual practice the members are welded or riveted together at their joints. yet for calculation purpose the joints are assumed to be hinged or pin jointed.

Types of trusses or Frames:-

The different types of trusses are,

1. Perfect trusses or Frames
2. Imperfect trusses or Frames.

(2)

Perfect frame (Nov / Dec 2013)

If a frame is composed of such members, which are just sufficient to keep the frame in equilibrium when the frame is supporting the external loads the the frame is known as perfect frame. $\boxed{m = 2j - 3}$

Imperfect frame:-

A frame in which number of members and Number of joints are not given by $m = 2j - 3$ is known as imperfect frame. This means that number of members in an imperfect frame will be either more or less than $(2j - 3)$

(NOV. / DEC 2013)

Deficient frame:-

If the number of member in a frame are less than $(2j - 3)$, then the frame is known as deficient frame.

$\boxed{m < 2j - 3}$

(3)

Redundant frame:—

If the number of member in a frame are more than $(2j-3)$, then the frame is known as redundant frame.

Assumptions:—

1) The frame is a perfect frame.

2) The frame carries load at the joints.

3) All the members are pin jointed.

Analysis of Forces in a Truss (or) Frame:—

Analysis of frame consists of

i) Determination of the reactions at the supports.

ii) Determination of the forces in the members of the frame.

The ~~most~~ methods for analysing the frame:— [(April/May 2010), (Nov/Dec. 2014)]

A frame is analysed by the

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following methods.

1) method of joints
2) Method of section
3) Graphical method.
4) Tension co efficient method.

Method of joints:-

In this method, after determining the reactions at the supports, the equilibrium of every joint is considered. This means that the sum of all the vertical forces as well as the horizontal forces acting on a joint is equated to zero.

Method of sections:-

When the forces in few members of a truss are to be determined, then the method of section is mostly used. This method is very quick as it does not involve the solution of other joints of the truss.

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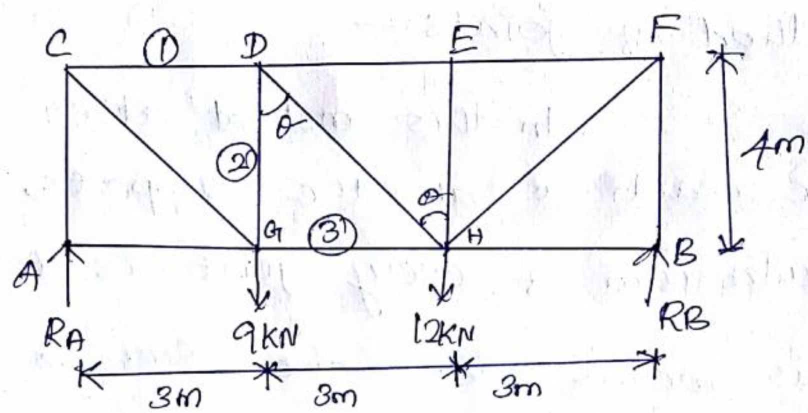
Probelem in analysis of pin jointed plane determinate trusses by method of sections:-

A truss of span 9m is loaded as shown in figure. Find the reactions and forces in the member marked (1), (2), (3).



Solution:-

Let us first calculate the reactions R_A and R_B

Taking moment about A, we get

$$R_B \times 9 - 9 \times 3 - 12 \times 6 = 0$$

$$9R_B = 99$$

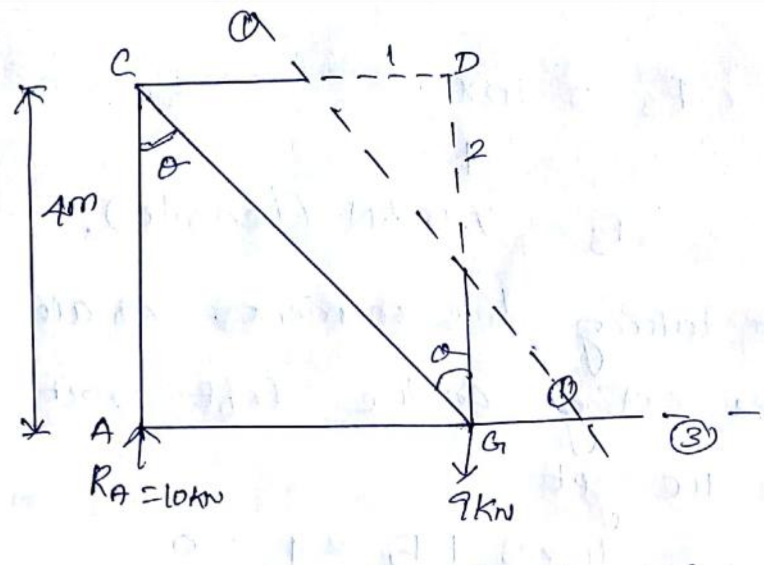
$$R_B = 11\,kN$$

$$R_A + R_B = 9 + 12$$

$$R_A + 11 = 21$$

$$R_A = 21 - 11$$

$$R_A = 10\,kN$$



Now draw a section line ① – ① cutting the members ①, ② & ③ in which forces are to be determined consider the equilibrium of the left part of the truss (because it is smaller than the right part). The part is shown in figure. Let F_1 F_2 and F_3 are the forces members about 1, 2 and 3 respectively. Let their directions are assumed as shown in figure.

Taking moments of all the forces actings on the left part about point D, we get

$$10 \times 3 = F_3 \times 4$$

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$$F_3 = \frac{10 \times 3}{4}$$

$$F_3 = 7.5 \text{ kN (tensile)}.$$

Now taking the moments of all the forces acting on the left part about point we get

$$10 \times 3 + F_1 \times 4 = 0$$

$$F_1 = -\frac{30}{4} = -7.5 \text{ kN (compressive)}$$

Negative sign shows that forces F_1 is compressive.

Now taking the moments about the point 'C' we get

$$F_2 \times 3 - 4 \times 3 + F_3 \times 4 = 0$$

$$F_2 \times 3 - 27 + 7.5 \times 4 = 0$$

$$F_2 = \frac{27 - 7.5 \times 4}{3} = \frac{-3}{3} = -1 \text{ kN}$$

$$F_2 = -1 \text{ kN}$$

Negative sign shows that forces F_2 is compressive

$$F_2 = -1 \text{ kN (compressive)}$$

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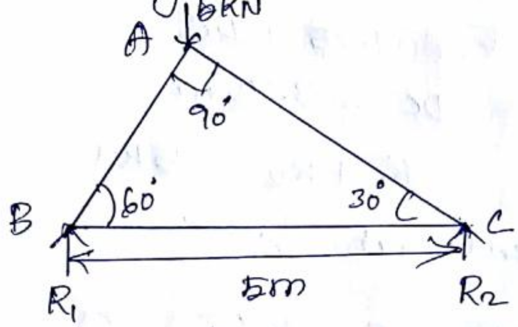
Probelem on Analysis of pin jointed plane determinate trusses by method of joint

A truss with a span of 5m is carrying a load of 5 kN at its apex as shown in figure. Find the forces in all the members by method of joints.

[NOV./DEC 2013]



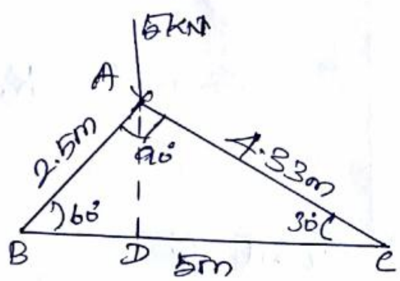
Solution:—

To find AB:—

$\sin 30° = \frac{AB}{BC}$

$AB = 5 \times \sin 30°$

$AB = 2.5m$



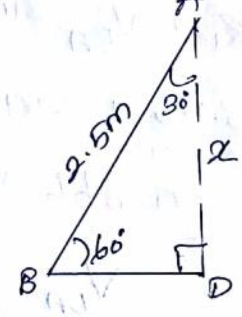
To find AC:—

$\sin 60° = \frac{AC}{BC}$

$5 \times \sin 60° = AC$

$AC = 4.33m$

$\sin 60° = \frac{x}{AB}$



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$x = 2.5 \times \sin 60^\circ$

$= 2.165 m$

$\sin 30^\circ = \frac{BD}{AB}$

$BD = 2.5 \times \sin 30^\circ$

$BD = 1.25 m$

$BC = BD + DC$

$5 = 1.25 + DC$

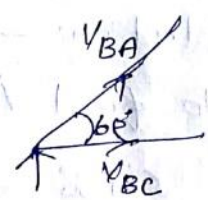
$DC = 3.75 m$

$R_1 + R_2 = 5 kN$

Take moment at C

$5R_1 - 5 \times 3.75 = 0$

$R_1 = 3.75 kN$



$R_1 + R_2 = 5$

$R_2 = 1.25 kN$

Joint B

$\Sigma V = 0$

$R_1 + V_{BA} \sin 60^\circ = 0$

$3.75 + \sin 60^\circ V_{BA} = 0$

$0.866 V_{BA} = -3.75$

$V_{BA} = -4.33$ (Compression)

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$\Sigma H = 0$

$V_{BA} \cos 60^\circ + V_{BC} = 0$

$V_{BA} \cos 60^\circ + V_{BC} = 0$

$V_{BC} = 4.33 \times \cos 60^\circ$

$V_{BC} = 2.165 \text{ kN (Tension)}$

Joint A

$\Sigma V = 0$

$5 + V_{AB} \sin 45^\circ + V_{AC} \sin 45^\circ = 0$

$V_{AB} + V_{AC} = -5$

$4.33 + V_{AC} = -5$

$V_{AC} = -9.33 \text{ kN (compression)}$

Result:-

$V_{BA} = 4.33 \text{ kN (T)}$

$V_{BC} = -2.165 \text{ kN} = 2.165 \text{ (C)}.$

$V_{AC} = -9.33 \text{ kN} = 9.33 \text{ (C)}$

——— x ——— x ———

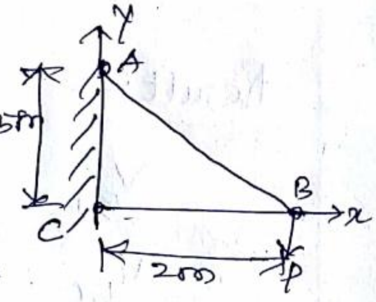
Analysis of pin jointed plane determinate trusses by method of tension coefficient

In this method, first introduced by Prof. R.V. Southwell is in effect a neat and ~~symm~~ systematic presentation of the method of joints. The method is particularly useful to space frames in which other methods prove to be cumbersome and tedious.

Problem

A plane frame consists of two members AB and CB, hinged at A & C to the wall as shown in figure. Determine the forces in the two members due to a vertical force P applied at joint P

Solution:-



Let us take the origin at joint C, and Cx and CY be the axes of reference. The coordinates of three joints are.

C(0, 0); B(2, 0) A(0, 1.5).

There are only two members and therefore, there will be only two tension coefficient

(12)

Let us therefore take joint B and set two equations at that joint, assuming that every member is in a state of tension, exerting a pull on the joint, though in the present case member BA will be in tension while member BC will be in compression. The tension coefficient for BC will automatically work out to be negative

Length $L_{BA} = \sqrt{(0-2)^2 + (1.5-0)^2} = 2.5m$

$L_{BC} = 2m$ (Given)

At the joint B, we have the following two equations in x and y direction.

$t_{BA}(x_A - x_B) + t_{BC}(x_C - x_B) + 0 = 0$

and $t_{BA}(y_A - y_B) + t_{BC}(y_C - y_B) - P = 0$

(Negative sign has been placed before P since force P acts in the negative y-direction)

Substituting the values we get

$t_{BA}(0-2) + t_{BC}(0-2) = 0$

or $t_{BA} + t_{BC} = 0$

and $t_{BA}(1.5-0) + t_{BC}(0-0) - P = 0$

or $1.5 t_{BA} = P$

Solving (1) & (2) we get (13)

$t_{BA} = \frac{P}{1.5}$ KN/m and $t_{BC} = -P/_{1.5}$ KN/m

Minus sign suggests that member BC will be in compression.

$\therefore$ Force in member BA $= T_{BA} = t_{BA} \cdot L_{BA}$

$= \frac{P}{1.5} \times 2.5 = 1.6667\,P$ (tension) Ans.

and Force in member BC $= T_{BC} = t_{BC} \times L_{BC}$

$= -\frac{P}{1.5} \times 2 = -1.333\,P = 1.333P$ (comp)

Analysis of space trusses by tension Coefficient method:—

Probelem:—

A space frame consists of six members AF, BE, BF, FE, EC and FD. The frame is pinned to a vertical wall at ABCD in such a way that ABCD from a square as shown in figure. Also, ABEF is a rectangle in a horizontal plane. Using method of tension coefficients find forces in each member due to a load of 100KN applied at E acting towards the joint D

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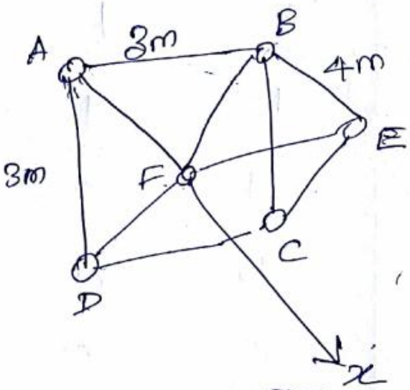
Solution :-

Select the origin at A Let x-axis be directed along AF y axis be directed along AB and z-axis be directed vertically through A.

Length $FD = \sqrt{3^2+4^2}$

$= 5m$



angle $\theta = \tan^{-1}\frac{3}{5}$

$\theta = 30.964°$

$\sin\theta = 0.5145$ and $\cos\theta = 0.8575$

Resolved component of the force EF

$EF = 100\,\theta = 100 \times 0.5145 = 51.45kN$

$EC = 100\cos\theta = 100 \times 0.8575 = 85.75kN$

$\cos\alpha = \frac{BE}{EC} = 4/5$

$\sin\alpha = \frac{BC}{EC} = \frac{3}{5}$

Resolved component in EC & EB

$= 85.75 \times 4/5 = 68.6kN$

Resolved component of force in EC, along Vertical direction $= 85.75 \times 3/5 = 51.45kN$ (15)

$P_x = -68.6kN$, $P_y = -51.45kN$,

$P_z = -51.45 kN$

Co-ordinates point	x	y	z
E	4	3	0
F	4	0	0
A	0	0	0
B	0	3	0
C	0	3	-3
D	0	0	-3

Joint E

The equation at the point.

$$t_{EF}(x_F - x_E) + t_{EC}(x_C - x_E) + t_{EB}(x_B - x_E) + P_x = 0$$

$$t_{EF}(y_F - y_E) + t_{EC}(y_C - y_E) + t_{EB}(y_B - y_E) + P_y = 0$$

$$t_{EF}(z_F - z_E) + t_{EC}(z_C - z_E) + t_{EB}(z_B - z_E) + P_z = 0$$

$$t_{EF} = -17.15$$

$$t_{EC} = -17.15$$

$$t_{EB} = 0$$

$T_{EF} = -17.15 \times 3 = -51.45 kN$; $T_{EC} = -17.15 \times 5 = -85.75 kN$

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Joint F The three equations are

$$t_{FA}(x_A - x_F) + t_{FB}(x_B - x_F) +$$
$$t_{FD}(x_D - x_F) + t_{FE}(x_E - x_F) = 0$$

$$t_{FA}(y_A - y_F) + t_{FB}(y_B - y_F) + t_{FD}(y_D - y_F)$$
$$+ (y_E - y_F) t_{FE} = 0$$

$$t_{EA}(z_A - z_F) + t_{FB}(z_B - z_F) + t_{FD}(z_D$$
$$- z_F) + t_{FE}(z_E - z_F) = 0$$

$$T_{FD} = 0$$

$$T_{FB} = +17.15 \times 5 = +85.75 kN$$

$$T_{FA} = -17.15 \times 4 = -68.60 kN$$

——— x ———

(17)